CENTER FOR ADVANCED AVIATION SYSTEM DEVELOPMENT (CAASD)

Modeling Effects of Ionospheric Delay on GNSS Availability

The Atmosphere and its Effect on GNSS Systems

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CAASD MITRE

Service Availability Modeling Approach

- To generate accurate service availability estimates, a Service Volume Model (SVM) must replicate relevant computations from the user equipment
  - Horizontal (Vertical) Protection Level(s), HPL (VPL)
    - Using appropriate characterizations of residual range errors
      - Ionospheric, tropospheric, clock and ephemeris, measurement noise
- In the case of an SBAS-based service, SVM must also replicate relevant computations from the SBAS ground system
  - To generate GIVEs, User Differential Range Errors (UDREs), and associated degradation factors

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Ionospheric Error Models

- This briefing is limited to the ionospheric error model
  - One of the component needed to compute HPL (VPL)

- Three different ionospheric models are covered
  - Model for GPS Single-Frequency (L1) User
  - Model for GPS/SBAS Single-Frequency (L1) User
  - Model for GPS Dual-Frequency User (L1/L5)

Associated Flight Operations

- **Model for GPS Single-Frequency (L1) User**
  - En route through non-precision approach (ER/NPA) operations
    - GPS receiver (TSO-C129)
    - SBAS receiver (TSO-C145/146) outside the SBAS APV service area
- **Model for GPS/SBAS Single-Frequency (L1) User**
  - LNAV/VNAV, LPV, LP operations
    - SBAS receiver (TSO-C145/146) inside the SBAS APV service area
- **Model for GPS Dual-Frequency User (L1/L5)**
  - ER/NPA, LNAV/VNAV, LPV, LP operations

LP: Horizontal version of LPV (no vertical guidance). Similar to localizer-only for ILS.

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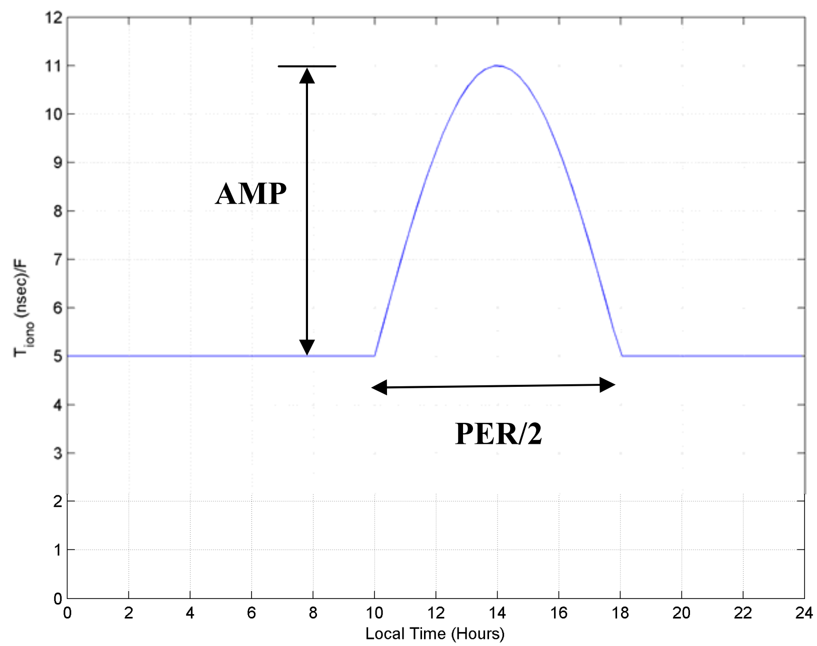
GPS Single-Frequency (L1) User Receiver

GPS Single-Frequency Model

1 of 2

- **Half-cosine model with peak at 14:00 local time**
- **Vertical delay is computed from broadcast coefficients and user IPP location**
- **Delay formulas are specified in the GPS Interface Specification document**
- **Error formulas are specified in the SBAS MOPS**



