13th MEETING OF THE AIR TRAFFIC SERVICES/AERONAUTICAL INFORMATION SERVICES/SEARCH AND RESCUE SUB-GROUP (ATS/AIS/SAR/SG/13)

(Bangkok, Thailand 23 June 2003)

STRATEGIC LATERAL OFFSET

(Presented by the United States of America)

Agenda Item 8 – Other Business - Flight Standards and Air Traffic Management (ATM) Issues

Summary

This paper will address the Proposed Lateral Offset Amendment of Regional Supplementary Procedures ICAO Doc 7030/4.
(Serial No. APAC-S oo/4 – MID/ASIA/PAC RAC)

1.0 Introduction

1.1 An Amendment to the Pacific Regional Supplementary Procedures (ICAO Doc 7030 PAC/RAC) document has been proposed by Australia on the subject of Strategic Lateral Offsets.

2.0 Background

2.1 The ICAO Separation and Airspace Safety Panel (SASP), has a history of discussions and working papers on strategic lateral offsets that go back to Brussels in 1997 and Toulouse in 1998. This issue has been worked extensively by this ICAO panel. The North Atlantic is completing the ICAO Doc 7030 at this time for the implementation of strategic lateral offsets in their region. The West Atlantic Route System (WATRS), has completed one year of testing using lateral offsets successfully with a decrease in risk associated with this operation. They have extended the use of these procedures for an additional year and will most likely make them permanent. The procedures used in the WATRS and eventually North Atlantic will be implemented using the 0, 1, or 2 NM offset, to the right. This will be for all aircraft including non-GNSS.

3.0 Discussion

3.1 The United States has not implemented any lateral offset scheme in the Pacific and does not plan to implement the use of the 1NM offset at this time. ICAO has proposed a phased implementation of the 2 NM offset option. Phase Two will use the 0,1, or 2 NM offset procedure after an analysis of the planned study in the North Pacific scheduled during the next 6 months. The United States will wait for the results of this study and plans to implement when the risk analysis is complete.

3.2 A previous risk analysis covers in depth the lateral overlap problem and takes into account all aircraft such as RNP 4 or RNP 10 capable aircraft. We believe that the bi-directional route systems possibly present the greatest risk of collision given the accuracy of the navigation systems in use today. Human error on the part of the flight crew or air traffic controller would be the most likely scenario in the bi-directional system. The system that uses the randomness of the 0, 1, or 2 NM offsets as a strategic operation will provide the greatest protection in the instance of an operational error. The tactical procedure of offsetting due to wake turbulence, ACAS alerts or GPWS alerts, is up to 2 NM either direction. The strategic offset procedure when implemented will require that the wake turbulence procedure be changed to offset only to the right.

3.3 Background information is attached as Appendix 1 and 2. Appendix 1 is the IP on Strategic Lateral Offsets in the North Atlantic and Watrs area. This was presented to IPACG/18 in Tokyo in October 2002. Appendix 2 is the letter that the FAA sent to Dr. Warren Jones of the Civil Aviation Safety Authority, Australia.

3.4 The Separation and Airspace Safety Panel (SASP) just completed their 3rd meeting of the Working Group of the Whole in London, England on May 23, 2003. Appendix 3 is included to explain the results of the Working Group on Lateral Offsets with the final report. The conclusions reached are that the Risk Analysis that has been accomplished shows a decrease in Risk utilizing the Strategic Lateral Offset procedure with the randomness of 0, 1, or 2 NM right of track in the 50 NM separation standard. The report also indicates that States can implement based on the State Letter which will be revised to include the 2 NM offset. This procedure will also include the Wake Turbulence procedure with offsets only allowed to the right.

CONCLUSION

4.1 The meeting is invited to note the FAA position.

APPENDIX 1

THE EIGHTEENTH MEETING OF THEINFORMAL PACIFIC AIR TRAFFIC CONTROL COORDINATING GROUP (IPACG/18)

Tokyo, Japan, 8-11 October 2002

STRATEGIC LATERAL OFFSET PROCEDURES FOR THE NORTH ATLANTIC (NAT) REGION

(Presented by the Federal Aviation Administration)

Agenda Item 2 - Air Traffic Management (ATM) Issues

Summary

This paper describes the procedures to be used in applying a "Strategic Lateral Offset" in the NAT ICAO Region. These procedures have been validated through a 1-year trial period in the West Atlantic Route System (WATRS).

1.0 Introduction

1.1 ICAO, through a State letter, has indicated that it was appropriate for regions to institute some form of regulated offset, as individual pilots were applying offsets with no guidance from ATC. Any offset procedure should be applied in a uniform manner throughout the world. It is important that each of the ICAO Regions attempt to harmonize any such procedure developed.

1.2 As is commonly known, with increasing use of GPS, the "core" lateral navigational performance will continue to increase - with resulting influence on the lateral overlap probability, and the vertical risk estimate. This, along with the implementation of reduced vertical separation minimum (RVSM), is why the development of lateral offset procedures is necessary in order to ensure continued safe operations.

2.0 Discussion

2.1 The NAT SPG has undertaken steps to "develop lateral offset procedures that would mitigate the impact on risk of the increasing lateral overlap probability which would be applicable in NAT Reduced Vertical Separation Minimum airspace." Additionally, an amendment proposal to the ICAO Doc 7030 NAT Regional Supplementary Procedures (SUPPS) for the use of lateral offsets to mitigate risk has been developed.

2.2 The NAT Implementation Management Group (NAT IMG) proposed to the NAT SPG that the NAT SUPPS be amended to include lateral offsets up to a maximum of 2 NM to the right of track. Concerns were raised by the International Business Aviation Council (IBAC) related to the effects of wake turbulence on business aircraft and it was decided that the proposal for amendment should include a note to reference the pilots' responsibility to use his/her judgement to determine the most appropriate action in any given circumstance.

2.3 With the implementation of RVSM in the West Atlantic Route System (WATRS) on 1 November 2001, the FAA began a 1-year operational trial of an offset procedure that incorporated both wake turbulence avoidance with the mitigation of lateral overlap risk.

2.4 The resulting procedure, as contained in the NAT SUPPS proposal, incorporates both wake turbulence avoidance with the mitigation of lateral overlap risk. The proposal is as follows:

NAT/RAC 7.5 Special procedures for lateral offsets within NAT airspace

Note: The following incorporates lateral offset procedures for both the mitigation of the increasing lateral overlap probability and wake turbulence encounters.

7.5.1 It has been determined that allowing aircraft conducting oceanic flight to fly lateral offsets not to exceed 2 NM right of centerline will provide additional safety margin and mitigate the risk of conflict when non-normal events such as aircraft navigation errors, height deviation errors and turbulence induced altitude-keeping errors occur.

7.5.2 This procedure provides for offsets within the following guidelines. Along a route or track there will be three positions that an aircraft may fly: centreline or one or two miles right. Offsets will not exceed 2 NM right of centreline. The intent of this procedure is to reduce risk (add safety margin) by distributing aircraft laterally across the three available positions.

a) Aircraft without automatic offset programming capability must fly the centerline.

b) Operators capable of programming automatic offsets may fly the centreline or offset one or two nautical miles right of centerline to obtain lateral spacing from nearby aircraft. (Offsets will not exceed 2 NM right of centerline). An aircraft overtaking another aircraft should offset within the confines of this procedure, if capable, so as to create the least amount of wake turbulence for the aircraft being overtaken.

c) Pilots should use whatever means is available to determine the best flight path to fly.

d) Pilots should also fly one of the three positions shown above to avoid wake turbulence. Aircraft should not offset to the left of centreline nor offset more than 2 NM right of centreline. Pilots may contact other aircraft on frequency 123.45, as necessary; to coordinate the best wake turbulence offset option.

Note: It is recognized that the pilot will use his/her judgement to determine the action most appropriate to any given situation and has the final authority and responsibility for the safe operations of the aeroplane

e) Pilots may apply an offset outbound at the oceanic entry point and must return to centerline at the oceanic exit point.

f) Aircraft transiting oceanic radar areas may remain on their established offset positions.

g) There is no ATC clearance required for this procedure and it is not necessary that ATC be advised.

3.0 Recommendation

3.1 The meeting is invited to take into account the lateral offset procedures proposed for the NAT Region when developing similar procedures for the Pacific.

APPENDIX 2.

Letter to Dr. Warren Jones
Principal Researcher
Airspace, Air Traffic & Aerodrome Standards Branch
Aviation Safety Standards Division
Civil Aviation Safety Authority, Australia

Warren,

Sorry for the delay in replying to your message. We felt that internal coordination and consultation was necessary to prepare our response to your request for information about the implementation of lateral offsets in individual FIRs. The United States has not implemented any lateral offset scheme in the Pacific and does not plan to implement the use of the 1NM offset implemented by Australia.

The United States has interests in several Oceanic FIRs in addition to those in the Pacific Region. We implemented a lateral offset procedure a year ago in our West Atlantic Route System (WATRS) that fully addresses all the issues, to the extent that the North Atlantic Systems Planning Group has adopted an identical procedure for the entire North Atlantic Region.

We have reviewed the proposed amendment to Doc 7030 and find that it falls short of our requirements in several aspects:

1. It fails to address the issue of aircraft avoiding wake turbulence. These procedures are in place today, for both same direction and opposite direction traffic.

2. It excludes non-GNSS aircraft from participation that we feel would definitely enhance the safety of the system when opposite direction traffic is separated by 1,000ft, and in same direction systems.

3. We do not agree that the offset procedure should be limited to airspaces in which track spacing is at least 93km (50NM). SASP-WG/WHL/1-WP/5 indicated that with one exception, even the WATRS offset procedure could safely be applied to RNP-4 routes separated by 30NM. The exception is for routes that carry traffic in opposite directions at the same flight level, that have a significant portion of their traffic consisting of non-GNSS airplanes, and on which each route’s traffic has the opposite-direction traffic to its right. This one exception should not prevent us from applying an offset procedure to other RNP-4 routes that operate with 30NM spacing.

4. We feel that options of 0, 1, or 2NM offsets provide a randomness that greatly enhances safety by reducing the collision risk. They also provide a mechanism for “rolling” the wake turbulence procedure into the lateral offset.

5. The procedure should be available in other than “OCA i.e. airspace where there are no ground-based navigation aids”. Regions where large separations are

applied, such as remote regions of Africa and South America, certainly have a need for this procedure.

We do not concur with the proposal for an amendment to ICAO Doc 7030/4. The United States will not implement the lateral offset proposed in ISPACG/16-WP/16.

This response has been coordinated between FAA Air Traffic, Flight Standards and the Technical Center. Questions should be referred to Robert Swain <Robert.Swain@faa.gov>.

Regards,
Leslie

Leslie McCormick
Acting Deputy Manager, ATS International Staff, AAT-30
Phone: 1.202.267.7646
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- END –

APPENDIX 3

SEPARATION AND AIRSPACE SAFETY PANEL (SASP) 3rd MEETING OF THE WORKING GROUP OF THE WHOLE LONDON, ENGLAND (May 2003)

REPORT OF PROJECT TEAM 9 – LATERAL OFFSETS

Participants (including part-time attendants):

Rob Butcher, Steve Kemp, Didier Malescot, Yoshiki Imawaka, Stephen Creamer, Robert M. Tegeder, Bob Miller, Mark Denney, Valeri Mkhitaryan, Brian Colamosca, Heinz Frühwirth (Coordinator).

During the first part of the morning session also the following members of the MSG attended the PT9 meeting: Geert Moek, Bennett Flax, Dr. Barry Silverman, Dr. David Anderson, Dr Karim Mehadhebi; Dr Sakae Nagaoka, Dr. Marcus Dacre, Dr. Paul Cowell

1. Introduction

PT 9 met on Wednesday, 21st May. As considerable discussions were held, the initially planned timeframe (morning session only) was extended to continue in the afternoon, where a considerable number of PT-members was still available.

