2. Summary of Agenda Item 2: Review of developments regarding the application of reduced vertical seperation to aircraft operating above FL 290 in the NAT Region

2.1 Introduction

2.1.1 A proposal for a mathematical approach to estimating the collision risk due to loss of vertical separation, for the case of 1000 feet vertical separation over the North Atlantic Ocean, was given in a Working Paper by the United Kingdom. The mathematical model was derived along the same lines as that for loss of lateral separation agreed at the Fourth Meeting of the Group. Many of the parameters were also taken from the data used at that meeting. Attention was concentrated on a system in which the 1000 feet standard would be applied only to same-direction aircraft; aircraft flying in opposite direction were still required to be separated by 2000 feet. The Group accepted the mathematical model and the values given for a number of parameters in the equation. These are given in Appendix 2-A. A general discussion on the interpretation of the equation and of its different parameters is given in Appendix 2-B. It was found that there were differences of opinion within the Group on the values which should be accorded to some of the parameters. These parameters were:

(a) The estimate for the probability of vertical overlap $P_z(1000)$.

(b) The estimate for the probability of lateral overlap $P_y(0)$.

(c) The value of the relative lateral speed $|\bar{\dot{y}}|$.

(d) Independence of occupancy E_z.

2.1.2 The introduction of vertical separation of 1000 feet over the North Atlantic would cause ATC problems in the transition areas. This problem is discussed in some detail under Agenda Item 3 (para. 3.8) and is believed to be more critical in the context of a reduction in vertical separation than in the composite case.

2.1.3 It should be noted that the value applied to the many parameters could only be applied to the North Atlantic case.

2.2 The estimate for the probability of vertical overlap $P_z(1000)$

2.2.1 The value of this parameter must in principle be determined from a distribution of the vertical distance between proximate aircraft. This distribution can be obtained in two ways:

(a) By actually measuring the vertical distances to proximate aircraft, and comparing the difference with the difference in pressure altitude assigned to the aircraft by ATC.

(b) By determining the vertical deviations of aircraft from their assigned flight levels and convolving the distribution so obtained with a similar distribution for the aircraft flying at flight levels 1000 feet away.

2.2.1.1 The Group agreed that data of the first type can be obtained from measurements by ground based missile tracking radars or by radio or radar altimeters in the aircraft. Then the difference in pressure altitude can, in good approximation, be determined from the difference in geometric height of pairs of aircraft which were proximate horizontally and vertically. This difference in pressure altitude could then be compared to the difference in flight level assigned to the aircraft by ATC. The Group noted that a small number of such measurements had been made in the past (ref. 1). The number of these measurements was, however, too small for estimating $P_z(1000)$ and, moreover, no comparison with the difference in assigned flight levels had been made.

2.2.1.2 Direct measurements of the second type could be made by using measurements from pacer aircraft, and comparing the measured pressure altitudes to the flight levels assigned by ATC.

2.2.1.3 The Group agreed that it is desirable that such measurements should be made in the future. It noted, however, that the extremely high cost of such measurements and of their analysis would make it unlikely that sufficient data could be obtained by such measurements to define the tail of the distribution. It decided, therefore, that the only possible approach to obtaining a distribution of the altitude errors would be to use data on several types of altitude errors and to obtain the total distribution by convolving the distributions of these different types of error. It found that the altitude errors could conveniently be broken down into 4 groups which are discussed in some detail in Appendix 2-A, namely:

- random altimeter errors
- flight technical errors
- systematic altitude errors, excluding static pressure errors
- static pressure calibration errors

Note:-- This latter quantity has been defined as the static pressure error, corrected for those corrections which are applied automatically in the altimetry system or as a card correction by the pilots. In the calculations it has been assumed that these corrections are always applied.

2.2.2 Unlike the case of lateral separation discussed at the Fourth Meeting of the Group, there are very few large errors (over 500 feet) in the data available. Collisions of aircraft nominally separated by 1000 feet can only occur if one of the aircraft has an error of more than 500 feet. Also some of the data have been derived from a non-representative sample of operators and aircraft types in that certain types of civil transport, military and general aviation aircraft could not be included at this time. The confidence level of any distribution function obtained from these data will therefore be low, even if apparently very cautious assumptions were made on tail shapes and tail areas of the distribution.

2.2.3 It was found that the data on the first 3 groups of altitude errors indicated that the probability of gross errors attributable to these causes was very low. These data are discussed in Appendix 2-A. The Group noted that even relatively large changes in the distributions of these effects would hardly influence the estimated value of the total collision risk.

2.2.4 For the determination of the distribution of the static pressure error only very few measurements are available. In total the results of only 90 trailing-cone measurements have been used in the calculations. These showed no static pressure errors larger than 300 feet. The Group discussed several methods of deriving a cautious estimate for the distribution function from these data, the most cautious of which was based on an approach based on the following assumptions:

- all static pressure errors will persist from the moment they occurred first until the end of the service life of the aircraft without being detected unless the aircraft is flight-calibrated. Such errors can have been caused by some misalignment during manufacture or by later damage.

- the distribution has a level tail
- the level tail will extend to 1200 feet; this is the level tail which gives the largest value of the collision risk for a given tail area
- no aircraft which was tested by trailing cones has ever previously had a static pressure error larger than 300 feet which persisted for any significant time

2.2.4.1 These assumptions determine the tail shape and provide a basis for a cautious method for estimating the tail area. This method has been described in Appendix 2-A. The collision risk calculated on the basis of these assumptions is very much larger than the target levels of safety agreed by the Group during its Second Meeting. In Appendix 2-A it is shown, however, that this tail area could be reduced if the static pressure errors of all aircraft crossing the North Atlantic should be periodically checked. Figures 2.2a and 2.2b show the collision risks for same-direction traffic as a function of the period between recalibrations. If the confidence level obtained from the Monte-Carlo process described in Appendix 2-A should be 90%, then the lower target level of safety would be attained if the period between recalibrations was less than one year. For the higher target level this would be about 1 1/2 years.

2.2.5 In order to show how large the influence is of the assumptions of tail shape, two other calculations were made. In one it was assumed that all static pressure errors larger than 800 feet were detected within so few flying hours, that this effect on the collision risk could be neglected. In the other calculation there were no tails added at all to the distribution derived from the trailing-cone measurements. The results are shown in figures 2.4 and 2.5. It will be seen that, for the 90% confidence level according to Appendix 2-A, the period between recalibration will be between 2-1/2 to 5 years for the former case and that the collision risk is well below the lower target level even if no recalibrations of the static pressure error are made in the latter case.

2.2.6 It is possible to use well established aerodynamic knowledge to help resolve the problem of possible large static errors and to devise a method of calculating the possible effect of static errors on P_Z(1.000) based on engineering arguments. Static source errors depend on the external aerodynamics of the aeroplane concerned. They depend in a not too sensitive way on the shape and alignment of the forward fuselage and wing surface which are controlled in modern production plants to quite close tolerances. They also depend in a much more sensitive way on defects and irregularities in the surface of the aircraft in close proximity to the static port, both forward and aft.

2.2.6.1 The manufacturers have a great deal of factual data on these local surface effects based on wind-tunnel data, aerodynamic theory and flight test. These establish that static source errors arising from local defects depend on the nature of the defect and on the ratio of its size to the distance of the defect from the static source. Based on these aerodynamic data the manufacturers have established tolerances for the production standards of the surface in the vicinity of the static ports, and these standards in turn ensure that the variation between the static source error in individual aircraft as they leave the manufacturer's plant are within acceptable limits. The Douglas Aircraft Company for example makes the following statement:

> "The position error of DC-8 aircraft which meet the established production level of aerodynamic cleanness will meet the established position error within ± 250 feet with a very high probability."

This claim has been substantiated without exception so far by flight calibrations made by the airlines using trailing cones.

2.2.6.2 This leaves the question of whether serious changes in calibration can arise in service and remain undetected for long periods of time. Apart from very temporary effects due to ice accumulations, such changes in calibration can only arise from in-service damage or maladjustment to the area of the fuselage in the vicinity of the ports. However, to cause large errors of the magnitude of 800 to 1 200 feet the damage would have to be quite substantial and immediately evident to the flight crew in its pre-flight inspection or to ground staff. For example, misalignment of the cargo door in the DC-8 is one possible source of static calibration error on that aircraft since the door is only roughly 20 inches from the static vents. However, to generate a static error of 800 feet, the door would have to be out of alignment by about two inches. Equally, sharp edged skin misalignment five inches from the static ports would need to be roughly one inch deep, or high, to generate an 800-feet error. Moreover, since the crew's instruments are connected into a line which cross-connects symmetrical static ports on both sides of the aircraft, the errors seen by the instruments would only be roughly half that generated by the defect. Thus an error seen by the instruments of 400 feet would be associated with defects in the order of one or two inches in magnitude.

2.2.6.3 Since the effect of defects of a given type on static errors basically depends on the ratio of the total depth of the defect to the distance away from the static port, it is of course possible that much smaller defects than those cited above could have similar effects if they are sufficiently close to the static vent itself. Thus a skin lap defect 1/10 inch in height only 1/2 inch from the port would have approximately the same effect as the example cited, and even smaller defects much closer to the port could have similar effects. Although these very small defects very close to the ports are much less likely to be picked up by visual inspection, except during planned aircraft maintenance checks, protection against this type of large static source error is given by the fact that such a defect, by definition, has to be extremely close to one of the static ports and therefore must be remote from the other static port. Therefore, this type of defect is immediately shown up in a marked difference between the pilot and copilot instrument reading.

2.2.6.4 It was therefore suggested that in addition to the calculations set out in Appendix 2-A.3, P_z(1000) should also be recalculated on the basis that defects of the magnitude cited above would be detected and rectified within a matter of a few flying hours and that defects of smaller magnitude would be rectified perhaps over a long period of flying hours. It was therefore agreed that as a supplementary calculation to the calculations already presented, P_z(std) and the consequent values of N_{az} should also be calculated on the basis that errors of $\pm$ 800 feet would be detected and rectified on the first return to a main base - say within 15 hours of occurrence; errors of $\pm$ 400 feet would be detected and rectified within one month, say 300 hours flying; and that errors of the magnitude inherent in production aircraft $\pm$ 250 ft. would remain undetected until trailing cone calibration was carried out. The Royal Aircraft Establishment (RAE) of the UK was requested to develop a distribution of error and duration before rectification based on these guidelines.

2.2.6.5 In addition to the safeguards against persistent large errors in static calibration which are given by simple visual inspection of the static port area, a number of other factors were discussed which would enable the flight crew or the airline, from its analysis of flight records, to become aware of the existence of a large error. Details of these are given in Appendix 2-A.6 and are briefly enumerated below:

i) Errors in the static source affect not only the altimeter but also the airspeed indicator and Mach meter. Errors in static source therefore lead to erroneous indicated Mach numbers which in significant cases will show up in markedly excessive fuel flow or, on some occasions, in unusual handling properties.

ii) The differences in pilot's and copilot's indications will in some cases highlight a defect affecting one system more than the other.

iii) Pitch attitude - in extreme cases errors in apparent airspeed and Mach number - will reflect in a detectable discrepancy in the pitch attitude.

2.2.6.6 A number of other possible indications which might lead to the detection of static errors were also mentioned, such as instrument fluctuations, unusual behaviour during take-off and landing, discrepancies in wind speed between flight records and forecasts, pitch trim compensation, near misses, and so on. While the Group agreed that these could on some occasions be useful indications, they were not regarded as of sufficient significance to materially affect the issue.

2.3 The estimate for the probability of lateral overlap $P_y(0)$

2.3.1 The estimate of $P_y(0)$ given in Appendix 2-A is based on the track-keeping accuracy during the 1967-68 data collection. The Group recognized that an increase in track-keeping accuracy would increase the collision risk due to loss of vertical separation, because there was an increased probability that aircraft would collide if vertical separation should be lost. It was recognized that there might be a large increase in track-keeping accuracy in the near future, especially if inertial navigation systems come into more general use. It seems probable that this increase in track keeping accuracy would be accompanied by a decrease in $|\bar{\dot{y}}|$, which would counteract the increase in collision risk in some measure. The Group decided that it could not at the present time estimate a value of $P_y(0)$, which would be representative of future track-keeping accuracy. It requested further studies on this subject.

2.3.2 In the transition regions, where the aircraft are using ground-based navigation such as VOR/DME, the value of $P_y(0)$ could be appreciably higher than for the oceanic regions. If any flights separated vertically by 1 000 ft. should be continued into these regions, thus the collision risk could increase considerably although the period of time would be much shorter than that flown under oceanic conditions.