IPP = Ionospheric Pierce Point

GPS Single-Frequency Model

2 of 2

- **Vertical Residual Error Model (Appendix J.2.3, Reference [1])**
  - **When GPS-based Ionospheric Corrections are applied**

$$\sigma_{i,UIRE} = \max\left\{\left(\frac{cT_{iono}}{5}\right), F_{pp}\tau_{vert}\right\}$$

  - **UIRE = User Ionospheric Range Error**
  - **T_{iono} = Ionospheric Correction by Klobuchar model (ICD-GPS-200)**
  - **F_{pp} = obliquity factor**

$$\tau_{vert} = \begin{cases} 9.0m, 0 \le |\phi_m| \le 20 \\ 4.5m, 20 < |\phi_m| \le 55 \\ 6.0m, \qquad |\phi_m| > 55 \end{cases}$$

  - **ϕ_m = geomagnetic latitude (deg)**

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GPS/SBAS Single-Frequency (L1) User Receiver

SBAS Ionospheric Model

- As discussed in a previous briefing
  - SBAS broadcasts Ionospheric Grid Delays (IGDs) and Grid Ionospheric Vertical Errors (GIVEs) at Ionospheric Grid Points (IGPs) identified in a separately broadcast IGP mask
    - SVM must generate the GIVEs using the same algorithm as the SBAS being modeled
    - Note: SVM does not model the IGDs
  - For each line of sight, the user calculates the Ionospheric Pierce Point (IPP) where the line of sight intercept the thin shell (SBAS ionospheric grid)
  - The user equipment must then
    - Interpolate GIVEs
    - Convert interpolated vertical error sigma to the slant domain

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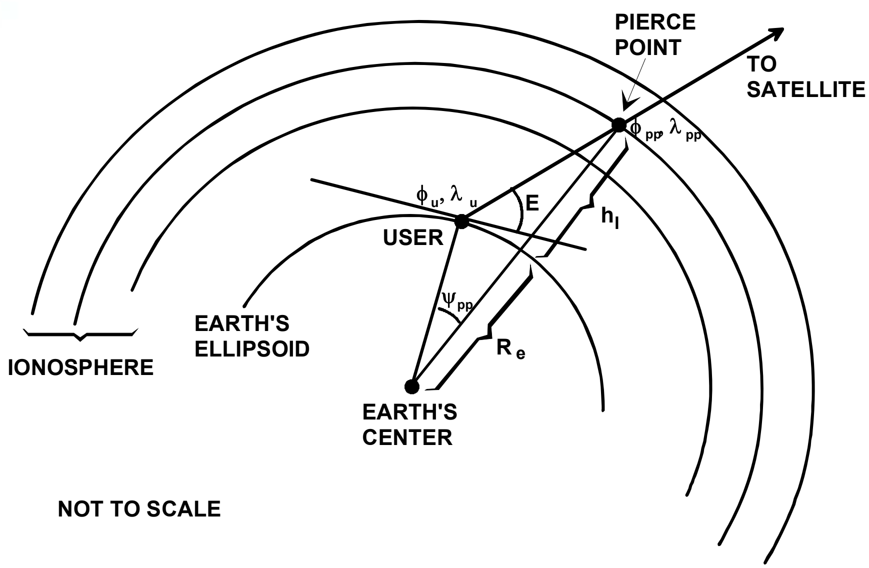
User Equipment Computations

- **The next 4 pages summarize the computations performed by the user equipment as required by the SBAS Minimum Operational Performance Standard (MOPS)**
  - **Reference [1]**

Calculation of the IPP Location



$$\phi_{pp} = \sin^{-1}\left(\sin\phi_u \cos\psi_{pp} + \cos\phi_u \sin\psi_{pp} \cos A\right)$$

$$\psi_{pp} = \frac{\pi}{2} - E - \sin^{-1}\left(\frac{R_e}{R_e + h_I}\cos E\right)$$