2. Working Papers considered

Dr. Nagaoka presented WP/33 that analysed the *Effect of Lateral Offsets on the Lateral Collision Risk of the NOPAC Routes.* The results contained in WP/33 were based on a hypothetical scenario of offset tracking on a portion of the NOPAC routes, data on flights from November 2002 on these routes and some parameters taken from previous studies. The results indicate that the estimated lateral collision risk for the assumed scenario is (slightly) less than that for the system without offsets.

The Project Team thanked Dr Nagaoka for the valuable work that had been undertaken and for the presentation of WP/33 and – taking also into account previous studies provided by various members of the SASP Math Sub Group – concluded that offsets up to 2 NM (to the right) would not significantly increase the lateral collision risk on route systems with 50 NM spacing between the centrelines.

Mr Flax than presented WP/2 that provided the results of calculating Lateral Overlap Probabilities of Opposite-Direction Airplanes Flying on a Bi-Directional Oceanic Route. The WP compared the overlap probability without any offset with five other options over a range of RNP types from 2 to 10. The results showed that significant reductions in lateral overlap probability would be achieved with any offset option. Reductions are largest, with compulsory offsets for GNSS equipped aircraft.

The Project Team recognized that the results of WP/2 were a valuable support for the work on lateral offset and thanked Mr Flax for his efforts and the presentation.

The Project Team noted the information given in WP/31 that the ICAO headquarters, after reviewing the proposal for amendment of Annex 2, had decided that no such amendment was necessary.

3. Regional Updates

The project team was informed that the implementation of the 1 NM offset according to the Phase I Guidelines in the South Pacific through an amendment proposal for the Regional Supplementary Procedures (ICAO Doc 7030) – the text of which had been agreed upon during the WG/WHL-2 Meeting – has been held back to try to achieve a coordinated implementation of the proposed 0,1,2-Offset Procedure over a larger part of the Pacific.

The Operational Trials in the WATRS Area of the North Atlantic Region had been extended to Nov 2003.

With the planned implementation of RVSM in the MID Area parallel uni-directional routes are considered to reduce passing frequency. In a particular portion of that airspace there is not enough room for the required spacing between these routes. Offset tracking on a single route might be an option to resolve that problem.

No information was available from other regions.

4. Discussion

The Project Team discussed whether it would be appropriate to **require** offset tracking for GNSS-equipped aircraft. It was the unanimous view of the members that any route or route system should be inherently safe, i.e. achieve the TLS without offset tracking. Mandatory offset tracking would have required a specific clearance and a means to monitor compliance. The optional offset tracking would enhance the achieved level of safety.

The area(s) for which offset tracking should be recommended were part of intensive discussion within the Project Team. No specific wording was found so far, but there was general agreement that benefits could be achieved in most areas apart from high-density radar controlled airspace with closely spaced routes. The exact wording for the Phase II Guidelines needs to be developed.

As the authorization of offset tracking according to the Phase I (revised) Guidelines is considered an important safety enhancement in many areas of the world, the Project Team recommends that ICAO encourge States (especially in Africa and South America) to make use of the Guidelines at any appropriate occasion (e.g. APANPIRG/13 Meeting in June 2003).

To develop the intended Phase II Guidelines a further study is required on the effect of 0,1,2 lateral offsets on the collision risk on routes spaced by 30 NM. This study should also take into account crossing track scenarios.

Offset Tracking should continue to be transparent to Air Traffic Controllers. When the passing of a waypoint is reported automatically, it needs to be assured that not the actual position on the offset track is reported. In advanced ATC systems, such as ADS, the effects of offset tracking on conflict detection and resolution systems needs to be taken into account. For this reason the (ground) system should have knowledge of the fact and amount of offset tracking.

Offset Tracking Phase II (0,1,2 NM to the right) would eliminate the need for a separate contingency procedure to mitigate wake turbulence encounters.

Pilot operating procedures for in-flight contingencies (e.g. engine failure, loss of cabin pressure, and weather deviation) should emphasize the possibility of adjacent aircraft being on one of the other offset options.

The Project Team intends to finalize Phase II Guidelines as soon as the 30 NM spacing study is concluded. In the meanwhile – based on information available so far – an update to the (revised) Phase I Guidelines is recommended to allow right offsets of up to 2 NM for all aircraft that have automatic offset tracking capability. It is envisaged that this revision could be relatively quickly produced by ICAO and for the basis of coordinated implementation in the whole Pacific area.

5. Conclusions and Recommendations

The WG/WHL is invited to agree that

- ICAO should urge States to make use of the *Guidelines on Lateral Offsets;*
- where offset tracking is authorized for an airspace, it should not be mandatory for pilots to apply the procedure;
- the required TLS should be achieved on any route/route system without taking into account any offset tracking;
- ICAO should upgrade the Revised Guidelines on the use of Lateral Offsets to allow all aircraft with automatic offset tracking capability to use right offsets of up to 2 NM.
- the Math SG should study, whether the 0,1,2-right-offset would be acceptable in a route system with 30 NM spacing, also taking into account crossing track situations
- further information on the effects of offset tracking in advanced and automated ATC systems should be made available to the Project Team
- the Project Team investigates the possibility of incorporating the Phase II Offset Guidelines in Doc 4444 PANS-ATM and whether any changes to Annexes 2 and 11 and/or any other ICAO Documents (especially Doc 9689/APM) are required/desirable.

SEPARATION AND AIRSPACE SAFETY PANEL (SASP)

THIRD MEETING OF THE WORKING GROUP OF THE WHOLE

London, England, 12 May– 23 May, 2003

Agenda Item 1:En-route Separation minima and procedures – horizontal

Agenda Item 4: Safety assessment methodologies for the future ATM environment

An Analysis on the Effect of Lateral Offsets on the Lateral Collision Risk of the NOPAC Routes

(Presented by Mr. H. Mito)
(Prepared by Dr S. Nagaoka and Mr O. Amai)

Summary

This paper investigates the effect of lateral offsets on the lateral collision risk for the existing NOPAC routes. As a typical case, lateral offsets of 2 NM to the right for the GPS-equipped aircraft on the predetermined flight levels are assumed for calculating the lateral collision risk. Traffic under consideration is based on the passing frequency values observed in the current NOPAC routes. The results indicate that the estimated lateral collision risk for the assumed scenario is less than that for the system without offsets.

1. INTRODUCTION

1.1 After the advent of GPS-equipped aircraft, concerns have been raised about adverse effects of high lateral navigation accuracy on collision risk [1] in the vertical and/or horizontal dimension. Accordingly, the need for revising the ICAO RVSM manual [2] was raised [3] and resulted in its revision. As a method for reducing the vertical collision risk, systematic use of parallel offsets has been proposed, and many studies associated with the offsets have been conducted [4]-[13].

1.2 At the last meeting (SASP-WG/WHL/2, Montreal), the need to investigate the effect of lateral offsets on the collision risk in a current route system, such as the North Pacific (NOPAC) routes, was raised. In response to a request, we carried out a survey on the passing frequencies of the recent NOPAC routes to assess the effect quantitatively. Then, the lateral collision risk was estimated for a hypothetical scenario where a parallel offset of 2NM to the right was applied to the current NOPAC route system.

1.3 This paper describes the method of the approach and some results.

2. AIRSPACE UNDER CONSIDERATION

2.1 Fig.1 shows the oceanic route system called the NOPAC routes. Currently, there are five routes for which route spacing is about 50 NM. Vertical separation of 1,000 ft has been used for the flight levels from FL290 to FL390 since 24 February 2000 and to FL410 since 5 October 2000. The two northernmost routes (R220 and R580) are used for westbound traffic, A590 for eastbound traffic; and R590 and G344 are bidirectional.

2.2 A survey [17] in 1999 showed that approximately 70% of the traffic is B747, and 40% of that is B747-400. Some part of the B747-400 and B777 fleets are equipped with GPS (FANS-1) and the rate of GPS equipage has been increasing. In a survey carried out in November 2000, 42% (1655 out of 3972 flights) of flights in the NOPAC routes were equipped with ADS. This means that these aircraft are also equipped with GPS.

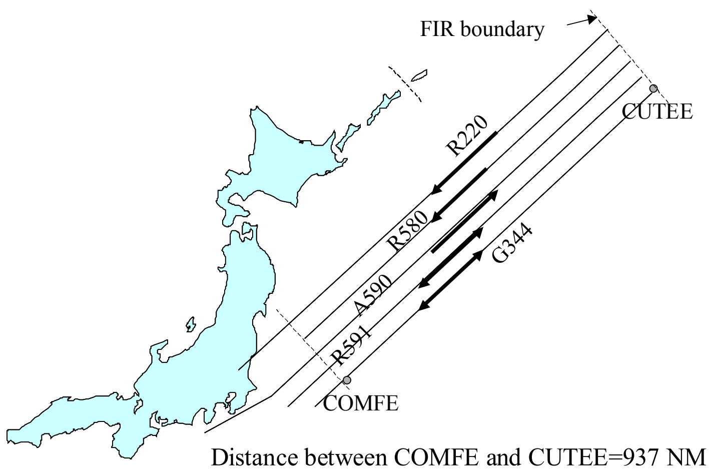

Fig.1 Configuration of the NOPAC routes

3. COLLISION RISK MODEL

3.1 The Reich model [1] has been used for assessing the safety of oceanic routes. The collision risk, due to loss of lateral separation between an aircraft pair nominally separated by S_y laterally and 0 vertically, is given by

$$N_{ay} = P_y(S_y)P_z(0)\times[N^y(s)K(s)+N^y(o)K(o)] \quad (1)$$

where

$$K(s)=[1+\frac{\lambda_x}{V_{rx}(s)}(\frac{V_{ry}}{\lambda_y}+\frac{V_{rz}}{\lambda_z})] \quad (2)$$

$$K(o)=[1+\frac{\lambda_x}{V_{rx}(o)}(\frac{V_{ry}}{\lambda_y}+\frac{V_{rz}}{\lambda_z})] \quad (3)$$

where

$P_y(S_y)$: the probability that two aircraft assigned to adjacent routes with route spacing S_y on the same flight level are in lateral overlap (the lateral overlap probability);

$P_z(0)$: the probability that two aircraft pair assigned to the same flight level are in vertical overlap (the vertical overlap probability);

$\lambda_x, \lambda_y, \lambda_z$: the length, width, and height of the aircraft, respectively;

$V_{rx}(s)$, $V_{rx}(o)$, V_{ry}, V_{rz} : the average relative longitudinal speed of the aircraft pair for same direction traffic, for opposite direction traffic, average relative lateral speed, and average relative vertical speed, respectively;

$N^y(o)$, $N^y(s)$: the opposite direction passing frequency and the same direction passing frequency of the whole route system, respectively.

3.2 The route system shown in Fig.1 consists of five tracks nominally separated by 50 NM (=S_y). There are four pairs of adjacent parallel routes. The route spacing between a pair of routes may change according to the way offsets are applied. For a system of five routes, the aggregated collision risk can be obtained by summing the risks for each route pair.

4. SURVEY ON THE PASSING FREQUENCIES IN THE NOPAC

4.1 We have investigated the passing frequencies of the NOPAC routes [18][20]. To know the current traffic situation, we calculated the number of passing events based on the reported time over position reporting fixes of flight progress data obtained in November 2002. The data were extracted from the Flight Data Processing (FDP) System. The airspace under investigation is within the Tokyo FIR. One side of the rectangular area in Fig.1 consists of a line connecting two fixes (CUTEE-COMFE: 937 NM) on G344. The passing events were counted for the rectangular area bounded by the FIR boundary and the perpendicular line down to each track from COMFE.