2.4 Estimates of $|\bar{\dot{y}}|(0)$

2.4.1 A question was raised whether or not a possible "hunting" mechanism associated primarily with non-Doppler navigation might raise $|\bar{\dot{y}}|(0)$ appreciably above the measured values. Such an assumed oscillatory behavior of an aircraft which is on-course was envisaged as a relatively high-frequency correction about track. From previous analysis on the effect of phugoid oscillations on $|\bar{\dot{z}}|$, it is known that any such contribution to $|\bar{\dot{y}}|$ would be proportioned to the product of the amplitude of such oscillations and their frequency. It was pointed out that two possibilities could exist: (1) if the amplitudes are sufficiently small to remain undetected in the smoothing during the processing of the radar measurements, then their net effect to increase $|\bar{\dot{y}}|$ would also be small, or (2) if sufficiently large to influence $|\bar{\dot{y}}|$ appreciably, then this contribution should be detectable by the radar. From an examination of the results of the 1968 radar measurements, as processed by RAE, it appears that case (2) has occurred. Both RAE curves showing a high correlation for $|\bar{\dot{y}}|$ with y (NAT/SPG Summary/4, pages 1-A-19 & 1-A-20) clearly indicate an appreciable non-zero residual for $|\bar{\dot{y}}|$ at $y = 0$. Measured values are 20 knots and 15 knots at 60 minutes and 15 minutes respectively for longitudinal separation between a pair of aircraft. By a slight extrapolation, this gives $|\bar{\dot{y}}|(0) = 11$ knots for zero longitudinal separation, showing that this residual "hunting" process for aircraft on-track contributes, on the average, about 11 knots to $|\bar{\dot{y}}|$. This value has already been included in all the NAT/SPG risk calculations. If this effect had been negligible, then the RAE correlation curves would have passed through the origin.

2.4.2 The question arose because of conflict which appeared to arise out of what seemed like a large collision risk when one aircraft in a staggered system passed between two other aircraft, one 1000 feet above and one 1000 feet below and what seemed like a low collision risk when one aircraft cruised permanently between two other aircraft, one 1000 feet above and one 1000 feet below (this section). Analysis shows that the mathematical model represented in Appendix 2-A was roughly equivalent to asserting that aircraft on the same track (but different flight levels) crossed laterally (i.e., had lateral overlap) on the average less than twice per ocean crossing. This appeared to conflict with pilot intuition and experience; for example, pilots report that the aircraft below on the same track "wanders" left and right with much higher frequency. This seems especially likely if one aircraft has Doppler with computer coupled to the autopilot and the other does not. However:

- Such apparent crossing of the observed aircraft may be an optical illusion due to change of heading of the aircraft above from which the observations are made.
- If aircraft really wander laterally with amplitudes of 20 miles or so (consistent with the P_y measured in 1967), such crossing frequency cannot be increased without an appropriate increase of $\overline{|\dot{y}|}$ beyond the measured 20 knots, which appears to be correct.
- If aircraft really wander laterally with much smaller amplitudes, then they would cross a datum line much more often, but would usually not overlap another aircraft at all.

In other words, subject to navigational accuracies measured in the 1967-68 data collection, the apparent conflict mentioned at the beginning of this section is not a genuine paradox. However, this analysis underlines the point (para. 2.3) that if lateral navigation improves, vertical collision risk increases.

2.5 Independence of Occupancy

The basic equation for determining collision risk involves multiplying several factors, which assumes (according to the theory of probability) stochastic independence. Attention was drawn to the possibility that aircraft on adjacent paths (horizontally or vertically) might not be introduced at random. While there is no evidence of non-randomness, nor even any theoretical reason to expect it, even an undetectably small dependence could have large effects, changing the effective value of the quantity $\frac{\lambda_x E_z}{S_x}$ in either direction. It was decided that no conclusion could be reached without detailed study, both analytical and operational.

2.6 Methods for measuring static pressure error

2.6.1 General

2.6.1.1 The Group briefly discussed its competency for discussion of this subject, which is also on the agenda of the Airworthiness Committee. It was found that the interests in investigating this same subject were different for the two bodies. The Group decided that it was completely free to discuss the subject as far as would be necessary in the accomplishment of its work programme, but that problems and conclusions should be brought to the attention of the Airworthiness Committee.

2.6.2 Trailing-cone method

2.6.2.1 A report (Ref. 2) analysed the results of all trailing cone-tests available to the Group (90 tests on 82 different aircraft). The conclusions of this report, which were provisional because of the smallness of the sample and because a large part of the measurements were made by non-standard trailing-cone systems did not so far appear to support the claims of high accuracy which are made for this method. One member suggested that it was possible that trailing-cone assemblies made according to the PAMC issued by the Airworthiness Committee, but produced by different manufacturers could give significantly different results. It was agreed that this should be brought to the attention of the Airworthiness Committee.

2.6.2.2 Experts on this subject, who attended the meeting, stated that about 1000 trailing-cone tests had been made by a number of manufacturers. These included comparisons with other methods for measuring static pressure errors:

- tower method (at low altitude)
- trailing bombs
- radar method
- kine-theodolite method

These experts stated that these results proved the high accuracy obtainable by this method. The Group requested that as much as possible of this data should be made available to its members. Further conclusions of the Group will only be made after these new data have been analysed.

2.6.3 Performance check method

2.6.3.1 A report (Ref.3) showed results of performance-check analysis of two groups of 9 aircraft of one type, operated by two different airlines, and of two groups of 9 aircraft of two different types operated by a third airline. It was pointed out that this was much less than what had been requested by the Group during its Third Meeting: ten types of aircraft operated by ten different operators. This was mainly due to lack of response of the operators to make available data for the evaluation programme. The Netherlands Member stated that he could only continue work on this programme if the Group stated an interest in the method and if more operators supplied data. In response, the Group concluded that continuation of these tests cannot be justified from the point of view of effecting a reduction in vertical separation in the NAT Region.

2.7 Conclusions

1. The Group concluded that there was at the present time insufficient evidence for showing that a 1000 feet vertical separation standard would be safe for application on the present track system over the North Atlantic.

2. If further progress is to be made in this direction, the following actions would be necessary:

   - the aerodynamic effects of different types of damage or misalignment on the static pressure error should be further investigated.

   - efforts should be made to establish a relation between the magnitude of a static pressure error and the period before it was detected. This would be the basis for a more realistic tail shape for the static pressure error.

   - an estimate should be made of the central part of the distribution of track-keeping errors in the near future in order to obtain a realistic estimate of $P_y(0)$.

   - an effort should be made to obtain data on all errors which are more representative of the total population of aircraft types and operators flying across the North Atlantic.

3. States should be encouraged to make measurements of total altitude error as described in para. 2.2.

APPENDIX 2-A

1. THE COLLISION RISK EQUATION

1.1 Collision risks are estimated from an equation similar to that agreed by the group for the lateral case (equation (2) of Ref.1). The corresponding equation for the vertical case is

$$N_{az} = 10^7 P_y(0) \cdot P_z(\text{std}) \cdot \frac{\lambda_x}{S_x} \left\{ E_z(\text{same}) \left[\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] + E_z(\text{opp}) \left[\frac{\bar{V}}{\lambda_x} + \frac{\overline{|\dot{y}|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] \right\} \quad (1)$$

1.2 N_{az} is the expected number of accidents per 10 million flying hours, due to the failure of vertical separation. Each of the terms on the right-hand side of equation (1) is defined and discussed briefly below. One collision is counted as two accidents.

1.3 $P_y(0)$: Probability of lateral overlap

1.3.1 $P_y(0)$ is the probability of lateral overlap (i.e. lateral separation less than a wing-span) for a pair of aircraft assigned to the same track. This quantity was calculated from a distribution of track-keeping errors calculated at the 4th NAT/SPG Meeting as representing the navigation performance over the area. The distribution, calculated as described in sections 1.4.3.1.2 and 1.4.3.1.3 of Ref.1 is a weighted average of distributions observed by different radars. From this distribution the value $P_y(0) = 0.0012$ was calculated.

1.3.2 The fact that this value is so small shows that aircraft derive considerable protection from "vertical collisions" because of their poor track-keeping ability in the NAT region. The risks would be larger if navigation were more precise. The increasing use of Doppler would increase the risk only slightly, but the effects of introducing inertial navigation may be very large. Some thought should be given to the beginning and end of each track where aircraft will navigate more accurately by the use of VOR, although this will be outside the oceanic control area. It is possible $p_z(0)$ may have been slightly underestimated owing to radar measurement errors.

1.4 E_z(same): Same direction vertical occupancy

1.4.1 E_z(same) is defined as the average number of same-direction aircraft occupying segments of length $2S_x$ on the flight levels adjacent to that of the typical aircraft, on the same track.

1.4.2 E_z(same) was represented by the probability distribution shown in Fig.9 of Ref.2, taking the average daily flow as 280, in accordance with traffic forecast for the period 1969-74.

1.4.3 For the simple calculations in section 2 the central estimate (50% value) of 0.73 was used.

1.4.4 E_z(same) would be increased if aircraft could be packed more closely onto each path either by reducing the longitudinal separation standard or by improving the accuracy of boundary ETAs thus enabling the existing standard to be used more efficiently.

1.4.5 E_z(same) could be either increased or decreased as a result of the increased number of flight levels which would be made available by reducing the vertical separation standard. Appendix 2-A.6 shows that the effect is likely to be small, within the range ±20%.

1.4.6 No allowance was made for any increase in E_z(same) arising as described in the preceding two paragraphs. Since the collision risk is directly proportional to E_z(same) the results could be modified without difficulty.

1.5 E_z(opp): Opposite direction lateral occupancy

1.5.1 E_z(opp) is defined as the average number of opposite-direction aircraft occupying segments of length $2S_x$ of the flight levels adjacent to that of the typical aircraft, on the same track.

1.5.2 In section 4 E_z(opp) was represented by the 50% point of the probability distribution shown in Fig.10 of Ref.2, again taking the forecast average daily flow as 280, thus calculating a value of 0.02.

1.5.3 In calculations where the 1000 ft standard is not applied to opposite-direction traffic, E_z(opp) is given a value of zero.

1.6 S_x: Along track criterion of proximity

1.6.1 The collision risk depends on the ratios E_z/S_x and these ratios are almost independent of S_x. Thus S_x may be chosen almost arbitrarily. S_x was taken as 120 NM.

1.7 $\overline{|\Delta V|}$: Average relative speed

$\overline{|\Delta V|}$ is the average relative along track speed of a pair of same-direction aircraft colliding nose-to-tail. A value of 13 knots was taken, as agreed for the lateral case in paragraph 1.4.2.4.2 of Ref. 1.

1.8 P_z(std): Probability of vertical overlap

P_z(std) is the probability of vertical overlap (vertical separation less than aircraft thickness) for a pair of aircraft assigned to adjacent flight levels. The question of numerical values for P_z(std) will be discussed later in this paper.

1.9 λ_x: Aircraft length

The collision risk equation in Ref.3 used a parameter Λ_x which was intended to represent the length of an average aircraft, adjusted if necessary to allow for fatal accidents caused by vortex penetration. The evidence from results of flight tests conducted by the United States and Canada did not reveal significant hazards. Further investigation is required before firm conclusions can be reached. The NAT/SPG agreed at its 4th Meeting to assume that the vortex hazard is negligible and take Λ_x as equal to λ_x the metallic length of an average aircraft which is taken as 0.025 NM (150 ft) as specified in paragraph 4.15.13.1 of Ref.3.

1.10 $\overline{|\dot{z}|}$: Average relative vertical speed

$\overline{|\dot{z}|}$ is the average relative vertical speed of two aircraft about to collide. $\overline{|\dot{z}|}$ is taken as 1 knot as specified in paragraph 4.4.3.2.3 of Ref. 1.

1.11 $\overline{|\dot{y}|}$: Average relative across track speed

A value of $\overline{|\dot{y}|}$ can be derived from Figs.1.16 and 1.17 of Ref.1, taking a value appropriate to a pair of aircraft which have not lost any lateral separation. A value of 20 knots is taken as agreed[1] in paragraph 4.4.2.1 for the longitudinal case.

1.12 λ_y: Average effective wing-span

λ_y is taken as 0.025 NM (150 ft) as laid down in paragraph 4.15.13.1 of Ref. 3.

1.13 λ_z: Average effective height of an aircraft

λ_z is taken as 0.0066 NM (40 ft) as laid down in paragraph 4.15.13.1 of Ref.3.