If $\phi_u > 70°$, and $\tan\psi_{pp}\cos A > \tan(\pi/2 - \phi_u)$
or if $\phi_u < -70°$, and $\tan\psi_{pp}\cos(A + \pi) > \tan(\pi/2 + \phi_u)$

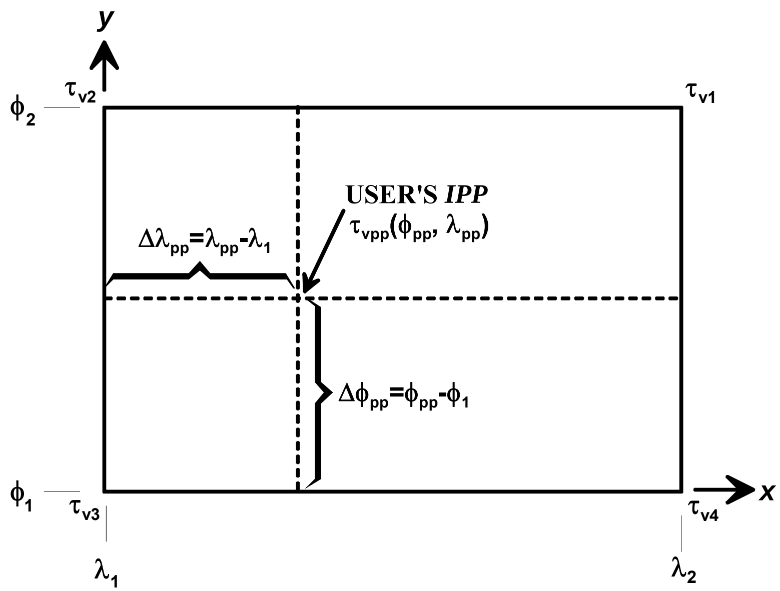
$$\lambda_{pp} = \lambda_u + \pi - \sin^{-1}\left(\frac{\sin\psi_{pp}\sin A}{\cos\phi_{pp}}\right)$$

Otherwise:

$$\lambda_{pp} = \lambda_u + \sin^{-1}\left(\frac{\sin\psi_{pp}\sin A}{\cos\phi_{pp}}\right)$$

Reference 1

4-Point MOPS Bilinear Interpolation Formula



$$\sigma^2_{UIVE} = \sum_{n=1}^{4} W_n\left(x_{pp}, y_{pp}\right) \cdot \sigma^2_{n,ionogrid}$$

$$f(x,y) = xy \qquad \Delta\lambda_{pp} = \lambda_{pp} - \lambda_1$$

$$W_1 = x_{pp} y_{pp} \qquad \Delta\phi_{pp} = \phi_{pp} - \phi_1$$

$$W_2 = (1 - x_{pp}) y_{pp}$$

$$W_3 = (1 - x_{pp})(1 - y_{pp})$$

$$W_4 = x_{pp}(1 - y_{pp})$$

$$\sigma^2_{ionogrid} = \begin{cases} \left(\sigma_{GIVE} + \varepsilon_{iono}\right)^2, & if \ RSS_{iono} = 0 \ (Message\ Type\ 10) \\ \sigma^2_{GIVE} + \varepsilon^2_{iono}, & if \ RSS_{iono} = 1 \ (Message\ Type\ 10) \end{cases}$$

$$\varepsilon_{iono} = C_{iono\_step}\left\lfloor \frac{t - t_{iono}}{I_{iono}} \right\rfloor + C_{iono\_ramp}(t - t_{iono})$$

= degradation term due to missing a message (not modeled in SVM)

C_{iono_step} = the bound on the difference between successive ionospheric grid delay values determined from Message Type 10

t = the current time

t_{iono} = the time of transmission of the first bit of the ionospheric correction message at the GEO