4.2 Aircraft speeds were assumed to be constant between the reported points or points used for calculation. A passing event observed at the edge of the segments was counted as 1/2 to avoid double counting. Time information was rounded to the nearest 0.1 minute. The number of passing events and total flight time in hours were counted for each route pair on each flight level between [290,410] inclusive. The results for each flight level are shown in Tables A1-A4 in Appendix 2. Table 1 shows the distribution of passing events and flight time for each route pair. The passing events were obtained for the even and odd flight levels separately.

4.3 The passing frequency of the $i-\text{th}$ sub-system (route pair) can be given by

$$N_i^y(o/s) = 2 \cdot n_i(o/s) / T_i^p \qquad (4)$$

where, $N_i^y(o/s)$: the passing frequency of $i-\text{th}$ sub-system ,

$n_i(o/s)$: the number of passing events observed in the $i-\text{th}$ sub-system,

T_i^P : the total flight time in the $i-\text{th}$ sub-system.

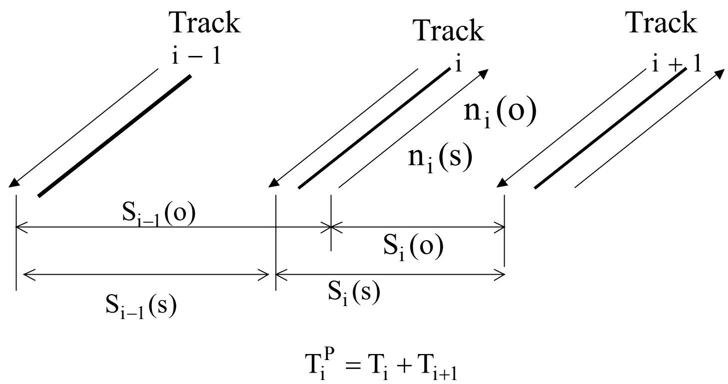

Fig.2 Concept of sub-system (route pair) for calculating the passing frequencies. Only the tracks on an assigned flight level are shown. T_i is the total flight time for all the traffic on the i-th route.

Table 1 Observed passing events and flight hours for each route pair and route. (Based on the flight progress data of November 2002.)

Route pair		Opposite direction traffic		Same direction traffic		Flight time spent in the route pair
No i	Pair i	Number of opposite direction passing events $n_i(o)$	Lateral separation between intended paths (NM)	Number of same direction passing events $n_i(s)$	Lateral separation between intended paths (NM)	$T_i^P = T_i + T_{i+1}$ T_i T_{i+1} (Hours)
1	R220-R580	0		61 24 (odd FL) 37 (even FL)	50 (odd FL) 50(even FL)	$T_1^p = 7124.79$ 5780.13(R220) 1344.66(R580)
2	R580-A590	207 95.5(odd FL) 111.5(even FL)	50 (odd FL) 54 (even FL)	0		$T_2^p = 4519.10$ 1344.66(R580) 3174.44(A590)
3	A590-R591	32 11(odd FL) 21(even FL)	50(odd FL) 46 (even FL)	10.5 10.5 (odd FL) 0 (even FL)	50(odd FL) 50(even FL)	$T_3^p = 3743.14$ 3174.44(A590) 568.71(R591)
4	R591-G344	0		0		$T_4^p = 768.53$ 568.71(R591) 199.83(G344)
		Total $\sum_{i=1}^{4} n_i(o) = 239$ $N(o) = 2\sum_{i=1}^{4} n_i(o) / H = 0.0432$		Total $\sum_{i=1}^{4} n_i(s) = 71.5$ $N(s) = 2\sum_{i=1}^{4} n_i(s) / H = 0.0129$		$H = \sum_{i=1}^{5} T_i$ =11067.77 hrs

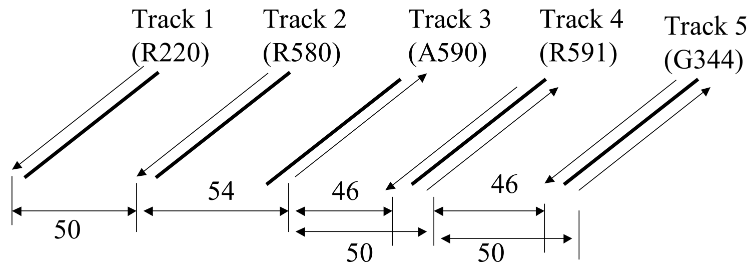

Fig.3 Effective assigned separations for each track of the NOPAC routes when an offset of 2NM to the right is applied.

5. COLLISION RISK OF THE SYSTEM WITH OFFSETS

5.1 A hypothetical system where 2 NM offsets to the right were applied for (GPS-equipped) aircraft on the predetermined flight levels was considered as an extreme example. As to the system, the following assumptions were made for simplicity:

(1) All the aircraft have the same navigational performance (the lateral path keeping errors of the aircraft are assumed to be independent and identically distributed).

(2) The parallel offset of 2NM to the right is applied only to all aircraft flying on the even (or odd) flight levels.

5.2 Fig.3 shows the effective separation between the intended paths of aircraft pairs on adjacent routes at the same flight level. This separation varies according to the combination of lateral offsets to be applied as shown in Fig.2. An estimate of the lateral collision risk for a route system composed of five parallel routes, such as the NOPAC route can be given by

$$E[N_{ay}] = P_z(0)\sum_{i=1}^{4}\frac{T_i^p}{H}\times[K(s)\cdot N_i^y(s)P_y(S_i(s)) + K(o)\cdot N_i^y(o)P_y(S_i(o))] \qquad (5)$$

where,

$S_i(o/s)$: the effective separation for opposite/same direction aircraft on the i-th adjacent route pair consisting of the i-th and $(i+1)-\text{th}$ routes,

$N_i^y(o/s)$: the opposite/same direction passing frequency for the traffic on the $i-\text{th}$ pair,

$H=\sum_{i=1}^{5}T_i$: the total flight time for all traffic on the five-route system,

T_i : the total flight time for all traffic on the i-th route,

T_i^p : the total flight time for all traffic on the i-th pair ($=T_i+T_{i+1}$),

Substitution of Eq. (4) for Eq. (5) yields

$$E[N_{ay}] = P_z(0)\sum_{i=1}^{4}\frac{2}{H}\times[K(s)\cdot n_i(s)P_y(S_i(s)) + K(o)\cdot n_i(o)P_y(S_i(o))] \qquad (6)$$

5.3 The product of the lateral overlap probability and the number of passing events in Eq.(6) can be apportioned into two categories of flight levels, i.e., even (300, 320,…., 400) and odd (290, …., 410) levels. Then, the following equations can be satisfied;

$$P_y(S_i(o/s))n_i(o/s) = P_y(S_i(o/s)_{odd})n_i(o/s)_{odd} + P_y(S_i(o/s)_{even})n_i(o/s)_{even} \qquad (7)$$

where the subscript "even" and "odd" represents the operation for even and odd flight levels, respectively. Substitution of Eq.(7) for Eq.(6) yields

$$E[N_{ay}] = \frac{2P_z(0)}{H}\sum_{i=1}^{4}[K(s)\{P_y(S_i(s)_{odd})\cdot n_i(s)_{odd} + P_y(S_i(s)_{even})\cdot n_i(s)_{even}\} + K(o)\{P_y(S_i(o)_{odd})\cdot n_i(o)_{odd} + P_y(S_i(o)_{even})\cdot n_i(o)_{even}] \quad (8)$$

5.4 The lateral collision risk for the system without offsets is given by

$$E[N_{ay}]_{NO} = \frac{2P_z(0)P_y(S_y)}{H}\sum_{i=1}^{4}[K(s)n_i(s) + K(o)n_i(o)] \quad (9)$$

where S_y=50 NM. Then, the ratio of the lateral collision risks is given by

$$R = \frac{E[N_{ay}]}{E[N_{ay}]_{NO}} \quad (10)$$

This is a function of parameters such as K(o/s), $P_y(S)$ ($S = S_y, S_y \pm 4$ NM), and $n_i(o/s)$.

6. OVERLAP PROBABILITY

6.1 The observation by an ARSR on the R220 route in the NOPAC [16] suggests that the lateral path keeping errors of conventional aircraft can be modeled by the following double-double exponential (DDE) distribution

$$f(y) = (1-\alpha)DE(y|\lambda_{core}) + \alpha DE(y|\lambda_{tail}) \quad (11)$$

where λ_{core} is the core parameter, λ_{tail} is the tail parameter, α is the mixing parameter, and

$$DE(y|\lambda) = (2\lambda)^{-1}\exp(-|y|/\lambda) \quad (12)$$

6.2 This is the probability density function of the double exponential (DE) distribution. The fitted model parameters [16] are $\lambda_{core}=\lambda_c$=0.816 NM, λ_{tail}=5.26 NM and α=7.26x10^{-4}. The core, in general, is related to normal errors and the tail is associated with abnormal errors due to failures, malfunctions or blunders, etc.

6.3 The observations [17][19] suggest that the standard deviation of path keeping errors of the GPS equipped aircraft seems was about one order of magnitude smaller than that of the conventional aircraft. We assume that the core distribution of the GPS-equipped aircraft follows a DE distribution with standard deviation 0.063 NM(=σ_{GPS}) as in references[14][15]. The core parameter of the GPS-equipped aircraft was assumed to be $\lambda_{core} = \lambda_g = \sigma_{GPS}/\sqrt{2}$ (=0.044 NM).

6.4 Herein, the lateral overlap probabilities of interest are $P_y(50)$, $P_y(54)$, and $P_y(46)$. The lateral overlap probability for pairs nominally separated by S is given by

$$P_y(S) = \int_{S-\lambda_y}^{S+\lambda_y}\int_{-\infty}^{\infty} f(y)\cdot f(y+u)\,dy\,du \quad (13)$$

$$\cong 2\lambda_y C(S;\lambda_{core},\lambda_{tail},\alpha) \quad (14)$$

where

$$C(y;\lambda_1,\lambda_2,\alpha) = \frac{(1-\alpha)^2}{4\lambda_1^2}(|y|+\lambda_1)\exp(-|y|/\lambda_1) + \frac{\alpha^2}{4\lambda_2^2}(|y|+\lambda_2)\exp(-|y|/\lambda_2) + \frac{\alpha(1-\alpha)}{(\lambda_2^2-\lambda_1^2)}[\lambda_2\exp(-|y|/\lambda_2) - \lambda_1\exp(-|y|/\lambda_1)] \quad (15)$$

6.5 Substitution of the values in Table 1 for the terms within square brackets in Eq.(8) becomes

$$\begin{aligned}&K(s)\{61\cdot P_y(50)+10.5\cdot P_y(50)\}+K(o)\{(95.5\cdot P_y(50)+111.5\cdot P_y(54)+11\cdot P_y(50)+21\cdot P_y(46)\}\\&=K(s)\cdot 71.5\cdot P_y(50)+K(o)\cdot\{111.5\cdot P_y(54)+21\cdot P_y(46)+106.5\cdot P_y(50)\}\end{aligned} \quad (16)$$

7. OTHER PARAMETERS

7.1 The parameters shown in Table 2 were used for the calculation. The parameters associated with aircraft size are based on observations on the NOPAC routes. Other parameters are from the Reference [9].

Table 2 Parameters used for calculation

Parameter	Estimated value	Basis
$P_z(0)$	0.5	Reference [9]
λ_x	0.036 NM	Observed in NOPAC[20]
λ_y	0.032 NM	Ibid
λ_z	0.010 NM	Ibid
$V_{rx}(s)$	13 kt	Reference [9]
V_{ry}	75 kt	Ibid
$V_{rx}(o)$	960 kt	Ibid
V_{rz}	1.5 kt	Ibid

8. RESULTS OF CALCULATION

8.1 In this calculation, one of the major contributors to collision risk is the mixing parameter, α, of the DDE distribution model. The results strongly depend on the magnitudes of λ_{tail} and α. Therefore, we have to be cautious in choosing the tail parameters of the DDE distribution. The lateral collision risk for the system with offsets was calculated by Eq.(8). The collision risk for the system without offsets was calculated for comparison using Eq.(9). Table 3 shows lateral overlap probabilities for $\lambda_{tail} = 50\text{NM}$. For comparison, the cases of several core parameters are shown.