2 SAME-DIRECTION AND OPPOSITE-DIRECTION RISK

2.1 Substituting the numerical values above into equation (1):

$$N_{az} = 10^7 \; 0.0012 \; P_z \; \frac{0.025}{120} \left\{ 0.73 \left[\frac{13}{0.05} + \frac{20}{0.05} + \frac{1}{0.0132} \right] + 0.02 \left[\frac{480}{0.025} + \frac{20}{0.05} + \frac{1}{0.0132} \right] \right\}$$

$$N_{az} = P_z \; (1340 + 980) \qquad (2)$$

$$N_{az} = 2320 \; P_z \qquad (3)$$

2.2 From equation (2) it can be seen that the 3% of flights which are opposite-direction traffic contribute 42% of the total risk. Because it seems unreasonable to allow a small proportion of the traffic to cause such a high proportion of the risk, the study concentrated on a system with 1000 ft separation applied only to same-direction traffic, thus effectively making E_z(opp) equal to zero. Substituting zero for E_z(opp) in equation (1) and the numerical values listed for all the other quantities in equation (1) it is found:

$$N_{az} = 1840 \; E_z(\text{same}) \; P_z(\text{std}) \quad . \qquad (4)$$

2.3 In section 1.4 a method has already been described of treating E_z(same) as a distributed variable which can be used for Monte Carlo calculations based on equation (4). In sections 3 and 4 the calculation of P_z(std) will be discussed.

3. COMPONENTS OF HEIGHT-KEEPING ERROR

3.1 Introduction

3.1.1 The problem of estimating P_z(std) corresponds to that of estimating P_y(std) for the lateral case, which was solved by direct measurement of a large number of lateral deviations from track over the North Atlantic Ocean.

3.1.2 The Group agreed that it is desirable that direct measurements of height-keeping should be made in the future. It noted, however, that the extremely high cost of such measurements and of their analysis would make it unlikely that sufficient data could be obtained by such measurements to define the tail of the distribution. It decided therefore, that the only possible approach to obtaining a distribution of the altitude errors was as follows:

3.1.3 By adopting the method of separately studying a number of effects which contribute to loss of vertical separation, one derives the distribution of the total effect by appropriate statistical calculations (convolutions). This method has the danger that some significant effect may be underestimated or even overlooked. In practice it is found, however, that the estimate for P_z(std) is dominated by the effects of static calibration errors and changes in assumptions regarding other sources of error have little effect.

3.1.4 The effects separately studied are:

(a) random instrument errors, i.e. altitude errors which are not common to the pilot's and co-pilot's altimetry systems in the same aircraft,

(b) flight technical errors, i.e. deviations from the assigned altitude which are indicated to the pilots,

(c) systematic altitude errors, i.e. all errors which are common to the pilot's and co-pilot's systems, except the static pressure calibration error. Four different error components of this type are considered in sections 3.3 to 3.6 of this appendix,

(d) the static pressure calibration error, i.e. the difference in altitude equivalent to the static pressure error, after application of the corrections automatically applied in the altimeter systems.

On the first 2 types of error a relatively abundant number of data is available. The systematic altitude errors include a number of effects which for a large part can be proved to have a negligible effect on the collision risk. For the static pressure calibration error a special method of analysis is introduced, in which as much information as possible is derived from the relatively scarce data.

3.1.5 It must be pointed out that the errors in the different groups are not entirely independent. For instance a part of the static pressure calibration error is included in the random instrument errors. This will have a slightly cautious effect on the estimate for $P_z(0)$.

3.2 Random instrument errors

3.2.1 ΔE (random altimetry) is defined as the separation error due to all those measuring errors which are not common to the pilot's and co-pilot's altimetry systems in the same aircraft. Leaks in the static system are included in this category. Data relating to this component were collected by IATA in 1965, analysed by R.A.E., and reported in Ref.4. Further data were collected by IFALPA in 1967. The IFALPA data for the North Atlantic area were consistent with the IATA data and were therefore combined with it as described in Appendix 2A.1 to form the central part of the required distribution.

3.2.2 This leaves the now familiar problem of fitting a tail to the distribution. In Ref.5 a level tail was fitted from 400 ft to 1200 ft. This is a very cautious assumption, because if very large differences between the pilot's and co-pilot's altimeters should occur, the pilots will in general either find out which altimeter is wrong or they will request a larger separation. Therefore the shape of this tail is extremely improbable. The area under the tail, i.e. the proportion of large errors, was taken as the 97½% upper confidence limit. However, the NAT/SPG subsequently decided that simply combining upper estimates for each "vague" parameter gave a too pessimistic result. To be consistent with the "Monte Carlo" approach adopted at the 2nd meeting the tail area should be represented by a parameter which would be drawn from an appropriate probability distribution.

3.2.3 If the model assumed that random instrument errors were the major factor in loss of separation this would be the only reasonable solution, but in Ref.5 it was shown that the results obtained by using the too pessimistic confidence level method were not very different from the optimistic results obtained by neglecting the tail area altogether. Thus any reasonable approximation to the "randomised" parameter representation of the tail area should be sufficiently accurate, and the tail area can be represented by the mean value of its "confidence distribution".

3.2.4 The distribution used is shown in Table 3.1.

Table 3.1 Distribution assumed for ΔE (random altimetry)

Size of error (ft)	Proportion
0 to 50	0.617 769
50 to 100	0.286 569
100 to 150	0.082 225
150 to 200	0.009 838
200 to 250	0.002 782
250 to 300	0.000 609
300 to 350	0.000 185
350 to 400	0.000 027
400 to 450	0.000 007
450 to 500	0.000 007
⋮	⋮
1150 to 1200	0.000 007
over 1200	0.0

This error distribution is assumed to be symmetric about zero, so that loss or gain of separation is equally likely.

3.3 Differences in instrument panel temperatures

3.3.1 It has been suggested that the temperature beneath the instrument panel in modern pressurised aircraft can reach high values and differences in instrument temperature between aircraft could contribute to loss of separation. In Ref.12 the results of temperature tests on 8 modern altimeters (3 type III altimeters and 5 type II altimeters) are published. The maximum errors found at cruising altitudes were a change of calibration of 150 ft for a temperature of 45°C below room temperature and a change of calibration of 100 ft for a temperature of 25°C above room temperature. In practice temperatures behind instrument panels are unlikely to differ by more than 10°C, corresponding to about 40 ft (or less, if the effect is non-linear). Also this difference was observed on a type II altimeter. For type III altimeters the effect was even smaller. As temperature differences of 10°C between aircraft will occur only on rare occasions and as the errors in the other altimeters were even smaller, the effect on the collision risk will be very small. They were therefore neglected in the calculations.

3.4 Altimeter drift and ageing

3.4.1 Altimeter drift. At one of the earlier meetings of the NAT/SPG it has been pointed out that altimeters tend to drift if they are held at high altitude for a considerable time. This effect has also been investigated and results reported in Ref.12. It was found that 2 altimeters purchased in 1955 showed a considerable drift during the first hour at 50000 ft, which continued with a decreasing rate after the first hour. After 4 hours the drift was 70 ft for one of the altimeters and 80 ft for the other altimeter. The other 8 altimeters, some of which were purchased in 1957, showed a drift of 5 ft (which is the reading accuracy) after 4 hours at 50000 ft altitude. This would indicate that the effect of drift need hardly to be considered for the types of altimeters used for crossing the North Atlantic. Drift was therefore neglected in the calculations.

3.4.2 Altimeter ageing. In Ref.11 some tests are reported on altimeter ageing, i.e. a gradual systematic change of calibration during the period between overhauls. This effect could become important if the 2 altimeters in the aircraft had not been changed for a considerable time. For this purpose KLM executed for some time calibrations of the altimeters coming back to the workshop, before they were overhauled. Although in some cases the altimeters showed a considerable change in calibration (which may have been the cause for sending them back to the workshop), the average values did not show a significant trend to change in a specific direction. This would indicate that the ageing effects are random and are therefore included in the data on random altimeter errors discussed in section 5.2.

3.5 Deviation of outside air temperatures from ISA values

3.5.1 The separation standard applied would not be a geometric one of 1000 ft but a barometric one of 10 flight levels equivalent to 1000 ft at ISA temperature. While on the average the resultant geometric separation would be about 1000 ft, on cold days it would be equivalent to a smaller distance and on warm days to a larger one. The Institute of Aeronautical and Space Research of the Netherlands (NLR) has calculated a distribution for the resultant loss or gain of separation. This distribution is shown in Table 3.2. This component was assumed to be independent of other components. This is a pessimistic assumption since most of the other error components would tend to be smaller on a cold day, being in fact errors in pressure altitude.

3.5.2 Details of the method of calculation are described in Appendix 2-A.2.

Table 3.2

Distribution of loss of separation due to ambient temperature variations

(NAT Region)

Loss of separation (ft)	Proportion
-200 to -150	0.000 001
-150 to -100	0.000 967
-100 to -50	0.074 961
-50 to 0	0.516 307
0 to 50	0.379 043
50 to 100	0.028 536
100 to 150	0.000 180

3.6 Errors in altimeter calibration

3.6.1 Another possible source of error lies in the calibration of altimeters. Such errors would not always be reflected in the P-CP error distribution since often both altimeters would have been calibrated in the same workshop possibly with the same calibration equipment. In the past some large errors have occurred in calibration (not necessarily to North Atlantic aircraft) but there is no technical difficulty in obtaining reference equipment of adequate performance, and the problem of ensuring correct procedures appears no more difficult than the equivalent problems for engine or control system maintenance.

3.6.2 The pilots of all civil airlines check their altimeters against the control tower QNH and usually QFE readings at zero altitude and zero speed. This provides a safeguard against some types of large calibration errors.

3.6.3 It seems that this type of error can be made negligible by suitable procedures and accordingly it has been neglected in the calculations.

3.7 Flight technical error

3.7.1 ΔE (flight technical) is defined as the separation error resulting from the "flight technical errors" of a pair of aircraft, i.e. due to the deviation from the required flight levels as indicated to the pilots.

3.7.2 Relatively abundant data is available on flight technical errors, from the following sources:

(a) The United States NASA flight recorder study[6] of deviations from steady cruise level by transport aircraft on North Atlantic and other routes. These cover some 5500 hours of jet operations.

(b) The deviations from assigned flight level inferred from pilots' reports in the 1965 flight deck data collection carried out by IATA at the request of the 1965 Special NAT Meeting, and described in Ref.4.

(c) Data collected by IFALPA in 1967.

Fig.1 shows a comparison between the distributions obtained from each source.

3.7.3 In Ref.5 it was found that the effect on P_z (and therefore on collision risk) of choosing one set of data rather than another is very small, provided 1200 ft tails are used for the static calibration error. Thus it is unnecessary to worry over much about the relative merits of the data and one can stay on the safe side by calculating the tails of the distribution from the NASA data collection. The centre of the distribution, which has even less effect on the final result, was taken from sources (b) and (c). Table 3.3 shows the distribution calculated. Appendix 2A.3 describes the method of calculation in more detail.

Table 3.3

Distribution calculated for flight technical errors

Size of errors (ft)	Proportion	Size of errors (ft)	Proportion
0 to 50	0.814 933	600 to 650	0.000 020
50 to 100	0.149 072	650 to 700	0.000 015
100 to 150	0.029 657	700 to 750	0.000 012
150 to 200	0.004 885	750 to 800	0.000 010
200 to 250	0.000 570	800 to 850	0.000 008
250 to 300	0.000 280	850 to 900	0.000 004
300 to 350	0.000 191	900 to 950	0.000 004
350 to 400	0.000 116	950 to 1000	0.000 003
400 to 450	0.000 083	1000 to 1050	0.000 003
450 to 500	0.000 062	1050 to 1100	0.000 002
500 to 550	0.000 035	1100 to 1150	0.000 001
550 to 600	0.000 029	1150 to 1200	0.000 001
		over 1200	0.0

3.7.4 ΔE (flight technical) is the difference of two independent flight technical errors from the distribution shown in Table 3.3. The distribution of ΔE (flight technical) was therefore calculated by convolving the distribution of Table 3.3 with itself. Table 3.4 shows the result of this convolution.