Reference [1]

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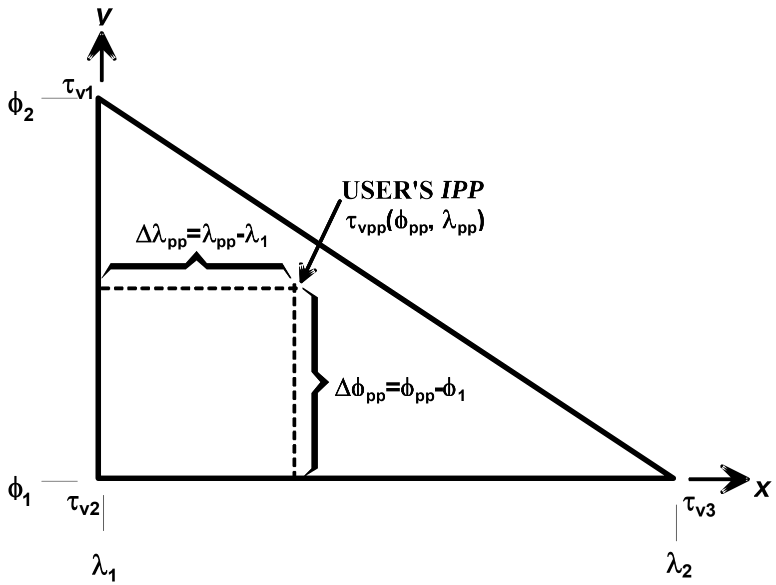
3-Point MOPS Interpolation Formula



$$\sigma_{UIVE}^2 = \sum_{n=1}^{3} W_n\left(x_{pp}, y_{pp}\right) \cdot \sigma_{n,ionogrid}^2$$

$$W_1 = y_{pp}$$

$$W_2 = 1 - x_{pp} - y_{pp}$$

$$W_3 = x_{pp}$$

Reference [1]

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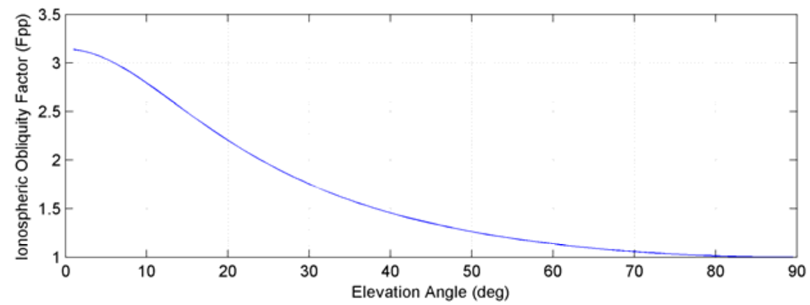
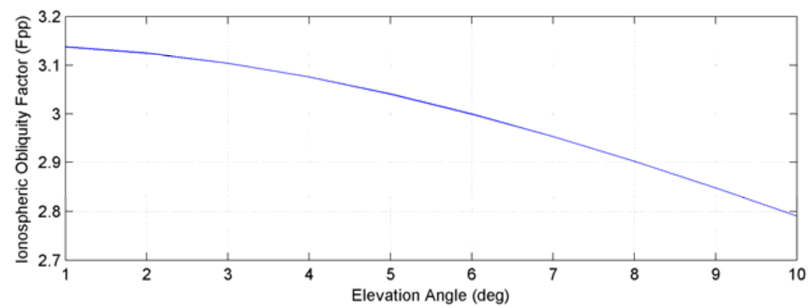
Obliquity Factor (F_{pp}) Formula

- **Converts slant delay to vertical delay and visa versa using the thin shell model**

$$F_{pp} = \left[1 - \left(\frac{R_e \cos E}{R_e + h_I} \right)^2 \right]^{-\frac{1}{2}}$$