8.2 The effect of core parameter on the lateral overlap probabilities such as $P_y(50)$ is not significant up to RNP4.

Table 3 Lateral Overlap Probabilities ($\lambda_{tail} = 50\text{NM}$)

	λ_{core}	α	$P_y(54)$	$P_y(50)$	$P_y(46)$
GPS	0.044	8.37×10^{-5}	3.638×10^{-8}	3.941×10^{-8}	4.269×10^{-8}
Conventional	0.816	8.37×10^{-5}	3.639×10^{-8}	3.942×10^{-8}	4.271×10^{-8}
RNP4	1.34	8.37×10^{-5}	3.641×10^{-8}	3.944×10^{-8}	4.272×10^{-8}
RNP12.6	4.21	8.37×10^{-5}	1.779×10^{-7}	3.798×10^{-7}	8.577×10^{-7}

8.3 The lateral collision risk based on the parameters in the shaded line results in $E[N_{ay}]=2.92\times 10^{-9}$[accidents/hour]. The lateral collision risk of the system without offsets, $E[N_{ay}]_{NO}$, is 2.94×10^{-9}[accidents/hour]. The ratio of the lateral collision risks, R, is 0.991. This means that the lateral collision risk for the route system under consideration is slightly reduced due to the application of offsets. The bulk of opposite passing events is for the route pair 2 (R580-A590). This pair has an effective assigned route spacing of 54 NM due to the application of offsets, resulting in a slight reduction in the risk of the route system.

8.4 For example, if we use the values based on the model parameters fitted to the data obtained in the NOPAC [16], the lateral overlap probabilities result in those shown in Table 4. The fitted model parameters associated with the tail are not as reliable as those obtained from the NAT region due to the smaller sample size (about 18,000).

Table 4 Lateral Overlap Probabilities ($\lambda_{tail} = 5.26\text{NM}$)

	λ_{core}	α	$P_y(54)$	$P_y(50)$	$P_y(46)$
GPS	0.044	7.26×10^{-4}	3.077×10^{-10}	6.583×10^{-10}	1.408×10^{-9}
Conventional	0.816	7.26×10^{-4}	3.153×10^{-10}	6.744×10^{-10}	1.433×10^{-9}
RNP4	1.34	7.26×10^{-4}	3.29×10^{-10}	7.038×10^{-10}	1.505×10^{-9}
RNP12.6	4.21	7.26×10^{-4}	1.491×10^{-7}	3.414×10^{-7}	8.172×10^{-7}

In this case, we obtained $E[N_{ay}] = 4.87\times10^{-11}$ [accidents/hour], $E[N_{ay}]_{NO} = 4.92\times10^{-11}$ [accidents/hour], and R=0.953. These estimates may be too optimistic because of the relatively small sample size.

9. CONCLUSIONS

9.1 The number of passing events in four route pairs of the NOPAC routes was calculated using flight progress data from one month. The lateral collision risk for a hypothetical scenario was then estimated on the basis of available information on the current collision risk model parameters. In the scenario a lateral offset of 2NM to the right was applied only for airplanes on even flight levels. This produced assigned track separations of 46, 50, and 54 NM. A double-double exponential (DDE) distribution is assumed for lateral deviations. The tail parameters, λ_{tail} =50NM (this maximizes $P_y(50)$) , and $\alpha = 8.37\times10^{-5}$ (based on an estimate for NAT in 1990s) were assumed to obtain a conservative estimate.

9.2 The estimate of the lateral collision risk for the assumed scenario is 2.92×10^{-9} [accidents/hour]. This estimate is less than the TLS. The ratio of the lateral collision risk is 0.991. This means that the lateral collision risk of the whole route system is slightly reduced by applying offsets for the assumed scenario. These results suggest that a 2 NM lateral offset for airplanes equipped with navigation performance of RNP4 or better does not impose significant adverse effects on the lateral collision risk of the NOPAC route system.

9.3 The absolute risk value strongly depends on the choice of tail parameters of the assumed lateral deviation model. Therefore, empirical data collection of large lateral deviations from the assigned track in the airspace under consideration would be needed to obtain a reliable estimate of the tail parameters.

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[APPENDIX 1] ESTIMATING σ_{GPS}

We expressed the standard deviation (sd) of the path keeping errors of GPS equipped aircraft, σ_{GPS}, by the following root sum squares

$$\sigma_{GPS} = (\sigma_A{}^2 + \sigma_{FTE}{}^2)^{1/2} \qquad \text{(a-1)}$$

where σ_{FTE} is the sd of the flight technical error of GPS equipped aircraft, and σ_A is the sd of positioning error.

Assuming the use of FANS-1 aircraft [15] , we take σ_{FTE}=0.0625NM. Reference [14] suggests that the 95% radial error of positioning for FANS1 aircraft, R_{95}. is 0.05~0.09NM. This corresponds to lateral error of $\sigma_A = R_{95} / 2.448$. Therefore, we get σ_A=0.02~0.37NM. Since May 2000, GPS Selective Availability (SA) has been turned off. The accuracy of 0.005NM(2drms) can be expected with SA off. Therefore, we assume σ_A=0.002NM. The substitution of these values into Eq.(a-1) then yields $\sigma_{GPS} = 0.063$ NM.

[APPENDIX 2] Tables of Passing Events (Data: Flight progress data of November 2002)

Table A1 Passing events on the route pair; R220-R580.

	Number of opposite direction passing events	Number of same direction passing events	Total flight light time on R220 [hours]	Total flight time on R580 [hours]
FL410			66.50	5.40
FL400		1	153.12	84.42
FL390		2.5	490.20	101.01
FL380		16.5	680.84	328.32
FL370		9	688.96	191.53
FL360		14.5	973.54	292.11
FL350		12.5	902.47	164.75
FL340		4	834.54	131.01
FL330			457.64	7.09
FL320		1	392.28	20.86
FL310			41.85	1.88
FL300			96.11	14.68
FL290			2.10	1.61
Total	0	61	5780.13	1344.66
Odd FL	0	24		7124.79
Even FL	0	37		

Table-A2 Passing events on the route pair; R580-A590

	Number of opposite direction passing events	Number of same direction passing events	Total flight time on R580 [hour]	Total flight time on A590 [hour]
FL410	3		5.40	23.57
FL400			84.42	0.00
FL390	1		101.01	6.88
FL380			328.32	0.00
FL370	8		191.53	56.04
FL360	1		292.11	10.02
FL350	67.5		164.75	290.10
FL340	97.5		131.01	438.46
FL330	14		7.09	1075.86
FL320	13		20.86	541.71
FL310	2		1.88	494.13
FL300			14.68	141.79
FL290			1.61	95.89
Total	207	0	1344.66	3174.44
Odd FL	95.5	0		4519.10
Even FL	111.5	0		

Table A3 Passing events on the route pair; A590-R591

	Number of opposite direction passing events	Number of same direction passing events	Total flight time on A590 [hour]	Total flight time on R591 [hour]
FL410			23.57	5.21
FL400			0.00	10.03
FL390			6.88	18.72
FL380			0.00	78.17
FL370			56.04	42.46
FL360			10.02	82.22
FL350	11	1	290.10	62.88
FL340	17		438.46	42.32
FL330		9.5	1075.86	113.15
FL320	4		541.71	23.45
FL310			494.13	71.13
FL300			141.79	8.28
FL290			95.89	10.71
Total	32	10.5	3174.44	568.71
Odd FL	11	10.5		3743.14
Even FL	21	0		

Table A4 Passing events on the route pair; R591-G344

	Number of opposite direction passing events	Number of same direction passing events	Total flight time on R591 [hour]	Total flight time on G344 [hour]
FL410			5.21	2.35
FL400			10.03	2.00
FL390			18.72	1.16
FL380			78.17	19.50
FL370			42.46	22.05
FL360			82.22	21.38
FL350			62.88	36.92
FL340			42.32	17.50
FL330			113.15	50.79
FL320			23.45	9.09
FL310			71.13	13.62
FL300			8.28	1.07
FL290			10.71	2.41
Total	0	0	568.71	199.83
Odd FL	0	0		768.53
Even FL	0	0		

9/6/03

INTERNATIONAL CIVIL AVIATION ORGANIZATION (ICAO)

SEPARATION AND AIRSPACE SAFETY PANEL (SASP)

WORKING GROUP OF THE WHOLE

London, 12 May - 23 May, 2002

Agenda Item 1: en-route separation minima and procedures – horizontal (lateral offsets)

Lateral Overlap Probabilities of Airplanes Flying in Opposite Directions on a Bi-Directional Oceanic Route

Presented by Mr. S. Creamer

(Prepared by B. Flax, FAA Technical Center)

SUMMARY

When airplanes headed toward each other on a bi-directional route lose their planned vertical separation, they avoid colliding only if they pass each other with adequate random separation in the vertical or lateral dimension. Allowing or requiring such airplanes to apply lateral offsets to the right of the route's center line increases the likelihood of their having adequate lateral separation. For each of several offset procedures, the present working paper derives a formula for the lateral overlap probability experienced by a pair of randomly chosen airplanes flying in opposite directions on the route; and it computes numerical values of the overlap probabilities.

1 Introduction

1.1 When airplanes that are headed toward each other on a bi-directional route lose their planned vertical separation (as might happen if one of them is trying to fly at the wrong altitude) they avoid colliding only if they pass each other with adequate random separation in the vertical or lateral dimension. Since the probability of vertical overlap for airplanes trying to maintain the same flight level is quite high – approximately ½ – it is far more likely that when a collision is avoided, it is avoided because the airplanes have enough random lateral separation. However, even in the lateral dimension the probability of achieving sufficient random separation is becoming more difficult. In recent years, navigation improvements – especially increased use of the Global Positioning System (GPS) – have significantly increased the probability that airplanes assigned to the same route have laterally overlapping positions. Therefore, the SASP has begun considering means of increasing the lateral dispersion of airplanes assigned to the same route. In particular, it has undertaken several studies of the effects of requiring or allowing airplanes to follow paths laterally offset from the center lines of their assigned routes. Since offsets to the same side with respect to the direction of flight (for example, offsets to the right) move opposite-direction airplanes away from each other, they are among the most promising of the many offset procedures that have been proposed.

1.2 Section 2 of this working paper derives a basic formula (equation 5) for computing the probability that two randomly chosen airplanes, flying in opposite directions along a bi-directional route, are in lateral overlap. Section 3 and appendix A derive general formulas for the probability that the two airplanes are in a given navigational class and have a given planned separation. Those formulas are then simplified according to the assumptions of the offset procedure under which the route is assumed to operate. The present study considers the following six offset procedures:

(1) *Offsets are not permitted*. This is a "base case", included in the present study in order to show the effect of inaction, i.e., of not adopting *any* offset procedure.

(2) *Only GPS-equipped airplanes that have automatic offset capability are allowed to apply offsets, and the offsets must be 1 nm to the right of the route's center line. All other airplanes must try to fly along the center line.* This procedure is currently being proposed to ICAO for implementation on South Pacific routes.

(3) *Only GPS-equipped airplanes with automatic offset capability are allowed to apply offsets, but an offset may be 1 nm or 2 nm to the right of the route's center line. All other airplanes must try to fly along the center line.* This procedure extends procedure (2) by allowing 2 nm offsets.