Table 3.4

Distribution of ΔE (flight technical)

Size of error (ft)	Proportion	Size of error (ft)	Proportion
0 to 50	0.572 676	650 to 700	0.000 032
50 to 100	0.302 255	700 to 750	0.000 025
100 to 150	0.092 877	750 to 800	0.000 020
150 to 200	0.024 135	800 to 850	0.000 015
200 to 250	0.005 404	850 to 900	0.000 011
250 to 300	0.001 263	900 to 950	0.000 008
300 to 350	0.000 484	950 to 1000	0.000 007
350 to 400	0.000 274	1000 to 1050	0.000 005
400 to 450	0.000 181	1050 to 1100	0.000 004
450 to 500	0.000 125	1100 to 1150	0.000 003
500 to 550	0.000 085	1150 to 1200	0.000 002
550 to 600	0.000 059	1200 to 1250	0.000 001
600 to 650	0.000 043	over 1250	0.0

3.8 Static pressure calibration error

3.8.1 The static pressure error is defined here as the difference in altitude, equivalent to the difference between the pressure in the static system near the altimeter and the pressure of the undisturbed air at the level flown. The static pressure calibration error is this static pressure error, corrected for any automatic or card corrections which are being applied to the altimeter indication during flight. This error includes:

(a) errors in the initial calibration by which the manufacturers determined the curves for the static pressure error under all kinds of flight regimes for a few prototype aircraft;

(b) errors due to production line variations, i.e. deviations of the static pressure error of the individual production aircraft from that measured on the prototypes;

(c) errors in applying the manufacturer's curves: although data for all kinds of flight regimes are supplied by the manufacturer, the applied corrections are in general only functions of Mach number and altitude, or of altitude alone (Smith's law). Effects due to yaw, incidence and major changes in configurations are generally neglected;

(d) errors due to in-service changes to the skin of the aircraft which do not give rise to a differential error between the two altimeters. These include accidental damage at or near the static vents or changes in fuselage shape in the neighbourhood of the static holes.

3.8.2 The effect of most of these types of error would be persistent: all errors of types (a) and (b) and most of those of type (c) will persist throughout the service life of the aircraft, unless definite action is taken to correct them. The same applies for many errors of type (d), except that these occur from the moment on when some permanent damage has been done to the fuselage. These persistent types of error will have the most significant influence on the vertical collision risk.

3.8.3 Some static pressure errors will, however, be transient. This would apply, for instance, to effects due to yaw and to effects caused by ice or dirt collected on the fuselage near the static holes. An example of such a transient error is described in section 16 of Appendix 2A.6. In general the effect of such transient errors on the collision risk will be negligibly small, because of their short duration.

3.8.4 It is, however, not completely certain that there may not be transient errors which could occur so often that they might make a significant contribution to the collision risk. One possible cause would be distortion of the fuselage under specific load conditions which will not occur when trailing-cone tests are being made on the aircraft. Significant errors of this type are, however, extremely improbable because they should have been detected during the very complete test series made by the manufacturer during the prototype tests. Another possible error of this kind was found in one type of aircraft regularly used in North Atlantic traffic. On this aircraft a cargo door is located at a very short distance behind the static holes. This door is normally recessed slightly below the aircraft surface at ground level, and should be flush with the fuselage when the pressure difference between inside and outside fuselage has its normal cruising value. It was found that misadjustment of this door will cause a difference in static pressure between 2 of the static holes in front of it, and this can be used for verifying during test flights whether the adjustment is correct. The real value of the static pressure caused by this effect is unknown. Trailing-cone tests will be made during the same flight, after the door position has been checked. If this door position should change shortly after this test flight, the corresponding static pressure error would never be detected by trailing-cone tests but would persist for the remainder of the period between overhauls.

3.8.5 In general it can be said that transient static pressure errors are either very infrequent or very improbable. In the calculation in which the distribution of the persistent static pressure errors is assumed to have a level tail to 1200 ft, the effect of such transient errors will have a negligible influence on the collision risk. It was, therefore, neglected in the calculations.

3.8.6 Estimates of static calibration errors have been made by the "trailing cone" method. 72 such measurements have been reported in Ref.10, and 90 (including these 72) in Ref.7.

3.8.7 In Ref.5 the central part of the distribution was represented by a uniform distribution between +300 and -300 ft. In the light of the ATA data it is possible to replace this by a more realistic distribution based on the trailing cone tests. These data may not be representative of the total population of aircraft flying the North Atlantic, but on the other hand the measurements are pessimistic because they include the effects of measurement errors. The results of the calculations will not be very sensitive to the centre of the distribution. It is therefore possible to use the recorded values to construct the centre part of the distribution and this has been done. Table 3.5 shows the actual values used.

Table 3.5

Distribution of static calibration error from ATA trailing cone tests

Size of error (ft)	Proportion
0 to 50	0.215 768
50 to 100	0.284 232
100 to 150	0.251 037
150 to 200	0.201 245
200 to 250	0.037 344
250 to 300	0.010 373
over 300	0.0

3.8.8 Because persistent large errors in static calibration by definition persist for a number of flights, measurements on the same airframe at different dates are not "statistically independent" and for this reason amongst others it is difficult to build up the large sample of data required to demonstrate the required level of freedom from large errors of this type.

Instead collision risks are estimated for a system which takes advantage of the persistent nature of such errors by calibrating each aircraft so that it enters service with only a small static pressure measuring error, and is thereafter checked at fixed intervals of service life, every t years. It is assumed that in such a system all the remaining large errors would be of type (d) above. Let "MTBF" represent the mean time between shifts of 300 ft or more in the "pressure error" at cruising altitudes. Let R incidents per aircraft year be the rate at which such shifts occur so that R = 1/MTBF. The proportion of aircraft having large errors will vary from zero for aircraft which have just been checked to Rt for aircraft just about to be checked, so that the average proportion will be 0.5 Rt. In the absence of information on tail shape level tails, from 300 ft to 1200 ft are assumed. The assumption of level tails seems to be a realistic assumption for damage due to collision with service vehicles at airports. The value of 1200 ft is chosen as being a "worst case".

3.8.9 The modified distribution of static calibration errors, i.e. with the level tails added, is shown in Table 3.6.

Table 3.6

Distribution calculated for static calibration errors

Size of error (ft)	Proportion
0 to 50	0.215 768
50 to 100	0.284 232
100 to 150	0.251 037
150 to 200	0.201 245
200 to 250	0.037 344
250 to 300	0.010 373
300 to 350	0.02778 tR
350 to 400	0.02778 tR
.	.
.	.
.	.
1150 to 1200	0.02778 tR
over 1200	0.0

This error distribution is assumed to be symmetrical.

3.8.10 Collision risk calculations were performed for various values of t, to determine the checking interval which would be necessary to demonstrate achievement of the target safety levels.

3.8.11 There are arguments for assuming a more optimistic distribution of large static calibration errors on the grounds that very large SC errors are likely to be detected by other means before the routine check. (See section 2.2.7 under Agenda Item 2.)

3.8.12 Accurate information on the value of the shift rate R is not available. However, as was pointed out in Ref.5 the IATA radio-altimeter data collection[9] together with dates at which aircraft came into service provides evidence of some 250 aircraft years without a large static calibration error. Further evidence is available from the 1965 ATA trailing-cone tests[10] and from later tests. After collating this evidence, to avoid counting the same aircraft twice, there is evidence of about 500 aircraft years without large errors.

3.8.13 Assuming this evidence can be regarded as representative of all North Atlantic operators then the "shift rate" of large static calibration errors can be represented by a value drawn at random from a distribution. Appendix F shows that an exponential distribution with mean 1/500 = 0.002 is appropriate. It should be noted that the assumption that the freedom-from-large-errors information from the IATA data and trailing cone tests is applicable to all NAT Jet traffic is important since the collision rate is approximately proportional to the number of these large errors.

4. CALCULATION OF P_z(std)

4.1 Distribution of ΔE (static calibration)

ΔE (static calibration) is assumed to be the difference between two independent errors from the distribution defined in Table 3.5. The distribution of ΔE (static calibration) was therefore calculated by convolving Table 3.5 with itself. Table 4.1 shows the result. R denotes the shift rate and t the interval in years between (trailing-cone or similar) checks.

Table 4.1

Distribution of ΔE (static calibration)

Size of error (ft)	Proportion
0 to 50	0.223 373 + 0.000 144 tR + 0.013 505 t^2R^2
50 to 100	0.199 269 + 0.000 807 tR + 0.012 734 t^2R^2
100 to 150	0.167 678 + 0.004 121 tR + 0.011 962 t^2R^2
150 to 200	0.138 491 + 0.010 403 tR + 0.011 190 t^2R^2
200 to 250	0.112 393 + 0.017 838 tR + 0.010 418 t^2R^2
250 to 300	0.081 189 + 0.024 783 tR + 0.009 647 t^2R^2
300 to 350	0.047 973 + 0.030 777 tR + 0.008 875 t^2R^2
350 to 400	0.021 346 + 0.037 722 tR + 0.008 103 t^2R^2
400 to 450	0.006 452 + 0.045 157 tR + 0.007 331 t^2R^2
450 to 500	0.001 586 + 0.051 439 tR + 0.006 560 t^2R^2
500 to 550	0.000 221 + 0.054 753 tR + 0.005 788 t^2R^2
550 to 600	0.000 027 + 0.055 416 tR + 0.005 016 t^2R^2
600 to 650	0.055 560 tR + 0.004 437 t^2R^2
650 to 700	0.055 560 tR + 0.004 052 t^2R^2
700 to 750	0.055 560 tR + 0.003 666 t^2R^2
750 to 800	0.055 560 tR + 0.003 280 t^2R^2
800 to 850	0.055 560 tR + 0.002 894 t^2R^2
850 to 900	0.055 560 tR + 0.002 508 t^2R^2
900 to 950	0.055 416 tR + 0.002 508 t^2R^2
950 to 1000	0.054 753 tR + 0.002 894 t^2R^2
1000 to 1050	0.051 439 tR + 0.003 280 t^2R^2
1050 to 1100	0.045 157 tR + 0.003 666 t^2R^2
1100 to 1150	0.037 722 tR + 0.004 052 t^2R^2
1150 to 1200	0.030 777 tR + 0.004 437 t^2R^2
1200 to 1250	0.024 783 tR + 0.004 823 t^2R^2
1250 to 1300	0.017 838 tR + 0.005 209 t^2R^2
1300 to 1350	0.010 403 tR + 0.005 595 t^2R^2
1350 to 1400	0.004 121 tR + 0.005 981 t^2R^2
1400 to 1450	0.000 807 tR + 0.006 367 t^2R^2
1450 to 1500	0.000 144 tR + 0.006 753 t^2R^2
1500 to 1550	0.006 753 t^2R^2
1550 to 1600	0.006 367 t^2R^2
1600 to 1650	0.005 981 t^2R^2
1650 to 1700	0.005 595 t^2R^2
1700 to 1750	0.005 209 t^2R^2
1750 to 1800	0.004 823 t^2R^2

4.2 Distribution of total relative height error

The relative height error of a pair of aircraft is the sum of the components due to random altimetry, deviation of ambient air temperatures from ISA values, flight technical errors, and static calibration errors (plus other components which we have neglected). Assuming these components to be mutually independent, the distribution of the relative height error is found by convolving the distributions given in Tables 3.1, 3.2, 3.4, and 4.1. Table 4.2 shows the part of the results of this convolution which gives losses of separation (the part relating to gains of separation is obviously of less interest).

4.3 Probability of vertical overlap $P_z(1000)$

$P_z(1000)$, the probability that two aircraft nominally separated by 1000 ft, overlap vertically, is the probability that their loss of separation was in the range $1000 - \lambda_z$ to $1000 + \lambda_z$, i.e. 960 to 1040 ft. From Table 6.2 this is calculated to be

$$0.000\,012 + 0.039\,042\,tR + 0.002\,626\,t^2R^2 \quad . \qquad (5)$$

The distribution, of which Table 4.2 forms a part, is not suitable for calculating the collision risk associated with a separation standard of 1500 or 2000 ft because it is based on assumptions about the cut-off points of level tails of distributions (paragraphs 3.2.2 and 3.6.5 refer) which were chosen so as to be cautious in assessing 1000 ft separation; if the separation standard being assessed had been 1500 ft different cut-off points would have been chosen.

5. CALCULATION OF COLLISION RISK AND DISCUSSION OF RESULTS

5.1 Using equation (5) to substitute for $P_z(\text{std})$ in equation (4) one finds

$$N_{az} = 1840\,(0.000\,012 + 0.039\,042\,tR + 0.002\,626\,t^2R^2)\,E_z \qquad (6)$$

and so

$$N_{az} = (0.022 + 71.8\,tR + 4.8\,t^2R^2)\,E_z \qquad (7)$$

and this gives the collision risk N_{az} as a function of (i) the checking interval t, (ii) R, the "failure rate" of the static calibration system, which is represented by a Monte Carlo variable, and (iii) $E_z(\text{same})$ the vertical same-direction occupancy, which is represented by a Monte Carlo variable.