- **R_e = 6378.137 km (semi-major radius of Earth)**
- **h_I = 350 km (height of ionosphere)**





Reference 1

SBAS Ground System Computations

- The next few pages summarize the computations performed by WAAS
  - Based on public domain information
  - WAAS is one particular SBAS implementation; EGNOS, for example, uses different algorithms

Planar Fit

- **The following computations are done for each IGP**
  - **WRS IPPs within a search area centered at the IGP are identified**
    - **The radius of the search area is variable to accommodate different conditions (e.g., edge or center of coverage)**
  - **IGD is obtained from plane (2-D 1st degree polynomial) fitted to the vertical delays at the IPPs in the search area**
  - **GIVE is obtained from the residual errors of the planar fit using a modified Chi-square computation**
    - **A Chi-square-based irregularity detector sets GIVE to a high value if the planar fit does not seem appropriate**
    - **A "threat model" derived from analysis of real severe storm data ensures adequate GIVE protection if the irregularity detector were close to tripping**

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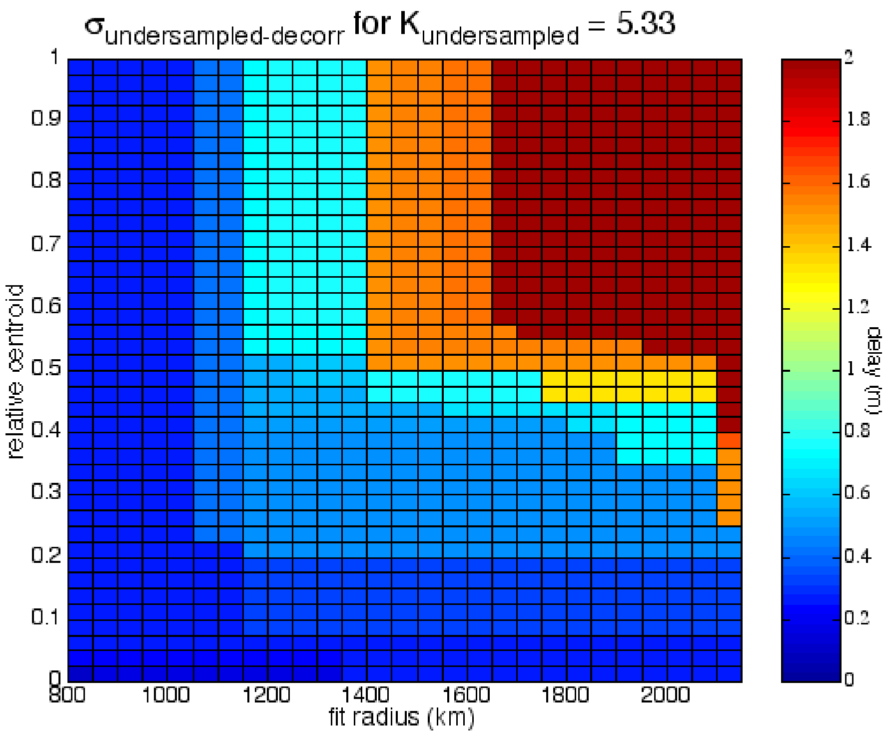
Example of a WAAS Ionospheric WAAS Threat Model (Generated by Stanford University)



- **See References [3,4] for the details of how the threat model can be generated offline using super truth data which are derived from L1/L2 GPS code and carrier data collected during severe storms**