(4) *All airplanes with automatic offset capability are allowed to apply offsets. An offset may be 1 nm or 2 nm to the right of the route's center line.* This is the procedure currently implemented on a trial basis in the West Atlantic Route System (WATRS).

(5) *GPS-equipped airplanes with automatic offset capability are required to apply offsets, which may be 1 nm or 2 nm to the right of the route's center line. All other airplanes must try to fly along the center line.* This procedure has been included in the present study in order to show the effect of eliminating the major source of lateral overlap probability, i.e., the use of the route's center line by GPS-equipped airplanes.

(6) *GPS-equipped airplanes with automatic offset capability are required to apply offsets, which may be 1 nm or 2 nm to the right of the route's center line. All other airplanes having automatic offset capability are allowed to apply offsets.* This procedure extends procedure (5) by allowing non-GPS-equipped airplanes to apply offsets.

1.3 The analysis of each procedure assumes that pilots follow the procedure's rules. In particular, if an airplane cannot automatically apply a lateral offset, the analysis assumes that its pilot tries to fly along the route's center line.

1.4 In the six procedures analyzed, the only permitted offsets are 0 nm, 1 nm, and 2 nm; and where offsets are permitted, they must be to the right of the route's center line. (Procedures that allow offsets to the left are among those considered in reference 5.) If two airplanes flying in opposite directions are randomly selected, they may have an intended separation of 0 nm, 1 nm, 2 nm, 3 nm, or 4 nm – though some procedures allow only some of these intended separations.

2 <u>Method of Analysis</u>

2.1 As in many prior studies of lateral risk, we assume that a typical airplane's lateral deviation from its intended flight path is a continuous random variable that can be described by a double double exponential (DDE) density function:

$$f(y) = \frac{1-\alpha}{2\lambda_1} e^{-\frac{|y|}{\lambda_1}} + \frac{\alpha}{2\lambda_2} e^{-\frac{|y|}{\lambda_2}} . \qquad (1)$$

Thus f is a weighted sum of two double exponential densities, a "core" density with parameter λ_1, and a "tail" density with parameter λ_2. The weighting parameter α is the fraction of flying time during which an atypical error is being committed. It is necessarily between 0 and 1. The shape parameters λ_1 and λ_2 are respectively $\frac{\sqrt{2}}{2}$ times the standard deviations of typical lateral errors and atypical errors; and $0 < \lambda_1 < \lambda_2$. We assume that aircraft using the routes are required to meet some level r of required navigation performance (RNP); and we take λ_1 to be the parameter implied by the requirement that 95% of typical deviations lie within r nm of the route center line, i.e., $\lambda_1 = \frac{r}{-\ln(.05)} \approx r/3$. In many studies of lateral risk λ_2 is taken to be the distance (in nautical miles) between adjacent parallel routes, a conservative value that maximizes the computed probability of lateral overlap between airplanes assigned to those routes. In the analyses done for the present working paper the value of λ_2 is of very little importance, since it has a negligibly small effect on the probability that airplanes assigned to the *same* path have overlapping positions. All of the computations done for the present working paper take $\lambda_2 = 50$ for the density function describing the deviations of GPS-equipped airplanes, and $\lambda_2 = 50 - \sqrt{2}\lambda_1$ for the density function describing the deviations of non-GPS-equipped airplanes.

2.2 If two airplanes with wingspan w have planned lateral separation of S nm, and the lateral deviations of both of them follow the (same) DDE density function f given by (1) above, then [as is shown in reference 1] their probability of lateral overlap, $P_y(S)$, is approximately equal to

$$2w\left\{\frac{(1-\alpha)^2}{4\lambda_1^2}e^{-\frac{S}{\lambda_1}}\left(\lambda_1+S\right)+\frac{\alpha^2}{4\lambda_2^2}e^{-\frac{S}{\lambda_2}}\left(\lambda_2+S\right)+\frac{\alpha(1-\alpha)}{2}\left[\frac{e^{-\frac{S}{\lambda_1}}+e^{-\frac{S}{\lambda_2}}}{\lambda_1+\lambda_2}+\frac{e^{-\frac{S}{\lambda_2}}-e^{-\frac{S}{\lambda_1}}}{\lambda_2-\lambda_1}\right]\right\}. \tag{2}$$

On the other hand, if one of the airplanes typically navigates far better than the other, then we characterize its lateral deviations by a DDE density with different parameters, e.g.,

$$g(y) = \frac{1-\beta}{2\kappa_1}e^{-\frac{|y|}{\kappa_1}}+\frac{\beta}{2\kappa_2}e^{-\frac{|y|}{\kappa_2}} . \tag{3}$$

In the most general case we assume that $\alpha \neq \beta$, $\lambda_1 \neq \kappa_1$, and $\lambda_2 \neq \kappa_2$. Since r', the RNP of the better-navigating airplane, is less than r, the RNP of the other airplane, it immediately follows that $\kappa_1 = \frac{r'}{-\ln(.05)} < \frac{r}{-\ln(.05)} = \lambda_1$. The lateral overlap probability of two airplanes whose lateral deviations have density functions f and g is [shown in reference 1 to be] approximately equal to

$$\begin{aligned}2w\Bigg\{&\frac{(1-\alpha)(1-\beta)}{4}\left(\frac{e^{-\frac{S}{\kappa_1}}+e^{-\frac{S}{\lambda_1}}}{\kappa_1+\lambda_1}+\frac{e^{-\frac{S}{\kappa_1}}-e^{-\frac{S}{\lambda_1}}}{\kappa_1-\lambda_1}\right)+\frac{\alpha(1-\beta)}{4}\left(\frac{e^{-\frac{S}{\kappa_1}}+e^{-\frac{S}{\lambda_2}}}{\kappa_1+\lambda_2}+\frac{e^{-\frac{S}{\kappa_1}}-e^{-\frac{S}{\lambda_2}}}{\kappa_1-\lambda_2}\right)\\&+\frac{(1-\alpha)\beta}{4}\left(\frac{e^{-\frac{S}{\kappa_2}}+e^{-\frac{S}{\lambda_1}}}{\kappa_2+\lambda_1}+\frac{e^{-\frac{S}{\kappa_2}}-e^{-\frac{S}{\lambda_1}}}{\kappa_2-\lambda_1}\right)+\frac{\alpha\beta}{4}\left(\frac{e^{-\frac{S}{\kappa_2}}+e^{-\frac{S}{\lambda_2}}}{\kappa_2+\lambda_2}+\frac{e^{-\frac{S}{\kappa_2}}-e^{-\frac{S}{\lambda_2}}}{\kappa_2-\lambda_2}\right)\Bigg\}.\end{aligned} \tag{4}$$

As was mentioned in paragraph 1.4, the only values of S used to compute lateral overlap probabilities in the present study are 0, 1, 2, 3, and 4.

2.3 If two airplanes traveling in opposite directions on the same route are selected at random, the pair can be in any of three navigational classes: class G, in which both airplanes have GPS; class N, in which neither airplane has GPS; and class M (as in "mixed"), in which one airplane has GPS but the other does not. Let C denote the pair's navigational class; i.e., $C = G$, N, or M. We shall also think of G, N, and M as mutually disjoint events associated with the "experiment" of choosing a pair of airplanes. When a pair is chosen, exactly one of those three events occurs.

2.4 Let $I_{S,C}$ denote the event that the randomly selected pair of airplanes has intended lateral separation S and navigational class C. We let O denote the event that the airplanes are in lateral overlap. Then $\mathrm{P}(O) = \sum_{S,C} \mathrm{P}(O|I_{S,C})\cdot\mathrm{P}(I_{S,C})$. (5)

2.5 When $C = N$ or $C = G$, i.e., when the two randomly chosen airplanes have the same distribution of lateral deviations, we use formula (2) to compute $\mathrm{P}(O|I_{S,C})$. On the other hand, when $C = M$ – i.e., when the two airplanes come from populations whose lateral deviations follow markedly different DDE densities – we use formula (4) to compute $\mathrm{P}(O|I_{S,C})$.

2.6 The probabilities $\mathrm{P}(I_{S,C})$ depend on the probabilities that airplanes apply offsets. Early observations of WATRS airspace, in which procedure (4) has been implemented on a trial basis, suggest that roughly half of the pilots eligible to apply offsets nonetheless continue to fly along the center lines of their assigned routes (reference 7). Thus, even if we consider only airplanes that are permitted to apply offsets, we should not expect the distribution of offset distances to be uniform.

2.7 We suppose that an airplane is chosen at random from those using the route, and we define some relevant events: O_0 = {the pilot is attempting to fly along the center line}, O_1 = {the pilot is attempting to fly 1 nm to the right of the center line}, and O_2 = {the pilot is attempting to fly 2 nm to the right of the center line}.

2.8 In analyzing the six proposed offset procedures we classify airplanes not only by their navigational capability, but also by their ability to fly along paths offset from the route's center line. When an airplane is chosen at random, exactly one of the following four events occurs:

E_1 = {the chosen airplane has neither automatic offset capability nor GPS};
E_2 = {the chosen airplane does not have automatic offset capability, but does have GPS};
E_3 = {the chosen airplane has automatic offset capability, but does not have GPS};
E_4 = {the chosen airplane has both automatic offset capability and GPS}.

For the sake of notational efficiency we let $e_i = \mathrm{P}(E_i)$ for each i = 1, 2, 3, 4; and we require that $e_1 + e_2 + e_3 + e_4 = 1$. In practice these probabilities should be estimated from data relevant to the flights that use (or may be expected to use) the route being studied. If we think of probabilities as abstractions of relative frequencies, then we can view each "equipage class" E_i as a subset of the set of all flights that use the route. The members of each E_i are defined by their equipage or non-equipage with GPS, and their equipage or non-equipage for automatic offsets; and each e_i is the fraction of the route's flights having the equipage combination needed for membership in E_i. We also assume that the e_i do not depend on the direction of travel of the selected airplane. That is, we assume that if we randomly choose an airplane from those headed in one direction, the e_i all have the same values that they would have if the choice were made from the airplanes headed in the opposite direction; and it then follows that the e_i would have the same values if the choice were made without regard to the direction of flight. (In determining the numerical values of the probabilities e_i we are interested in the equipage of the *flights* that use the route, rather than the equipage of the *airframes* that use it, since some airplanes may make far more flights than others.)

2.9 For $j = 0, 1,$ or 2, let n_j denote the probability that an airplane randomly chosen from class E_3 is applying an offset of j nautical miles, and let g_j denote the probability that an airplane randomly chosen from class E_4 is applying an offset of j nautical miles. That is, $n_j \equiv P(O_j \mid E_3)$ and $g_j \equiv P(O_j \mid E_4)$. Since every airplane in each E_i must be flying on one of the three available flight paths, $n_0 + n_1 + n_2 = g_0 + g_1 + g_2 = 1$. (Since pilots are assumed to follow the rules, and none of the procedures listed in paragraph 1.2 allows airplanes to apply (non-zero) offsets unless they have automatic offset capability, we don't need to define analogous quantities for airplanes in classes E_1 and E_2.) With these definitions it becomes easy to express the probability that a randomly chosen airplane is in the set E_i and is applying an offset of j nm. Since $E_1 \subseteq O_0$, it follows that $P(E_1 \cap O_0) = P(E_1) = e_1$, while $P(E_1 \cap O_1) = P(E_1 \cap O_2) = 0$. Similarly, since $E_2 \subseteq O_0$, we have $P(E_2 \cap O_0) = P(E_2) = e_2$, while $P(E_2 \cap O_1) = P(E_2 \cap O_2) = 0$. From the definitions above, we find that $P(E_3 \cap O_j) = P(O_j \mid E_3) \cdot P(E_3) = n_j \cdot e_3$ and $P(E_4 \cap O_j) = P(O_j \mid E_4) \cdot P(E_4) = g_j \cdot e_4$.