Table 4.2

Distribution of loss of vertical separation

Size of error (ft)	Proportion
0 to 50	$0.094\,955 + 0.001\,699\ tR + 0.006\,276\ t^2R^2$
50 to 100	$0.088\,979 + 0.002\,638\ tR + 0.006\,115\ t^2R^2$
100 to 150	$0.078\,994 + 0.004\,361\ tR + 0.005\,846\ t^2R^2$
150 to 200	$0.066\,577 + 0.006\,753\ tR + 0.005\,511\ t^2R^2$
200 to 250	$0.053\,126 + 0.009\,621\ tR + 0.005\,145\ t^2R^2$
250 to 300	$0.039\,777 + 0.012\,746\ tR + 0.004\,766\ t^2R^2$
300 to 350	$0.027\,574 + 0.015\,937\ tR + 0.004\,383\ t^2R^2$
350 to 400	$0.017\,451 + 0.019\,016\ tR + 0.004\,000\ t^2R^2$
400 to 450	$0.009\,971 + 0.021\,791\ tR + 0.003\,622\ t^2R^2$
450 to 500	$0.005\,112 + 0.024\,066\ tR + 0.003\,252\ t^2R^2$
500 to 550	$0.002\,352 + 0.025\,718\ tR + 0.002\,903\ t^2R^2$
550 to 600	$0.000\,981 + 0.026\,763\ tR + 0.002\,583\ t^2R^2$
600 to 650	$0.000\,380 + 0.027\,331\ tR + 0.002\,300\ t^2R^2$
650 to 700	$0.000\,145 + 0.027\,591\ tR + 0.002\,056\ t^2R^2$
700 to 750	$0.000\,060 + 0.027\,685\ tR + 0.001\,847\ t^2R^2$
750 to 800	$0.000\,030 + 0.027\,671\ tR + 0.001\,674\ t^2R^2$
800 to 850	$0.000\,018 + 0.027\,541\ tR + 0.001\,547\ t^2R^2$
850 to 900	$0.000\,013 + 0.027\,209\ tR + 0.001\,483\ t^2R^2$
900 to 950	$0.000\,010 + 0.026\,524\ tR + 0.001\,493\ t^2R^2$
950 to 1000	$0.000\,008 + 0.025\,318\ tR + 0.001\,574\ t^2R^2$
1000 to 1050	$0.000\,007 + 0.023\,485\ tR + 0.001\,708\ t^2R^2$
1050 to 1100	$0.000\,005 + 0.021\,052\ tR + 0.001\,875\ t^2R^2$
1100 to 1150	$0.000\,004 + 0.018\,171\ tR + 0.002\,058\ t^2R^2$
1150 to 1200	$0.000\,003 + 0.015\,041\ tR + 0.002\,247\ t^2R^2$
etcetera	etcetera

5.2 Since the appropriate distributions for R and E_z(same) have already been calculated, equation (7) can be used as the basis of a Monte Carlo calculation to find distributions of N_{az} for various values of t. Figs 2.2a and 2.2b show the results of such calculations, together with the NAT/SPG agreed target safety levels. Fig. 2.3 presents the results in a different way, showing the probabilities that a given checking interval t (years) will make it possible to meet the NAT/SPG target levels.

5.3 It is reasonable to suppose that, as further evidence from trailing cone checks becomes available, the estimated "shift rate" R will become larger, or at least more accurately defined, so that the checking interval t can be increased. When t becomes greater than the average service life of an aircraft it will no longer be necessary to re-check aircraft.

APPENDIX 2-A.1

DISTRIBUTION OF RANDOM ALTIMETRY ERRORS

1. The central portion of the distribution of ΔE (random altimetry) may be inferred from data collections in which simultaneous readings of pilots and co-pilots altimeter were recorded and distributions of the difference tabulated. IATA collected suitable data[10] for the North Atlantic area in 1965. IFALPA collected data on a world-wide basis in 1967. Table 1 shows two distributions, one inferred from the 1965 data and one from the part of the 1967 data which referred to the North Atlantic area. (Errors are worse in other areas.) An average (weighted according to the number of flights recorded) was taken as the centre portion of the distribution. The more important problem of the tail area is dealt with in sections 3.2.2 and 3.2.3 of Appendix 2A.

Table 1

Distributions of random altimetry (P-CP) errors

Size of errors (ft)	(1) From 1965 data	(2) From 1967 NAT data	(3) Weighted average
0 to 50	0.614 835	0.636 005	0.617 769
50 to 100	0.288 253	0.276 100	0.286 569
100 to 150	0.084 331	0.069 131	0.082 225
150 to 200	0.009 605	0.011 287	0.009 838
200 to 250	0.002 117	0.006 913	0.002 782
250 to 300	0.000 614	0.000 564	0.000 609
300 to 350	0.000 215	0.0	0.000 185
350 to 400	0.000 031	0.0	0.000 027
Number of observations	32586	3544	36130
Number of flights	11034	1775	12809

2. In Ref.4 it was found that large values of P-CP persisted for $1\frac{1}{3}$ flights, on average, so the equivalent number of independent observations is $\frac{3}{4}$ of the number of flights, or 9607.

APPENDIX 2-A.2

THE INFLUENCE OF AIR TEMPERATURE ON THE VERTICAL COLLISION RISK

1. Introduction

It has been suggested that the air temperature could have a significant influence on the vertical collision risk. This is because vertical separation is measured as a difference in pressure altitude, and the geometric altitude difference between aircraft for a given difference in pressure altitude will differ with the air temperature. This influence of air temperature on collision risk will be discussed in this Appendix. In this discussion a number of rough approximations will be used which could be refined by a more thorough analysis. It is believed, however, that the conclusions based on this approximate analysis are sufficiently definite to make it unnecessary to refine the analysis further.

2. Theory

A small difference in pressure altitude is related to the corresponding difference in geometric altitude by the equation

$$dH = \frac{273 + T}{273 + ISA}\, dh$$

or

$$dH = \left(1 + \frac{T - ISA}{273 + ISA}\right) dh \qquad (1)$$

where T is the air temperature in degrees centigrade and ISA the theoretical or standard temperature, h is the pressure altitude, and H is the geometric altitude. Strictly speaking the equation is only valid if H denotes the geopotential altitude but for the altitudes and altitude differences considered here the difference between geopotential and geometric altitude can be neglected.

For a finite difference Δh in pressure altitude, up to 1000 ft, the following equation is a sufficiently accurate approximation

$$\Delta H = r\,\Delta h \qquad (2)$$

where

$$r = \frac{T - ISA}{273 + ISA} \qquad (3)$$

3. Estimate of the probability distribution for r

3.1 An estimate of the probability distribution of r can be obtained from the data published in Ref.14. These maps are given, showing for January, April, July and October the average temperatures and the standard deviations of the temperatures at different altitudes over the earth.

3.2 Table 1 shows the average monthly temperatures for a number of regions over the North Atlantic read from the graphs of Ref.14. The regions are the same as those used in Fig.1.22 of Ref.1 for displaying the traffic densities over the North Atlantic Ocean. For each region the 4 monthly average temperatures are given at the 300 mb level (pressure altitude 30059 ft) and at the 200 mb level (pressure altitude 38644 ft).

Table 1

Average monthly air temperatures over the North Atlantic (degrees Kelvin)

Lat. (N)	Month	Longitude (degrees West)									
		300 mb					200 mb				
		55°	45°	35°	25°	15°	55°	45°	35°	25°	15°
60°	Jan.	220	220	220	222	222	219	220	220	219	217
	Apr.	222	225	225	225	225	223	223	223	223	223
	July	229	229	229	229	229	225	225	224	225	225
	Oct.	225	225	226	226	227	223	223	220	220	219
55°	Jan.	222	223	224	224	224	221	221	221	219	217
	Apr.	224	225	226	226	226	220	219	219	219	219
	July	230	230	230	230	231	223	223	223	223	223
	Oct.	226	227	228	228	228	222	221	220	219	218
50°	Jan.	224	224	224	224	224	221	221	219	217	215
	Apr.	224	226	226	227	227	221	219	218	217	217
	July	232	233	233	233	233	221	221	221	221	221
	Oct.	228	229	230	230	230	221	220	219	218	218
45°	Jan.	225	226	226	226	225	217	217	216	215	215
	Apr.	226	226	227	228	227	218	216	215	215	215
	July	235	234	234	234	235	219	219	219	219	219
	Oct.	231	231	231	231	231	219	218	217	217	217
40°	Jan.	228	228	228	227	227	215	215	215	214	214
	Apr.	228	229	229	229	229	215	215	214	215	215
	July	237	236	236	236	236	217	217	219	219	219
	Oct.	234	234	234	233	232	217	217	217	217	216

3.3 In order to determine the <u>average value of r per flight</u> these values must be weighted for the variation of traffic density with geographic position, for the time of the year and for the different flight altitudes. This has been done by using the following assumptions:

(a) the total amount of traffic in each geographic region has been assumed to be proportional to the number of minutes of flight given in each region in Figs.1.22 and 1.23 of Ref.1. This same assumption has been used in determining the "occupancy" figures for lateral separation in Ref.1 The relative traffic densities for each geographic region are given in Table 2.

Table 2

Normalised traffic density in different regions of the North Atlantic

Lat.	Longitude (°W)				
	55°	45°	35°	25°	15°
60°	0.0090	0.0222	0.0318	0.0306	0.0255
55°	0.0356	0.0760	0.0889	0.1045	0.0726
50°	0.0236	0.1223	0.0961	0.0770	0.0498
45°	0.0119	0.0327	0.0206	0.0148	0.0091
40°	0.0127	0.0075	0.0055	0.0094	0.0105

(b) The variation of traffic density over the year has been taken into account by assuming that 2/12 of all traffic will fly under the conditions for January, 3/12 under those for April, 4/12 under those of July and 3/12 under those of October. This is a good approximation to the distribution of traffic densities over the year provided by Shanwick ATCC to the NLR. It is assumed that this distribution is valid for each region over the North Atlantic.

(c) The temperature difference (T - ISA) between standard temperature and actual temperature has been assumed to vary linearly with altitude between the 300 mb and the 200 mb pressure levels. It is thought that this is a better assumption than to assume that the temperatures vary linearly with altitude because the tropopause lies often in the region between these 2 altitudes.

3.4 Under these assumptions average values of -0.002 at 300 mb and +0.016 at 200 mb were found for r. This means that on average a pressure altitude difference of 1000 ft is equivalent to a geometric altitude difference of 1007 ft.

3.5 In order to find the probability distribution for r, the variation of the temperatures about this average value must be taken into account. This variation can be broken down into 2 parts: the variation of the temperatures about the monthly average and the variation of the monthly averages about the yearly average. These can be estimated as follows:-

3.5.1 The standard deviations of the temperatures about the monthly averages are given in plates 45-76 of Ref.14. It is shown there that all distributions are nearly normal. The standard deviations over the North Atlantic are everywhere about 4°C at the 300 mb levels and about 6°C at the 200 mb level.

3.5.2 The distribution of the monthly average temperatures can be obtained from Table 1, if the frequency with which each value occurs is weighted by the relative traffic density in each region and during each season. If it is, as a first approximation, assumed that this distribution is normal, then the standard deviation can be estimated from the extreme values in the distribution. This would indicate a standard deviation of 3.5°C at the 300 mb level and 2.2°C at the 200 mb level.

3.6 The standard deviation of the complete distribution is equal to the square root of the sum of the squares of the 2 component standard deviations. After substitution in equation (3) it is found that the standard deviation of r is 0.023 at the 300 mb level and 0.029 at the 200 mb level. These values are probably somewhat on the small side, because the temperatures measured by the weather balloons were probably measured at fixed hours of the day, so that the variation over the day is not accurately included in the figures given above. They give, however, some indication of the magnitudes involved, and in the light of the following discussion this is thought to be sufficient for the present purpose.

3.7 In Table 3.2 of Appendix 2-A an approximate distribution for the total effect is given, assuming that the average effect is 7 ft and standard deviation 30 ft at all altitudes considered, and assuming that the shape of the distribution will be normal.

4. Discussion

4.1 In the previous section an approximate calculation was made of the difference between the geometric altitude difference and the pressure altitude difference between 2 aircraft at jet cruising altitudes. It was found that the geometric altitude difference is on the average 0.7% larger than the pressure altitude difference and that the standard deviation of the variation in the difference of these two is about 3% of the pressure altitude difference. Below the effect which these values can have on the collision risk due to loss of vertical separation will be analysed.