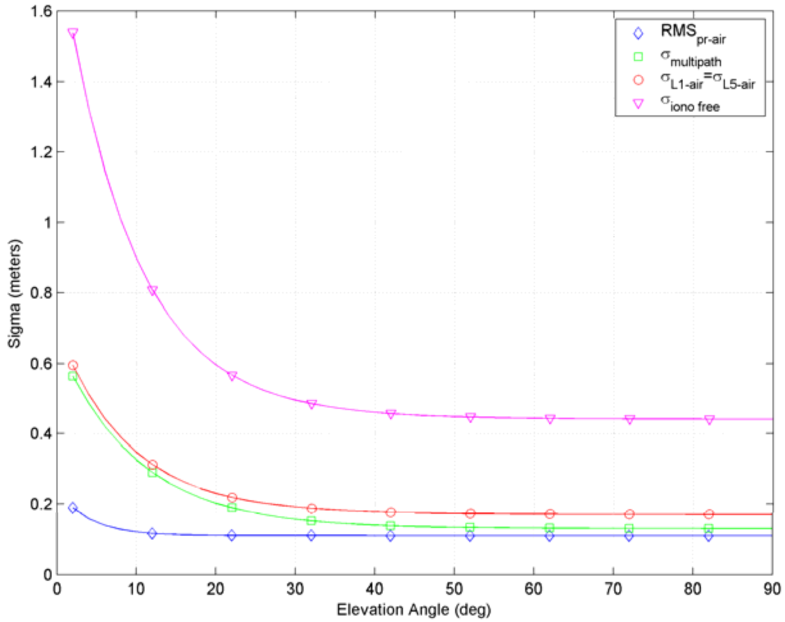
GPS Dual-Frequency User Ionospheric-Free Error Model

$$\sigma_{i,iono-free} = \sqrt{\left(\frac{f_1^2}{f_1^2 - f_5^2}\right)^2 \sigma_{i,L1,air}^2 + \left(\frac{f_5^2}{f_1^2 - f_5^2}\right)^2 \sigma_{i,L5,air}^2}$$

$$= 2.59\sigma_{i,L1,air}$$

Assume $\sigma_{i,L5,air} = \sigma_{i,L1,air}$

$f_1 = 1575.42$ MHz, $f_5 = 1176.45$ MHz



Where

$$\sigma_{i,L1,air} = \sigma_{i,L5,air} = \sqrt{RMS_{pr\_air,GPS}^2 + \sigma_{i,multipath}^2}$$