2.10 In computing the $P(I_{S,C})$ it is helpful to have an expression for the probability that a randomly chosen airplane has a given navigational equipage – GPS or non-GPS – and that it is attempting to apply a given offset – 0, 1, or 2 nm. Thus we need to derive expressions for the probabilities $P[(E_1 \cup E_3) \cap O_j)]$ and $P[(E_2 \cup E_4) \cap O_j)]$, for $j = 0, 1, 2$. Since the events E_i are disjoint, $P[(E_1 \cup E_3) \cap O_j)] = P(E_1 \cap O_j) + P(E_3 \cap O_j)$ and $P[(E_2 \cup E_4) \cap O_j)] = P(E_2 \cap O_j) + P(E_4 \cap O_j)$. Applying the results of the last paragraph then gives the following formulas:

$$P[(E_1 \cup E_3) \cap O_0)] = P(E_1 \cap O_0) + P(E_3 \cap O_0) = e_1 + n_0 e_3 ; \tag{6a}$$

$$P[(E_1 \cup E_3) \cap O_1)] = P(E_1 \cap O_1) + P(E_3 \cap O_1) = 0 + n_1 e_3 = n_1 e_3 ; \tag{6b}$$

$$P[(E_1 \cup E_3) \cap O_2)] = P(E_1 \cap O_2) + P(E_3 \cap O_2) = 0 + n_2 e_3 = n_2 e_3 ; \tag{6c}$$

$$P[(E_2 \cup E_4) \cap O_0)] = P(E_2 \cap O_0) + P(E_4 \cap O_0) = e_2 + g_0 e_4 ; \tag{6d}$$

$$P[(E_2 \cup E_4) \cap O_1)] = P(E_2 \cap O_1) + P(E_4 \cap O_1) = 0 + g_1 e_4 = g_1 e_4 ; \tag{6e}$$

$$P[(E_2 \cup E_4) \cap O_2)] = P(E_2 \cap E_2) + P(E_4 \cap O_2) = 0 + g_2 e_4 = g_2 e_4 . \tag{6f}$$

3 Computing $P(I_{S,C})$

3.1 In order to simplify the derivations we suppose (without loss of generality) that we view the route from one of its ends. If two airplanes, moving in opposite directions, are randomly selected from the route's traffic, we let *a* denote the airplane that's moving away from us, and we let *t* denote the airplane that's moving toward us. We also let *A* denote the lateral offset that the pilot of airplane *a* is trying to apply, and we let *T* denote the lateral offset that the pilot of airplane *t* is trying to apply. In general, the random variables *A* and *T* have the possible values 0, 1 and 2; but (as was noted above) some procedures do not allow *A* and *T* to assume all three of those values with non-zero probability. Since offsets must be applied to the right, it immediately follows that the intended separation between the airplanes is simply the sum of *A* and *T*. That is, $S = A + T$. Thus we may write each $P(I_{S,C})$ as a sum of three terms. For each of the three possible values of *C*, i.e., $C = G$, N, or M, and for each of the five possible values of *S*, i.e., $S = 0, 1, 2, 3,$ or 4, we can write

$$P(I_{S,C}) = \sum_{k=0}^{2} P(C,\ A = k,\ T = S - k); \qquad (7)$$

and we recognize that in most of the sums, one or two terms are 0.

3.2 We assume that the offset flown by airplane *a* is independent of the offset flown by airplane *t*. In other words, we assume that *A* and *T* are independent random variables. Each of the probabilities in a sum such as the right side of (7) can then be written in terms of equations (6a) through (6f). For example, $P(G, A = 1, T = 2)$

$$\begin{aligned}
&= P(a \in E_2 \cup E_4 \text{ and } A = 1) \times P(t \in E_2 \cup E_4 \text{ and } T = 2) \\
&= P[(E_2 \cup E_4) \cap O_1)] \times P[(E_2 \cup E_4) \cap O_2)] \\
&= g_1 e_4 \cdot g_2 e_4 = g_1 g_2 e_4^2 .
\end{aligned}$$

Similarly, $P(M, A = 0, T = 2)$

$$\begin{aligned}
&= P(a \in E_1 \cup E_3 \text{ and } A = 0) \times P(t \in E_2 \cup E_4 \text{ and } T = 2) \\
&\quad + P(a \in E_2 \cup E_4 \text{ and } A = 0) \times P(t \in E_1 \cup E_3 \text{ and } T = 2) \\
&= P[(E_1 \cup E_3) \cap O_0)] \times P[(E_2 \cup E_4) \cap O_2)] \\
&\quad + P[(E_2 \cup E_4) \cap O_0)] \times P[(E_1 \cup E_3) \cap O_2)] \\
&= (e_1 + n_0 e_3) \cdot g_2 e_4 + (e_2 + g_0 e_4) \cdot n_2 e_3 .
\end{aligned}$$

3.3 For the sake of completeness we list here all fifteen formulas for $P(I_{S,C})$. The formulas are derived in appendix A.

$$P(I_{0,N}) = (e_1 + n_0 e_3)^2 \qquad (8a)$$
$$P(I_{1,N}) = 2(e_1 + n_0 e_3) \cdot n_1 e_3 \qquad (8b)$$
$$P(I_{2,N}) = 2(e_1 + n_0 e_3) \cdot n_2 e_3 + (n_1 e_3)^2 \qquad (8c)$$
$$P(I_{3,N}) = 2 n_1 n_2 e_3^2 \qquad (8d)$$
$$P(I_{4,N}) = (n_2 e_3)^2 \qquad (8e)$$
$$P(I_{0,G}) = (e_2 + g_0 e_4)^2 \qquad (8f)$$
$$P(I_{1,G}) = 2(e_2 + g_0 e_4) \cdot g_1 e_4 \qquad (8g)$$

$P(I_{2,G}) = 2(e_2 + g_0e_4)\cdot g_2e_4 + (g_1e_4)^2$ (8h)
$P(I_{3,G}) = 2g_1e_4\cdot g_2e_4$ (8i)
$P(I_{4,G}) = (g_2e_4)^2$ (8j)
$P(I_{0,M}) = 2\cdot(e_1 + n_0e_3)(e_2 + g_0e_4)$ (8k)
$P(I_{1,M}) = 2[(e_1 + n_0e_3)\cdot g_1e_4 + n_1e_3\cdot(e_2 + g_0e_4)]$ (8l)
$P(I_{2,M}) = 2[(e_1 + n_0e_3)\cdot g_2e_4 + n_2e_3\cdot(e_2 + g_0e_4) + n_1e_3\cdot g_1e_4]$ (8m)
$P(I_{3,M}) = 2(n_1e_3\cdot g_2e_4 + n_2e_3\cdot g_1e_4)$ (8n)
$P(I_{4,M}) = 2\cdot n_2e_3\cdot g_2e_4$ (8o)

3.4 <u>$P(I_{S,C})$ for procedure (1)</u>

3.4.1 Under procedure (1) offsets are not permitted, and all pilots aim to fly along the route's center line, so that $n_0 = g_0 = 1$, and $n_1 = n_2 = g_1 = g_2 = 0$. Formulas (8a) through (8o) simplify to:

$P(I_{0,N}) = (e_1 + e_3)^2$ (9a)
$P(I_{1,N}) = 0$ (9b)
$P(I_{2,N}) = 0$ (9c)
$P(I_{3,N}) = 0$ (9d)
$P(I_{4,N}) = 0$ (9e)
$P(I_{0,G}) = (e_2 + e_4)^2$ (9f)
$P(I_{1,G}) = 0$ (9g)
$P(I_{2,G}) = 0$ (9h)
$P(I_{3,G}) = 0$ (9i)
$P(I_{4,G}) = 0$ (9j)
$P(I_{0,M}) = 2\cdot(e_1 + e_3)(e_2 + e_4)$ (9k)
$P(I_{1,M}) = 0$ (9l)
$P(I_{2,M}) = 0$ (9m)
$P(I_{3,M}) = 0$ (9n)
$P(I_{4,M}) = 0$ (9o)

3.5 <u>$P(I_{S,C})$ for procedure (2)</u>

3.5.1 Under procedure (2), only GPS-equipped airplanes with automatic offset capability are permitted to apply offsets; and the offsets must be 1 nm to the right of the center line. It follows that $n_0 = 1$, and $n_1 = n_2 = g_2 = 0$. Formulas (8a) through (8o) simplify to:

$P(I_{0,N}) = (e_1 + e_3)^2$ (10a)
$P(I_{1,N}) = 0$ (10b)
$P(I_{2,N}) = 0$ (10c)
$P(I_{3,N}) = 0$ (10d)
$P(I_{4,N}) = 0$ (10e)
$P(I_{0,G}) = (e_2 + g_0e_4)^2$ (10f)

$P(I_{1,G}) = 2(e_2 + g_0e_4)\cdot g_1e_4$ (10g)
$P(I_{2,G}) = (g_1e_4)^2$ (10h)
$P(I_{3,G}) = 0$ (10i)
$P(I_{4,G}) = 0$ (10j)
$P(I_{0,M}) = 2(e_1 + e_3)(e_2 + g_0e_4)$ (10k)
$P(I_{1,M}) = 2(e_1 + e_3)g_1e_4$ (10l)
$P(I_{2,M}) = 0$ (10m)
$P(I_{3,M}) = 0$ (10n)
$P(I_{4,M}) = 0$ (10o)

3.6 P($I_{S,C}$) for procedure (3)

3.6.1 Under procedure (3), only GPS-equipped airplanes with automatic offset capability are permitted to apply offsets; but the offsets may be 1 nm or 2 nm to the right of the center line. It follows that $n_0 = 1$, and $n_1 = n_2 = 0$. Formulas (8a) through (8o) simplify to:

$P(I_{0,N}) = (e_1 + e_3)^2$ (11a)
$P(I_{1,N}) = 0$ (11b)
$P(I_{2,N}) = 0$ (11c)
$P(I_{3,N}) = 0$ (11d)
$P(I_{4,N}) = 0$ (11e)
$P(I_{0,G}) = (e_2 + g_0e_4)^2$ (11f)
$P(I_{1,G}) = 2(e_2 + g_0e_4)\cdot g_1e_4$ (11g)
$P(I_{2,G}) = 2(e_2 + g_0e_4)\cdot g_2e_4 + (g_1e_4)^2$ (11h)
$P(I_{3,G}) = 2g_1e_4\cdot g_2e_4$ (11i)
$P(I_{4,G}) = (g_2e_4)^2$ (11j)
$P(I_{0,M}) = 2\cdot(e_1 + e_3)(e_2 + g_0e_4)$ (11k)
$P(I_{1,M}) = 2(e_1 + e_3)\cdot g_1e_4$ (11l)
$P(I_{2,M}) = 2(e_1 + e_3)\cdot g_2e_4$ (11m)
$P(I_{3,M}) = 0$ (11n)
$P(I_{4,M}) = 0$ (11o)

3.7 P($I_{S,C}$) for procedure (4)

3.7.1 Procedure (4) is the most general of the six offset procedures considered in this working paper, as all airplanes having automatic offset capability are permitted to apply offsets of 1 nm or 2 nm, and all are permitted to fly along the center line. Therefore, the equations for P($I_{S,C}$) are simply equations (8a) through (8o).