4.2 It must first be realised that almost all altitude errors are errors in pressure altitude, i.e. their magnitude in pressure altitude units is independent of air temperature. This is true for all static pressure errors and for all altimeter errors. It is possible that inertial effects may cause some of the dynamic motions in flight technical errors to be proportional to geometric altitude differences rather than to pressure altitude differences, but their proportion will be negligible.

4.3 Under these conditions it can be stated that the effect of the average value of r on the collision risk will be inversely proportional to r. In the case considered here this means that neglecting the temperature effect will cause an overestimate of the collision risk by 0.7%, which is certainly negligible.

4.4 The variations of the temperature about the average have, under the level-tail assumptions used for most of the altitude errors in the calculation of P_z (std), no effect at all on the value of the collision risk. The effects of positive and negative values of r will cancel out completely in the calculation.

5. Summary

5.1 A method has been indicated by which the effect of atmospheric temperature can be incorporated in the calculation of the collision risk due to loss of vertical separation.

5.2 It has been shown that the effect on the estimated collision risk is very small. It is therefore sufficient to calculate the effect by the approximate method described in this Appendix.

APPENDIX 2-A.3

DISTRIBUTION OF FLIGHT TECHNICAL ERRORS

1. Small flight technical errors

The NASA measurements[6] do not include the small errors which occur when initially "setting up" to the required steady cruise level. For this reason they give an optimistic picture of the small errors less than about 200 ft. Although it was shown in Ref.5 that the effect on the collision risk estimates is small, the centre of the distribution was calculated from the IATA data and the part of the IFALPA data which relates to the North Atlantic. Table 1 shows distributions from both these exercises, together with the "weighted average" distribution used.

Table 1

Distributions of small flight technical errors

Size of error (ft)	Proportions		
	IATA 1965	IFALPA 1967	Weighted average
0 to 50	0.820 629	0.762 556	0.814 933
50 to 100	0.145 338	0.183 409	0.149 072
100 to 150	0.028 448	0.040 773	0.029 657
150 to 200	0.004 266	0.010 581	0.004 885
Number of observations	32586	3544	36130

2. Large flight technical errors

Data on large flight technical errors is available from each of the three sources listed in section 3.7.2 of Appendix 2-A (NASA, IATA and IFALPA) although there is insufficient IFALPA data to estimate the shape of the "tails" for present purposes. If deviations of the indicated flight level from the required one, due to either pilot or autopilot, are likely to be transient, i.e. lasting a few minutes rather than several hours, then the numbers of effectively independent observations are of the following orders of magnitude:-

NASA 50000

IATA 33000

IFALPA 3500 (NAT region only)

3. The number of observations available from either the NASA or the IATA data would be sufficient to estimate the shape of the "tails" of the probability distribution. It can be seen from Fig.7 that the differences between the two sources are not large for errors between 200 and 600 ft, but the IATA data shows no errors over 600 ft.

4. The question thus arises: which, if either, of the two sets of data gives a true measure of large flight technical errors? On the one hand it may be argued that some of the large deviations recorded in the NASA study could have been made deliberately knowing that it was safe to do so, that conditions for accurate height-keeping were less favourable than they are now on the North Atlantic routes, and so on; i.e. that the NASA data would lead to unduly pessimistic estimates of flight technical errors in current North Atlantic operations. On the other hand, it may be argued that when aircrew are recording their own performance they are not operating in quite the usual way. The act of preparing to take an altimeter reading may itself lead to a greater than usual attention to altitude, and so to a reduced liklihood of large errors developing. On this argument the IATA data should be regarded as optimistic. If the computation of a safe standard depended on resolving this argument, a more closely controlled data collection would seem necessary, but fortunately this is not the case. The calculated safe standard depends much more on the assumptions used for estimating the incidence of large errors of pressure measurement (i.e. of static pressure calibration and to a lesser extent of "random altimetry") than upon the flight technical errors. Thus one can stay on the safe side by choosing to base the results on the NASA data (which shows the greater incidence of large deviations) without much affecting the final results. The effect on estimates of P_z of interchanging the data from, respectively, the NASA and the IATA collections was shown in Ref.5 to be small. This is mainly because the difference between the "tail" areas of the two distributions is small in comparison with the "tail" areas which have to be considered for the other component errors.

5. Accordingly the distribution of flight technical errors over 200 ft is taken from the NASA data, combining the autopilot and manual data in the proportions 993:7 since the IATA data showed that the autopilot was disengaged in only 0.7% of the observations.

Table 2

Distribution of large flight technical errors

Size of errors (ft)	Proportion
200 to 250	0.000 570
250 to 300	0.000 280
300 to 350	0.000 191
350 to 400	0.000 116
400 to 450	0.000 083
450 to 500	0.000 062
500 to 550	0.000 035
550 to 600	0.000 029
600 to 650	0.000 020
650 to 700	0.000 015
700 to 750	0.000 012
750 to 800	0.000 010
800 to 850	0.000 008
850 to 900	0.000 004
900 to 950	0.000 004
950 to 1000	0.000 003
1000 to 1050	0.000 003
1050 to 1100	0.000 002
1100 to 1150	0.000 001
1150 to 1200	0.000 001
over 1200	0.0

APPENDIX 2-A.4

DISTRIBUTIONS FOR OCCURRENCE RATE OF LARGE STATIC PRESSURE SHIFTS AND FOR TAIL AREA PARAMETER FOR STATIC CALIBRATION ERROR

1 Introduction

1.1 The sort of events which could cause a persistent large change in "pressure error" (e.g. collision with a service vehicle at an airport) are likely to occur at random and to conform to a "Poisson arrival pattern". It may therefore be assumed that the true probability P_n of n significant shifts occurring in a fixed period of T years is given by

$$P(n) = e^{-RT} (RT)^n/n! \qquad (1)$$

where R, the expected rate of shifts per year, is taken to be a constant which is characteristic of jet operations. The probability of n or fewer shifts is therefore

$$P(\leq n) = e^{-RT} (1 + RT + (RT)^2/2! + \ldots + (RT)^n/n!) \qquad (2)$$

1.2 The problem of finding a distribution for R is almost identical with that of finding a distribution for the tail area parameter β which occurred in the lateral separation case.

1.3 There is more than one method of deriving a probability distribution for a population parameter such as R from a distribution for an observable quantity such as the distribution given by equations (1) and (2). The philosophical differences between these methods are very subtle. The three possible methods are the confidence limit method, the Bayesian method, and the fiducial method. The consequences of all three methods have been explored as applied to equations (1) and (2) and all three led to the same solution. (This usually happens when only one parameter such as R is involved.)

1.4 A brief outline is given below of how each method was applied to the problem of finding a distribution of the shift rate R. (For a description of the philosophical background behind each method the reader should consult an advanced statistical text book for example Ref. 11.)

2 Confidence limit method

2.1 The confidence limit method consists essentially of the symbolic equation

$$\text{Prob(true rate} > R|n,T) = \text{Prob(observed number} \leq n|R,T) \quad . \qquad (3)$$

2.2 The method is designed so that if for example $R(0.10)$, the upper 10% value for R, is found by this method for a large number of problems then the proportion of problems for which the value of $R(0.10)$ found is less than the true value R is equal to or less than 10%. In this sense it is said that the probability that R is greater than $R(0.10)$ is equal to or less than 10%, thus defining one point on our cumulative distribution curve for R.

2.3 Using equation (2) to substitute for the right-hand side of (3) One has Prob(true rate > R)

$$= e^{-RT} (1 + RT + (RT)^2/2! + \ldots + (RT)^n/n!) \qquad (4)$$

and this defines the confidence distribution for the shift rate.

F.3 Bayesian method

3.1 In this method R is imagined to have a hypothetical "a priori" probability distribution before the experiment which is modified as a result of the experiment to give an "a posteriori" distribution which is taken as the distribution of R.

3.2 One takes the a priori distribution as a uniform distribution. The need for an arbitrary choice of this nature is the principal objection to the method.

3.3 The "a priori" probability density at each point is then multiplied by the "likelihood", i.e. by the probability of producing the observed result, and the "a posteriori" probability is proportional to this product.

3.4 Symbolically one writes

$$p(R)\, dR \propto \pi(R)\, dR \cdot P(n|R,T) \qquad (5)$$

where $p(R)$ is the a posteriori probability density, $\pi(R)$ is the a priori probability density which shall be taken as constant, and $P(n|R,T)$ the likelihood of observing n large errors, given by the Poisson distribution. Substituting the arbitrary distribution for $\pi(R)$, and the Poisson probability from equation (1) for $P(n|R,T)$ one has

$$p(R) = C\, e^{-RT}\, (RT)^n/n! \qquad (6)$$

3.5 The "constant of proportionality" C, is determined by the requirement that $p(R)$, as a probability density, must integrate to 1, i.e.

$$\int_0^{\infty} p(R)\, dR = 1 \qquad (7)$$

therefore

$$-\,C \left[\frac{e^{-RT}}{T} \left(1 + RT + \frac{(RT)^2}{2!} + \ldots + \frac{(RT)^n}{n!} \right) \right]_0^{\infty} = 1 \qquad (8)$$

therefore $C = T$ and equation becomes

$$p(R) = T\, e^{-RT}\, (RT)^n/n! \qquad (9)$$

3.6 The cumulative distribution curve for R is found as follows:-

$$P(> R) = \int_R^{\infty} p(x)\, dx$$

$$= \int_R^{\infty} T\, e^{-xT}\, (xT)^n/n!\, dx$$

$$= e^{-RT} \left[1 + RT + \frac{(RT)^2}{2!} + \ldots + \frac{(RT)^n}{n!} \right] \qquad (10)$$

which agrees with the result obtained from the confidence limit method.

4 Fiducial method

4.1 The essence of the fiducial method is that the distribution of R should depend only on the likelihood of obtaining the observed result, so that

$$p(R|n,T) = f(P(n|R,T)) \tag{11}$$

where f is some arbitrary function. It would seem reasonable for greater probabilities to be assigned to those values of R which make the observed number n most likely, so f should be a monotonically increasing function. For reasons which appear somewhat arbitrary the equation is replaced by

$$p(R|n,T) = C\, P(n|R,T) \tag{12}$$

i.e. the probability density is taken to be directly proportional to the likelihood, with constant of proportionality C. When the Poisson distribution term is substituted for $P(n|R,T)$ in equation (12), one obtains

$$p(R) = C\, e^{-RT}\, (RT)^n/n! \quad . \tag{13}$$

4.2 This equation is the same as equation (6) and the subsequent argument is the same as that for the Bayesian method. Again one ends up with the equation

$$P(> R) = e^{-RT}\left[1 + RT + \frac{(RT)^2}{2!} + \ldots + \frac{(RT)^n}{n!}\right] \quad . \tag{14}$$

5 No large errors observed

5.1 In the special case when $n = 0$, indicating that no large errors are observed in a period of T years, the equations take a simpler form. Equation (9) takes the form

$$p(R) = T\, e^{-RT} \tag{15}$$

and equation (10) takes the form

$$P(> R) = e^{-RT} \tag{16}$$

so that R has an exponential distribution with mean $1/T$.

APPENDIX 2-A.5

THE EFFECT OF ADDED FLIGHT PATHS ON OCCUPANCY

1 The occupancy figures given in Ref.1 are valid for the number of tracks and flight levels which were in use during the period in which the data were collected. If vertical separation should be reduced to 1000 ft over the North Atlantic Ocean, the number of possible flight paths for a given number of tracks would be approximately doubled. In this Appendix the effect of this large increase in flight paths on occupancy is investigated.

2 For a period in which the traffic flow is constant an analytic expression can be derived for the occupancy, which can be used conveniently for investigating changes in the flight path pattern. In this derivation the definitions given in Ref.2 are used. Let M be the number of aircraft which pass a certain point on a single track per hour, and m_i the number per hour passing the point on the i^{th} flight level on that track. If aircraft are regarded as proximate if they are on adjacent flight levels on the same track and within fifteen minutes flying time of each other longitudinally, then the average number of aircraft on the i^{th} level which are proximate to an aircraft on the (i-1)th level on the same track is $0.5\, m_i$. Taking length of the track as 3.25 hours flying time, the proximity per hour from these two paths will then be

$$T' = 3.25 \times 0.5\, m_i\, m_{i-1} \quad . \qquad (1)$$

For the p flight levels of this track the proximity will be

$$T = 1.625 \sum_{i=1}^{p} m_i\, m_{i-1} \quad . \qquad (2)$$

As the total number of flying hours made per hour is 3.25 M the occupancy during this hour for these p flight levels will be

$$E_z(p) = \frac{\sum_{i=2}^{p} m_i \cdot m_{i-1}}{M} \quad . \qquad (3)$$

The ratio of the occupancies for a track with p flight levels and for a track with q flight levels with the same number M of aircraft per hour will be

$$\frac{E_z(p)}{E_z(q)} = \frac{\sum_{i=2}^{p} m_i \cdot m_{i-1}}{\sum_{i=2}^{q} m_i \cdot m_{i-1}} \quad . \qquad (4)$$

3 If the vertical separation is reduced from 2000 ft to 1000 ft, then p will be equal to 2q. From equation (G-4) it can be found that, if the aircraft are divided equally between the flight levels, that

$$\frac{E_z(2q)}{E_z(q)} = \frac{2q - 1}{4(q - 1)} \quad . \qquad (5)$$

These ratios vary from 0.75 (if there were 2 flight levels per track at a separation of 2000 ft) to a limit value of 0.50 (for an infinite number of flight levels per track.