$$RMS_{pr\_air,GPS} = 0.11 + 0.13e^{(-El_i/4)}$$

$$\sigma_{i,multipath} = 0.13 + 0.53e^{(-El_i/10)}$$

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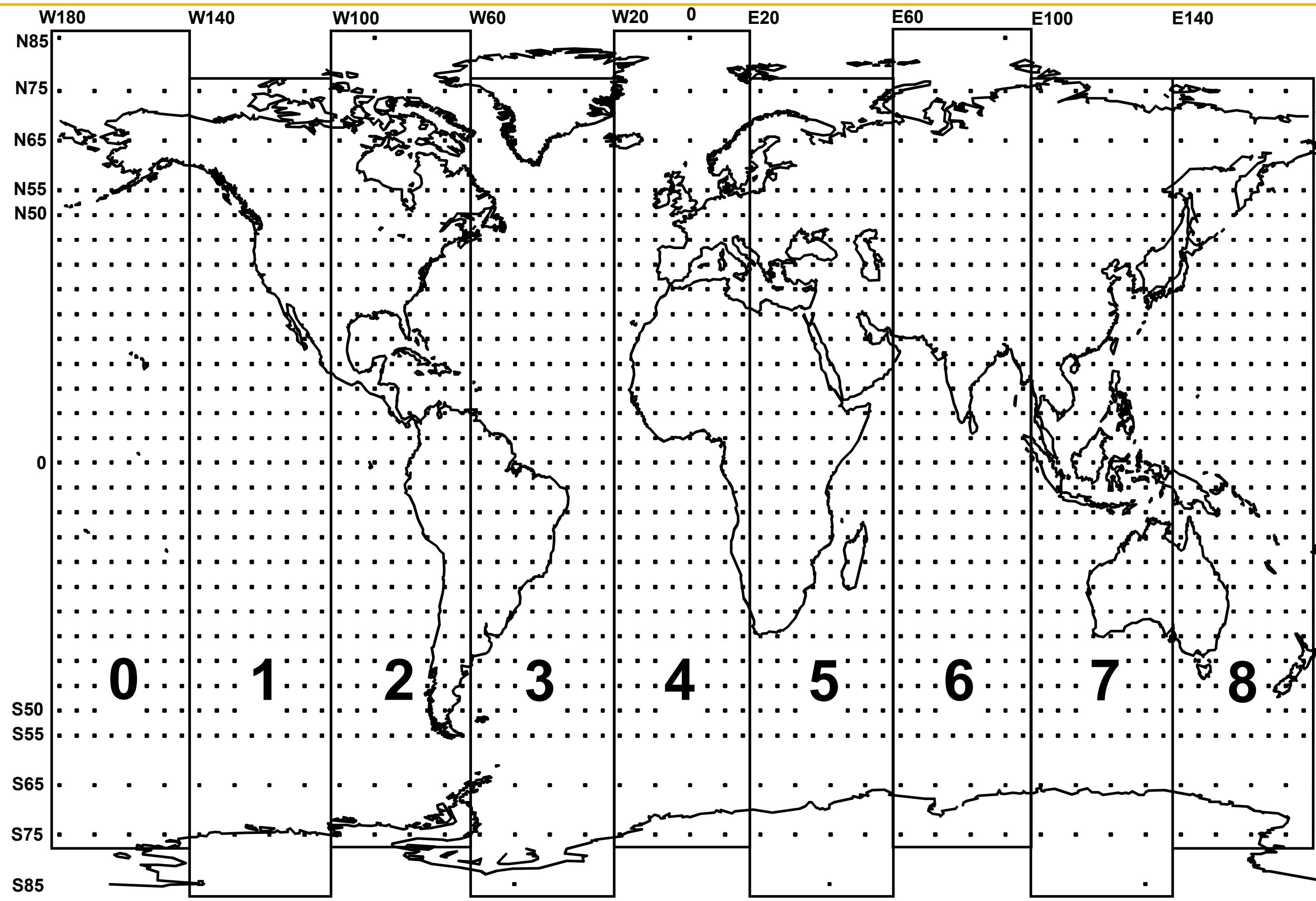
Additional Material

PREDEFINED GLOBAL IGP GRID (BANDS 9 AND 10 ARE NOT SHOWN)



Reference [1]

GIVE Table

$GIVEI_i$	$GIVE_i$ Meters	$\sigma^2_{i,GIVE}$ Meters^2
0	0.3	0.0084
1	0.6	0.0333
2	0.9	0.0749
3	1.20	0.1331
4	1.5	0.2079
5	1.8	0.2994
6	2.1	0.4075
7	2.4	0.5322
8	2.7	0.6735
9	3.0	0.8315
10	3.6	1.1974
11	4.5	1.8709
12	6.0	3.3260
13	15.0	20.7870
14	45.0	187.0826
15	Not Monitored	Not Monitored

Reference [1]

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References

1. RTCA, Inc., *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System*, DO-229D, RTCA, Inc., Washington, D.C., 2006.

2. Altshuler, E., R. M. Fries and L. Sparks, “The WAAS Ionospheric Spatial Threat Model,” ION-GPS-2001, Salt Lake City, UT, September 2001.

3. Datta-Barua, S., T. Walter, S. Rajagopal, “WAAS Ionospheric Undersampled Threat Model,” presented at GNSS Ionospheric Workshop, ICTP, Trieste, Italy, December 2006. (http://cdsagenda5.ictp.trieste.it/full_display.php?ida=a05234)

4. Pandya, N., M. Gran, and E. Paredes, “WAAS Performance Improvement with a New Undersampled Ionospheric Gradient Threat Model Metric,” The Institute Of Navigation-National Technical Meeting, San Diego, CA, January 22-24, 2007.

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