3.8 P($I_{S,C}$) for procedure (5)

3.8.1 Under procedure (5), GPS-equipped airplanes with automatic offset capability are required to apply non-zero offsets, and non-GPS-equipped airplanes must aim to fly along the center line. It follows that $n_0 = 1$, and $n_1 = n_2 = g_0 = 0$. Formulas (8a) through (8o) simplify to:

$$P(I_{0,N}) = (e_1 + e_3)^2 \quad (12a)$$
$$P(I_{1,N}) = 0 \quad (12b)$$
$$P(I_{2,N}) = 0 \quad (12c)$$
$$P(I_{3,N}) = 0 \quad (12d)$$
$$P(I_{4,N}) = 0 \quad (12e)$$
$$P(I_{0,G}) = e_2^{\,2} \quad (12f)$$
$$P(I_{1,G}) = 2e_2\, g_1\, e_4 \quad (12g)$$
$$P(I_{2,G}) = 2e_2\, g_2\, e_4 + (g_1 e_4)^2 \quad (12h)$$
$$P(I_{3,G}) = 2g_1 e_4 \cdot g_2\, e_4 \quad (12i)$$
$$P(I_{4,G}) = (g_2 e_4)^2 \quad (12j)$$
$$P(I_{0,M}) = 2\cdot(e_1 + e_3)e_2 \quad (12k)$$
$$P(I_{1,M}) = 2(e_1 + e_3)\cdot g_1 e_4 \quad (12l)$$
$$P(I_{2,M}) = 2(e_1 + e_3)\cdot g_2 e_4 \quad (12m)$$
$$P(I_{3,M}) = 0 \quad (12n)$$
$$P(I_{4,M}) = 0 \quad (12o)$$

3.9 P($I_{S,C}$) for procedure (6)

3.9.1 Under procedure (6), GPS-equipped airplanes with automatic offset capability are required to apply non-zero offsets, while non-GPS-equipped airplanes having automatic offset capability are permitted to aim for the center line or for either of the two offset tracks. It follows that $g_0 = 0$, so that formulas (8a) through (8o) simplify to:

$$P(I_{0,N}) = (e_1 + n_0 e_3)^2 \quad (13a)$$
$$P(I_{1,N}) = 2(e_1 + n_0 e_3)\cdot n_1 e_3 \quad (13b)$$
$$P(I_{2,N}) = 2(e_1 + n_0 e_3)\cdot n_2 e_3 + (n_1 e_3)^2 \quad (13c)$$
$$P(I_{3,N}) = 2n_1 e_3 \cdot n_2 e_3 \quad (13d)$$
$$P(I_{4,N}) = (n_2 e_3)^2 \quad (13e)$$
$$P(I_{0,G}) = e_2^{\,2} \quad (13f)$$
$$P(I_{1,G}) = 2e_2\, g_1\, e_4 \quad (13g)$$
$$P(I_{2,G}) = 2e_2\, g_2\, e_4 + (g_1 e_4)^2 \quad (13h)$$
$$P(I_{3,G}) = 2g_1 e_4 \cdot g_2 e_4 \quad (13i)$$
$$P(I_{4,G}) = (g_2 e_4)^2 \quad (13j)$$
$$P(I_{0,M}) = 2\cdot(e_1 + n_0 e_3)e_2 \quad (13k)$$
$$P(I_{1,M}) = 2[(e_1 + n_0 e_3)\cdot g_1 e_4 + n_1 e_3 e_2] \quad (13l)$$
$$P(I_{2,M}) = 2[(e_1 + n_0 e_3)\cdot g_2 e_4 + n_2 e_3 e_2 + n_1 e_3 \cdot g_1 e_4] \quad (13m)$$

$$P(I_{3,M}) = 2[n_1e_3{\cdot}g_2e_4 + n_2e_3{\cdot}g_1e_4)] \qquad (13n)$$
$$P(I_{4,M}) = 2{\cdot}n_2e_3{\cdot}g_2e_4 \qquad (13o)$$

4 Numerical Results

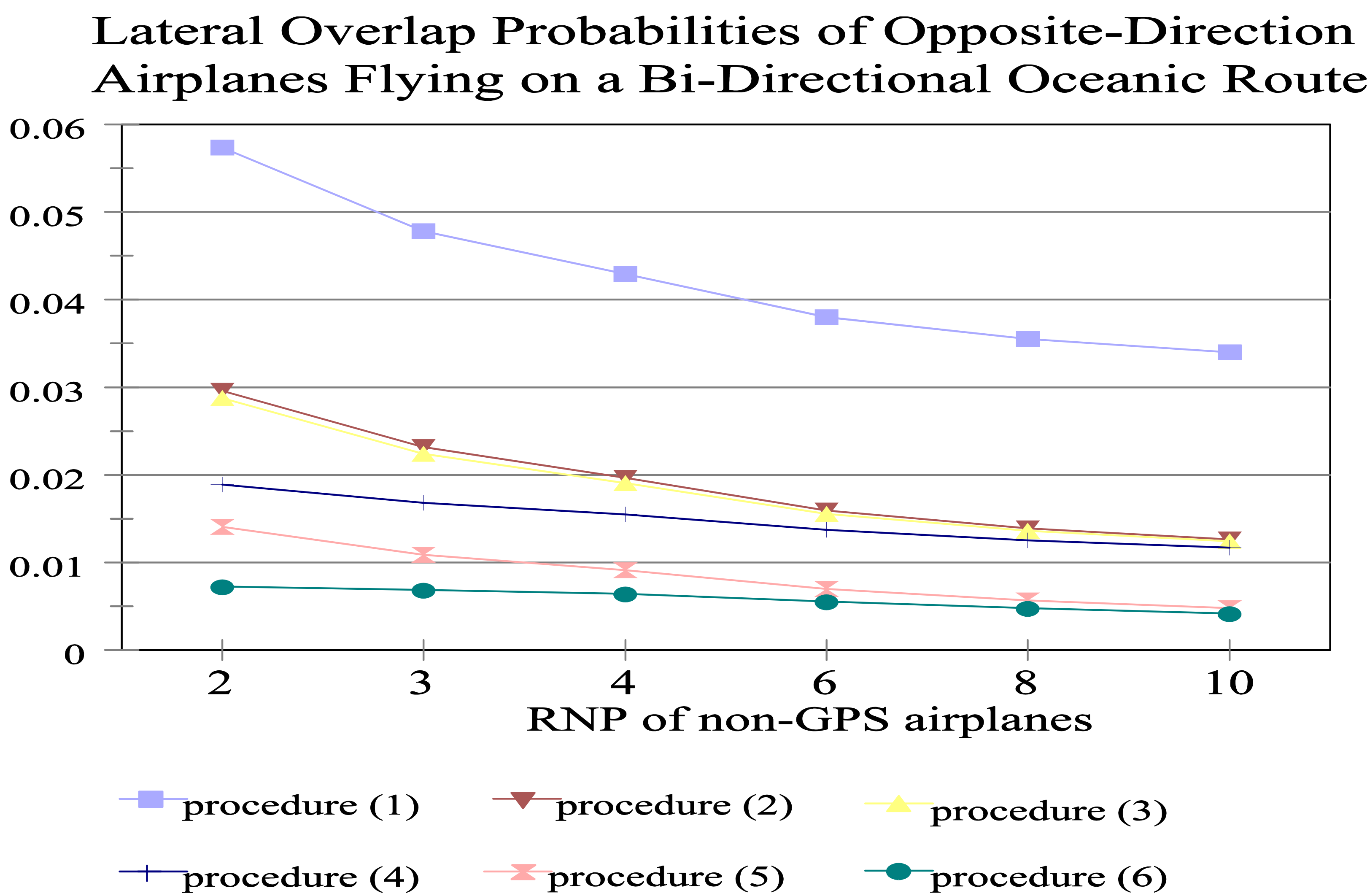
4.1 Table 1 shows lateral overlap probabilities computed from the formulas derived in section 3. Figure 1 shows the same data in graphical form. The following values of the parameters e_i were used in the computations: $e_1 = 0.05$, $e_2 = 0$, $e_3 = 0.65$, and $e_4 = 0.30$. In the most general case, i.e., for procedure (4), the parameters n_j and g_j had values $n_0 = g_0 = 0.5$, $n_1 = g_1 = 0.25$, and $n_2 = g_2 = 0.25$. These values are believed to be typical of the fleet that uses the West Atlantic Route System (WATRS), where procedure (4) has been implemented on a trial basis. In all of the computations, the wingspan of a typical airplane was taken to be 0.031 nm.

4.2 For each of the six offset procedures, table 1 and figure 1 show the lateral overlap probability computed when the non-GPS-equipped airplanes using the route (i.e., those in classes E_1 and E_3) navigate at each of six RNP levels: 2, 3, 4, 6, 8, and 10. GPS-equipped airplanes (those in classes E_2 and E_4) are assumed to navigate at a level equivalent to RNP 0.15.

4.3 In each column of table 1 – i.e., for each level of RNP of the non-GPS-equipped airplanes – the lateral overlap probabilities decrease from top to bottom. At most RNP levels, the decrease in overlap probability from procedure (1) to procedure (2) (i.e., from a policy of not allowing offsets to the policy currently being proposed for the South Pacific) is between 50% and 60%. Using procedure (3) rather than procedure (2) reduces overlap probability only slightly: between 1.5% and 3.3%. Procedure (4), the WATRS procedure, yields substantial reductions in overlap probability from procedure (3), ranging from 5.8% at RNP 10 to 34.4% at RNP 2. Procedure (5) gives further reductions from the probabilities obtained under procedure (4): between 25.5% at RNP 2 and 59.1% at RNP 10. Procedure (6) continues to reduce the overlap probability at every RNP level, with its most significant contributions being for low RNPs.

Table 1: Lateral Overlap Probabilities of Airplanes Flying
in Opposite Directions on a Bi-Directional Oceanic Route

	RNP:	2	3	4	6	8	10
procedure (1)		5.7378e-02	4.7827e-02	4.2947e-02	3.7995e-02	3.5490e-02	3.3978e-02
procedure (2)		2.9605e-02	2.3141e-02	1.9661e-02	1.5902e-02	1.3879e-02	1.2607e-02
procedure (3)		2.8754e-02	2.2383e-02	1.9053e-02	1.5514e-02	1.3617e-02	1.2420e-02
procedure (4)		1.8870e-02	1.6817e-02	1.5493e-02	1.3712e-02	1.2537e-02	1.1704e-02
procedure (5)		1.4059e-02	1.0870e-02	9.0886e-03	6.9644e-03	5.6742e-03	4.7924e-03
procedure (6)		7.2269e-03	6.8445e-03	6.4071e-03	5.5206e-03	4.7724e-03	4.1762e-03


Lateral Overlap Probabilities of Opposite-Direction Airplanes Flying on a Bi-Directional Oceanic Route
0.06
0.05
0.04
0.03
0.02
0.01
0
2
3
4
6
8
10
RNP of non-GPS airplanes
procedure (1)
procedure (2)
procedure (3)
procedure (4)
procedure (5)
procedure (6)

5 Recommendation

5.1 Discussions on the use of lateral offsets at previous meetings of the SASP and RGCSP have taken into account a great many important considerations of several proposed offset schemes, such as their technical feasibility and practicality, human factors, and the changes in various lateral overlap probabilities that would result from their implementation. However, the effects of offsets on the lateral overlap probabilities of airplanes flying in opposite directions on a bi-directional route have not previously been quantified at a SASP or RGCSP meeting. The working group is urged to take into account the results presented herein during its discussions of the application of lateral offsets.