4 If the traffic is not equally divided over the flight levels of the track, then the value of the ratio will depend on the distribution of the traffic over the individual flight levels for 2000 ft separation, and on the method in which ATC redistributes the traffic over the flight levels with 1000 ft separation. In general the distribution of the traffic over the levels with 2000 ft separation will be peaked near the central levels. If ATC redistributes the traffic so, that the aircraft which originally occupied one flight level with 2000 ft separation is equally divided over this same flight level and the added flight level 1000 ft below it, then the ratio will in general be somewhat higher than would follow from equation (5) . This can be readily verified from simple calculations using equation (4) . In theory the ratio can vary from 0 (if the added tracks are not used at all) to 1 (if the traffic pattern is compressed vertically so that the outer levels at 1000 ft separation are not used). For the more likely ways of redistribution of the traffic over the added levels, the ratio between the occupancies for 1000 ft and for 2000 ft separation will be somewhere in the region between 0.7 and 0.9 if the total traffic per track remains unaltered. If the number of tracks is reduced from 4 to 3, then the occupancy will in general increase. The value of this increase will depend on the distribution of the traffic over the original

4 tracks and on the way in which ATC will redistribute the traffic over the 3 tracks. For the case where the traffic was originally equally divided over the four tracks and will also be equally divided over the 3 tracks, the occupancy will increase by a factor of about 4/3 if the flight levels remain unchanged in the redistribution. In that case the total ratio would become somewhat more than 1, i.e. the occupancy for 1000 ft separation might become some 10 to 20% larger than the occupancy for 2000 ft with the same total amount of traffic. If it will be standard practice to reduce the number of tracks if the separation is reduced, then the use of the same value for the occupancy will be optimistic. It would be more cautious to increase the value of E_z by some 20%.

5 It must also be pointed out that a reduction of the average longitudinal separation can influence the value of the vertical (and the lateral) occupancy. In the first place it can lead to further reduction of the number of tracks, which would lead to increased occupancy. But also if the number of tracks remains the same, the occupancy can increase considerably if ATC increases the traffic on the busiest tracks.

6 No allowance has been made for any increase in E_z(same) arising as described in this Appendix. Since the collision risk is directly proportional to E_z(same) the results obtained could easily be modified to comply with any other value of E_z (same) which might be agreed.

APPENDIX 2-A.6

ALTERNATIVE TAIL SHAPES FOR THE STATIC PRESSURE CALIBRATION ERROR DISTRIBUTION

1. In section 3.8.5 of Appendix 2-A it was assumed that the distribution of the static pressure calibration error has a "level tail". This can be regarded as a cautious representation of the true distribution. The upper limit of the distribution is taken as 1200 ft. As for the lateral separation case, this value is obtained by a maximisation process which gives the largest possible value for the collision risk for a given area of a level tail. This is a second and very cautious assumption about the tail shape.

2. There are, however, a number of reasons to expect that the larger values of the static pressure error, say those above 800 ft, would be detected within a few flights after the large static pressure error started and will never become "persistent errors". A list of such reasons is given below. Although it cannot be proved that each of these reasons separately will always lead to an early detection, it seems very likely that in every case that a large static pressure error occurs it will be detected by one or more of the following causes within the next few flights. When interpreting these causes, it must be kept in mind that a static pressure error of 800 ft at cruising altitude is equivalent to an error in Mach number of about 0.04. It should also be kept in mind that a static system always consists of 2 interconnected sets of holes, one on each side of the fuselage. An error of 800 ft due to damage on one side of the fuselage must therefore be caused by an error on one side equivalent to 1600 ft.

3. Ground inspection

Persistent static pressure errors of 800 ft and more can only occur if the fuselage near the static holes has been seriously damaged. Such damage should be plainly visible during a ground inspection by the crew or by ground mechanics. As visual inspection of the aircraft before each flight is required by most operators, such damage would be detected if not before the first flight after the damage had occurred, then at least very shortly afterwards.

4. Cruising performance

If the indicated Mach number is 0.04 lower than the actual Mach number, a DC-8 aircraft cruising at an indicated Mach number of 0.81 at 35000 ft would have a fuel consumption which is 29% higher than normal. At an indicated Mach number of 0.80 this would be 17%. These are, in fact, extreme cases of the effects used in the performance-check method for monitoring static pressure error (Ref.13). The unusual engine settings experienced during such a flight would probably be reported by the pilot.

If the indicated Mach number is too high by 0.04 these effects will be much less pronounced. Then the pilots would, however, be disturbed by the fact that the speed stability was much reduced and might even become negative.

5. Difference between pilot's and co-pilot's indications

Even if the static holes of these 2 systems are located very near to each other, it is extremely likely that the effect of damage will cause differences in the static pressures of the 2 systems. For the large static pressure errors considered here, this could easily become so large that the pilots would become alarmed by the difference in indication of the 2 altimeters, airspeed indicators and Machmeters in the cockpit. This would lead to investigations of the altimeters and static pressure systems.

6. Fluctuations

If the static pressure error is due to damage to the fuselage, the flow behind it will in some cases not be very smooth. This will lead to fluctuating indications, especially on the rate-of-climb indicators. The Group was inclined to doubt the importance of this.

7. Take-off and landing

If such large static pressure errors exist during cruising, it is extremely likely that the altimeters and airspeed indicators will also show unusual behaviour during take-off (causing take-off at the wrong speed for example) and initial climb, and during approach and landing.

8. Near miss reports

If a static pressure error in excess of 800 ft should exist, then the aircraft would become involved in near misses, some of which would be reported. The investigation of such near misses, especially if they occurred several times with the same aircraft, would almost certainly lead to the detection of the large static pressure error.

9. Besides the effects mentioned above, which will be obvious to any pilot, there are a number of effects which will alarm some pilots, while they are not detected by others. Some of these effects are mentioned below.

10. Wind speed

Wind speeds determined from computed true airspeed and Doppler ground speed would differ from the forecast winds. For a static pressure error of

800 ft the error in computed wind speed would be of the order of 20 KT. Although such errors may initially be attributed to forecasting errors, the pilots would probably become suspicious if such an error persisted during a complete flight.

11. <u>Pitch attitude</u>

Some pilots use pitch attitude as an important monitoring parameter and they will certainly detect the attitude errors of the order of 1°, associated with an error in Mach number of 0.04.

12. <u>Pitch trim compensation</u>

The pitch trim indicator will have an unusual position if its detector for static pressure is connected to static holes which are not affected in the same way by the static pressure error as the pilot's instruments.

13. <u>"Coffin corner"</u>

If a heavily loaded aircraft is climbing near its maximum altitude, the pilots will experience serious trouble. The speed range between stall speed and high-speed buffet becomes very small.

14. <u>Glide path interception</u>

If the static pressure shows an appreciable error at low altitudes and airspeeds, the pilots might observe that the point of interception of the glide path would be unusual.

15. <u>Flight recorders</u>

Computer analysis of flight recorder data could enable some operators to detect the effects of large static calibration errors.

16. The following experience by KLM illustrates some of the effects mentioned above. On one of their DC-8 aircraft ice ridges had formed in front of the static holes on the ground. During take-off the pilots remarked that the build-up of the speed was unusual. During the initial climb they mistrusted the artificial horizon, but checking against the true horizon proved it correct. The rate-of-climb indicators fluctuated between +1000 ft/MIN and - 1000 ft/MIN. The diagnosis that something was wrong with the static pressure was reached very soon after take-off.

17. The effects described above would make it almost certain that large static pressure errors would be detected by the crew very soon after they had started. For aircraft equipped with flight recorders, they would also become apparent during the analysis of the recordings.

18. In order to show the influence on the collision risk of taking into account the effects mentioned above, calculations have been made using level tails cut off at 800 ft. The tail area was made the same as for the other calculations. The results are shown in Fig. 2.4.

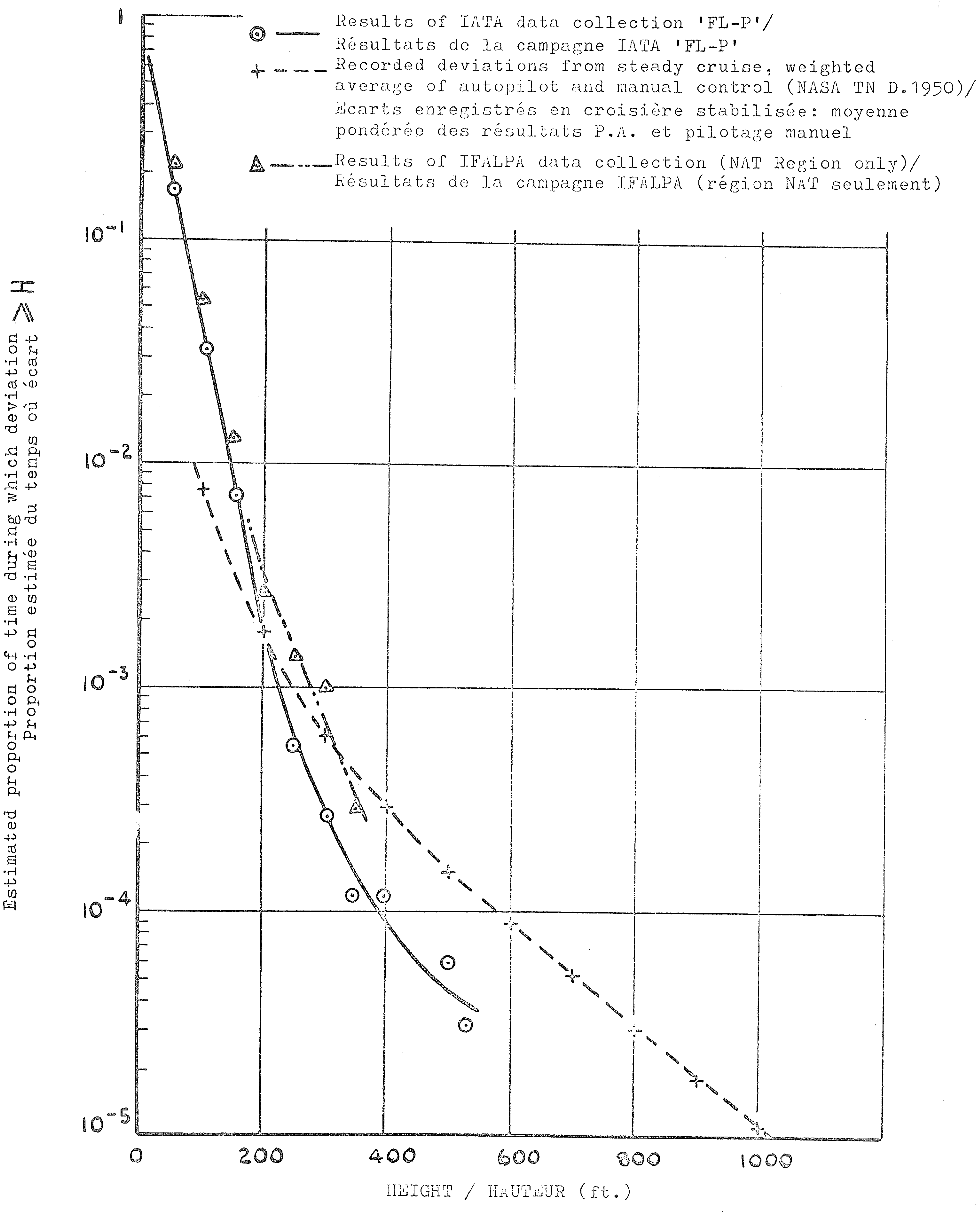

Fig. 2.1 DATA RELATING TO FLIGHT TECHNICAL ERRORS/
DONNEES SUR L'ERREUR "TECHNIQUE DE VOL"