6 References

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6. Flax, B., *Lateral Overlap Probabilities in Route Systems Where Lateral Offsets are Applied,* SASP-WG/WHL/1-WP/5, Canberra, May 2002

7. telephone conversation of February 19, 2003 between R. Swain, U.S. Federal Aviation Administration (AFS-430), and B. Flax, U.S. Federal Aviation Administration (ACB-310)

Appendix A: Derivations of $\mathrm{P}(I_{S,C})$

$$
\begin{aligned}
\mathrm{P}(I_{0,N}) &= \sum_{k=0}^{2} P(N,\ A=k,\ T=0-k) \\
&= \mathrm{P}(N,\ A=0,\ T=0) + \mathrm{P}(N,\ A=1,\ T=-1) + \mathrm{P}(N,\ A=2,\ T=-2) \\
&= \mathrm{P}(a \in E_1 \cup E_3,\ A=0)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=0) + 0 + 0 \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0] \\
&= (e_1 + n_0 e_3)^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{1,N}) &= \sum_{k=0}^{2} P(N,\ A=k,\ T=1-k) \\
&= \mathrm{P}(N,\ A=0,\ T=1) + \mathrm{P}(N,\ A=1,\ T=0) + \mathrm{P}(N,\ A=2,\ T=-1) \\
&= \mathrm{P}(a \in E_1 \cup E_3,\ A=0)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=1) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=1)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=0) + 0 \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] + \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0] \\
&= 2\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&= 2(e_1 + n_0 e_3)\cdot n_1 e_3
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{2,N}) &= \sum_{k=0}^{2} P(N,\ A=k,\ T=2-k) \\
&= \mathrm{P}(N,\ A=0,\ T=2) + \mathrm{P}(N,\ A=1,\ T=1) + \mathrm{P}(N,\ A=2,\ T=0) \\
&= \mathrm{P}(a \in E_1 \cup E_3,\ A=0)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=2) + \mathrm{P}(a \in E_1 \cup E_3,\ A=1)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=1) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=0) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] + \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&\quad + \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0] \\
&= 2\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] + \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&= 2(e_1 + n_0 e_3)\cdot n_2 e_3 + (n_1 e_3)^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{3,N}) &= \sum_{k=0}^{2} P(N,\ A=k,\ T=3-k) \\
&= \mathrm{P}(N,\ A=0,\ T=3) + \mathrm{P}(N,\ A=1,\ T=2) + \mathrm{P}(N,\ A=2,\ T=1) \\
&= 0 + \mathrm{P}(a \in E_1 \cup E_3,\ A=1)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=2) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=1) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] + \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&= 2\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&= 2 n_1 e_3 \cdot n_2 e_3
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{4,N}) &= \sum_{k=0}^{2} P(N,\ A=k,\ T=4-k) \\
&= \mathrm{P}(N,\ A=0,\ T=4) + \mathrm{P}(N,\ A=1,\ T=3) + \mathrm{P}(N,\ A=2,\ T=2) \\
&= 0 + 0 + \mathrm{P}(a \in E_1 \cup E_3,\ \ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ \ T=2) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&= (n_2 e_3)^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{0,G}) &= \sum_{k=0}^{2} P(G,\ A=k,\ T=0-k) \\
&= \mathrm{P}(G,\ A=0,\ T=0) + \mathrm{P}(G,\ A=1,\ T=-1) + \mathrm{P}(\mathrm{G},\ \mathrm{A}=2,\ T=-2) \\
&= \mathrm{P}(a \in E_2 \cup E_4,\ \ A=0)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=0) + 0 + 0 \\
&= \mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0] \\
&= (e_2 + g_0 e_4)^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{1,G}) &= \sum_{k=0}^{2} P(G,\ A=k,\ T=1-k) \\
&= \mathrm{P}(G,\ A=0,\ T=1) + \mathrm{P}(G,\ A=1,\ T=0) + \mathrm{P}(G,\ A=2,\ T=-1) \\
&= \mathrm{P}(a \in E_2 \cup E_4,\ \ A=0)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=1) \\
&\quad + \mathrm{P}(a \in E_2 \cup E_4,\ \ A=1)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=0) + 0 \\
&= \mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] + \mathrm{P}[(E_2 \cup E_4) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0] \\
&= 2\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] \\
&= 2(e_2 + g_0 e_4)\cdot g_1 e_4
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{2,G}) &= \sum_{k=0}^{2} P(G,\ A=k,\ T=2-k) \\
&= \mathrm{P}(G,\ A=0,\ T=2) + \mathrm{P}(G,\ A=1,\ T=1) + \mathrm{P}(G,\ A=2,\ T=0) \\
&= \mathrm{P}(a \in E_2 \cup E_4,\ \ A=0)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=2) + \mathrm{P}(a \in E_2 \cup E_4,\ \ A=1)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=1) \\
&\quad + \mathrm{P}(a \in E_2 \cup E_4,\ \ A=2)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ \ T=0) \\
&= \mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] \\
&\quad + \mathrm{P}[(E_2 \cup E_4) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0] \\
&= 2\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] \\
&= 2(e_2 + g_0 e_4)\cdot g_2 e_4 + (g_1 e_4)^2
\end{aligned}
$$

$$
\begin{aligned}
P(I_{3,G}) &= \sum_{k=0}^{2} P(G,\ A=k,\ T=3-k) \\
&= P(G, A=0, T=3) + P(G, A=1, T=2) + P(G, A=2, T=1) \\
&= 0 + P(a \in E_2 \cup E_4,\ A=1) \cdot P(t \in E_2 \cup E_4,\ T=2) \\
&\quad + P(a \in E_2 \cup E_4,\ A=2) \cdot P(t \in E_2 \cup E_4,\ T=1) \\
&= P[(E_2 \cup E_4) \cap O_1] \cdot P[(E_2 \cup E_4) \cap O_2] + P[(E_2 \cup E_4) \cap O_2] \cdot P[(E_2 \cup E_4) \cap O_1] \\
&= 2 \cdot P[(E_2 \cup E_4) \cap O_1] \cdot P[(E_2 \cup E_4) \cap O_2] \\
&= 2 g_1 e_4 \cdot g_2 e_4
\end{aligned}
$$

$$
\begin{aligned}
P(I_{4,G}) &= \sum_{k=0}^{2} P(G,\ A=k,\ T=4-k) \\
&= P(G, A=0, T=4) + P(G, A=1, T=3) + P(G, A=2, T=2) \\
&= 0 + 0 + P(a \in E_2 \cup E_4,\ A=2) \cdot P(t \in E_2 \cup E_4,\ T=2) \\
&= P[(E_2 \cup E_4) \cap O_2] \cdot P[(E_2 \cup E_4) \cap O_2] \\
&= (g_2 e_4)^2
\end{aligned}
$$

$$
\begin{aligned}
P(I_{0,M}) &= \sum_{k=0}^{2} P(M,\ A=k,\ T=0-k) \\
&= P(M, A=0, T=0) + P(M, A=1, T=-1) + P(M, A=2, T=-2) \\
&= P(a \in E_1 \cup E_3,\ A=0) \cdot P(t \in E_2 \cup E_4,\ T=0) + P(a \in E_2 \cup E_4,\ A=0) \cdot P(t \in E_1 \cup E_3,\ T=0) \\
&\quad + 0 + 0 \\
&= P[(E_1 \cup E_3) \cap O_0] \cdot P[(E_2 \cup E_4) \cap O_0] + P[(E_2 \cup E_4) \cap O_0] \cdot P[(E_1 \cup E_3) \cap O_0] \\
&= 2 \cdot P[(E_1 \cup E_3) \cap O_0] \cdot P[(E_2 \cup E_4) \cap O_0] \\
&= 2 \cdot (e_1 + n_0 e_3)(e_2 + g_0 e_4)
\end{aligned}
$$

$$
\begin{aligned}
P(I_{1,M}) &= \sum_{k=0}^{2} P(M,\ A=k,\ T=1-k) \\
&= P(M, A=0, T=1) + P(M, A=1, T=0) + P(M, A=2, T=-1) \\
&= P(a \in E_1 \cup E_3,\ A=0) \cdot P(t \in E_2 \cup E_4,\ T=1) + P(a \in E_2 \cup E_4,\ A=0) \cdot P(t \in E_1 \cup E_3,\ T=1) \\
&\quad + P(a \in E_1 \cup E_3, A=1) \cdot P(t \in E_2 \cup E_4, T=0) + P(a \in E_2 \cup E_4, A=1) \cdot P(t \in E_1 \cup E_3,\ T=0) \\
&\quad + 0 \\
&= P[(E_1 \cup E_3) \cap O_0] \cdot P[(E_2 \cup E_4) \cap O_1] + P[(E_2 \cup E_4) \cap O_0] \cdot P[(E_1 \cup E_3) \cap O_1] \\
&\quad + P[(E_1 \cup E_3) \cap O_1] \cdot P[(E_2 \cup E_4) \cap O_0] + P[(E_2 \cup E_4) \cap O_1] \cdot P[(E_1 \cup E_3) \cap O_0] \\
&= 2 \cdot \{P[(E_1 \cup E_3) \cap O_0] \cdot P[(E_2 \cup E_4) \cap O_1] + P[(E_1 \cup E_3) \cap O_1] \cdot P[(E_2 \cup E_4) \cap O_0]\} \\
&= 2[(e_1 + n_0 e_3) \cdot g_1 e_4 + n_1 e_3 \cdot (e_2 + g_0 e_4)]
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{2,M}) &= \sum_{k=0}^{2} P(M,\ A=k,\ T=2-k) \\
&= \mathrm{P}(M,\ A=0,\ T=2) + \mathrm{P}(M,\ A=1,\ T=1) + \mathrm{P}(M,\ A=2,\ T=0) \\
&= \mathrm{P}(a \in E_1 \cup E_3,\ A=0)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=2) + \mathrm{P}(a \in E_2 \cup E_4,\ A=0)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=2) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=1)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=1) + \mathrm{P}(a \in E_2 \cup E_4,\ A=1)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=1) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=2)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=0) + \mathrm{P}(a \in E_2 \cup E_4,\ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=0) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&\quad + \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] + \mathrm{P}[(E_2 \cup E_4) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&\quad + \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_0] + \mathrm{P}[(E_2 \cup E_4) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_0] \\
&= 2\{\mathrm{P}[(E_1 \cup E_3) \cap O_0]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_0]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&\quad + \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1]\} \\
&= 2[(e_1 + n_0 e_3)\cdot g_2 e_4 + n_2 e_3\cdot(e_2 + g_0 e_4) + n_1 e_3\ \cdot g_1 e_4]
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{3,M}) &= \sum_{k=0}^{2} P(M,\ A=k,\ T=3-k) \\
&= \mathrm{P}(M,\ A=0,\ T=3) + \mathrm{P}(M,\ A=1,\ T=2) + \mathrm{P}(M,\ A=2,\ T=1) \\
&= 0 + \mathrm{P}(a \in E_1 \cup E_3,\ A=1)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=2) + \mathrm{P}(a \in E_2 \cup E_4,\ A=1)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=2) \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=2)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=1) + \mathrm{P}(a \in E_2 \cup E_4,\ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=1) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_1]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&\quad + \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1] + \mathrm{P}[(E_2 \cup E_4) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_1] \\
&= 2\cdot\{\mathrm{P}[(E_1 \cup E_3) \cap O_1]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_1]\} \\
&= 2[n_1 e_3\cdot g_2 e_4 + n_2 e_3\cdot g_1 e_4)]
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}(I_{4,M}) &= \sum_{k=0}^{2} P(M,\ A=k,\ T=4-k) \\
&= \mathrm{P}(M,\ A=0,\ T=4) + \mathrm{P}(M,\ A=1,\ T=3) + \mathrm{P}(M,\ A=2,\ T=2) \\
&= 0 + 0 \\
&\quad + \mathrm{P}(a \in E_1 \cup E_3,\ A=2)\cdot\mathrm{P}(t \in E_2 \cup E_4,\ T=2) + \mathrm{P}(a \in E_2 \cup E_4,\ A=2)\cdot\mathrm{P}(t \in E_1 \cup E_3,\ T=2) \\
&= \mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] + \mathrm{P}[(E_2 \cup E_4) \cap O_2]\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2] \\
&= 2\cdot\mathrm{P}[(E_1 \cup E_3) \cap O_2]\cdot\mathrm{P}[(E_2 \cup E_4) \cap O_2] \\
&= 2\cdot n_2 e_3\cdot g_2 e_4
\end{aligned}
$$