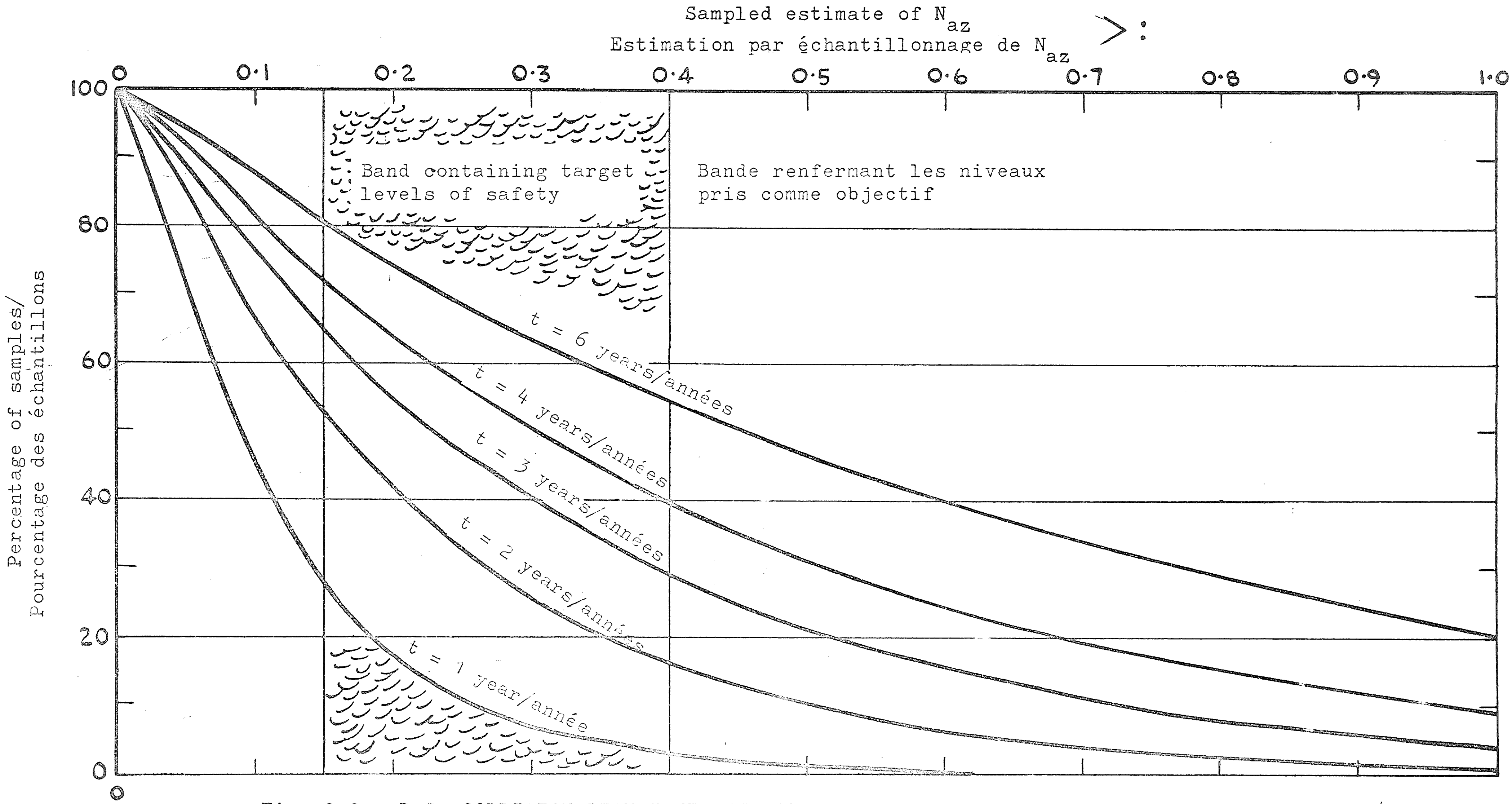

Fig. 2.2a D.S. COLLISION RISK ESTIMATES FOR DIFFERENT VALUES OF THE CHECKING INTERVAL t /

ESTIMATIONS "DS" DU RISQUE D'ABORDAGE POUR DIFFERENTES VALEURS DE L'INTERVALLE t ENTRE CONTROLES

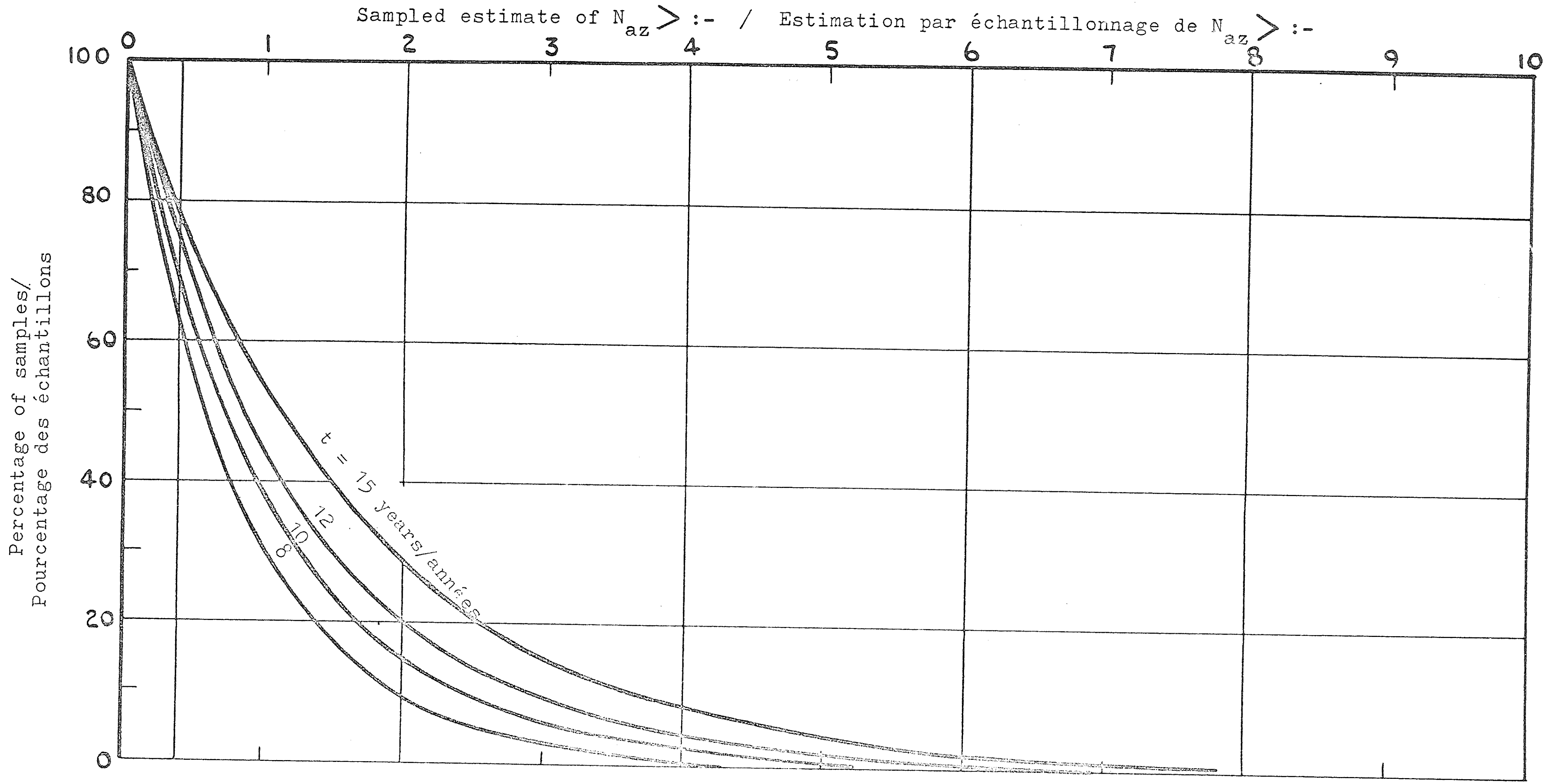

Fig. 2.2b D.S. COLLISION RISK ESTIMATES FOR DIFFERENT VALUES OF THE CHECKING INTERVAL t / ESTIMATIONS "DS" DU RISQUE D'ABORDAGE POUR DIFFERENTES VALEURS DE L'INTERVALLE t ENTRE CONTROLES

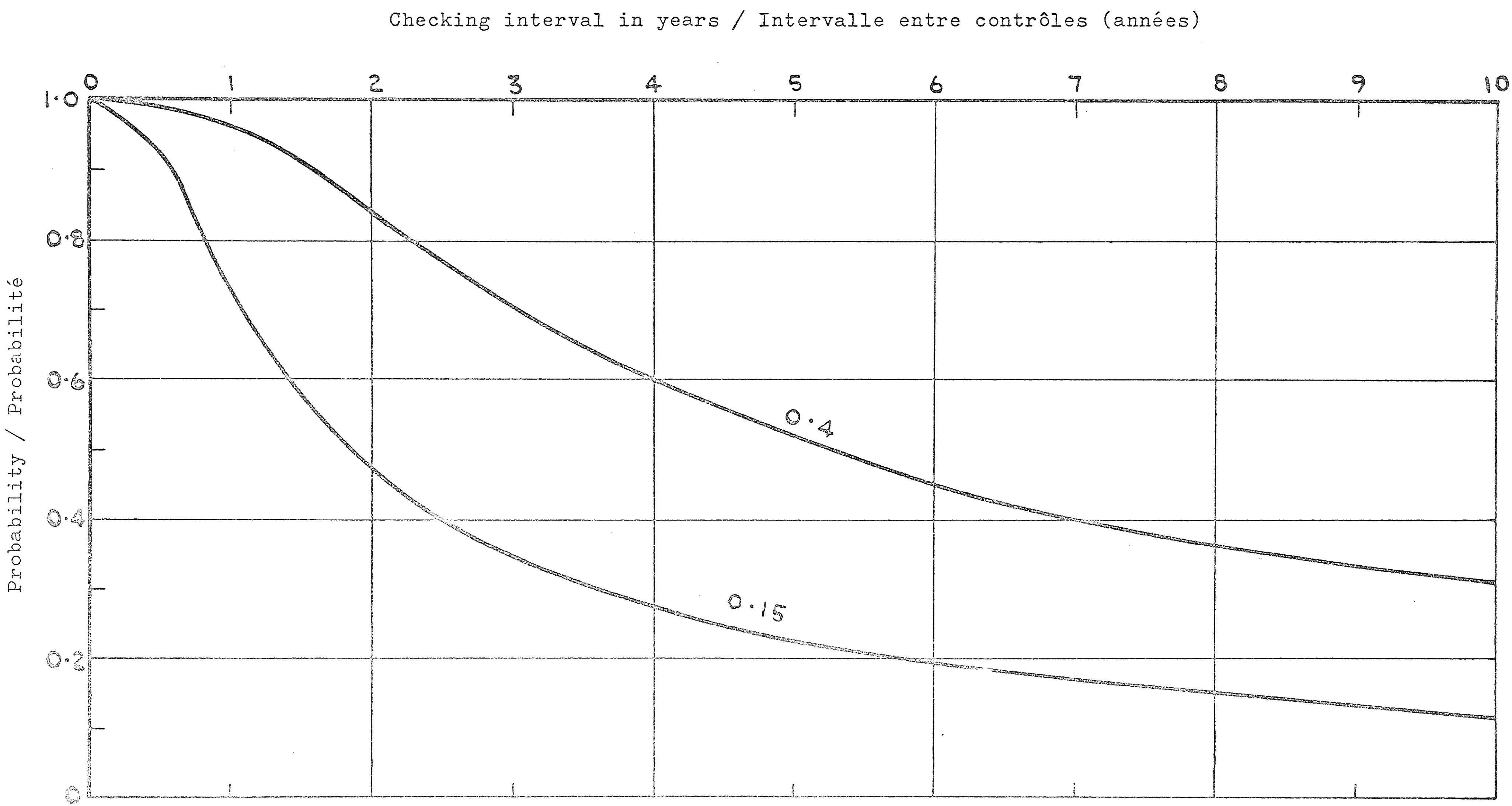

Fig. 2.3 PROBABILITY THAT CHECKING INTERVAL IS SUFFICIENT TO MEET TARGET SAFETY LEVELS OF 0.15 AND 0.4/

PROBABILITE POUR QUE LA PERIODICITE DES CONTROLES PERMETTE D'ATTEINDRE LES NIVEAUX DE SECURITE VISES 0,15 ET 0,4

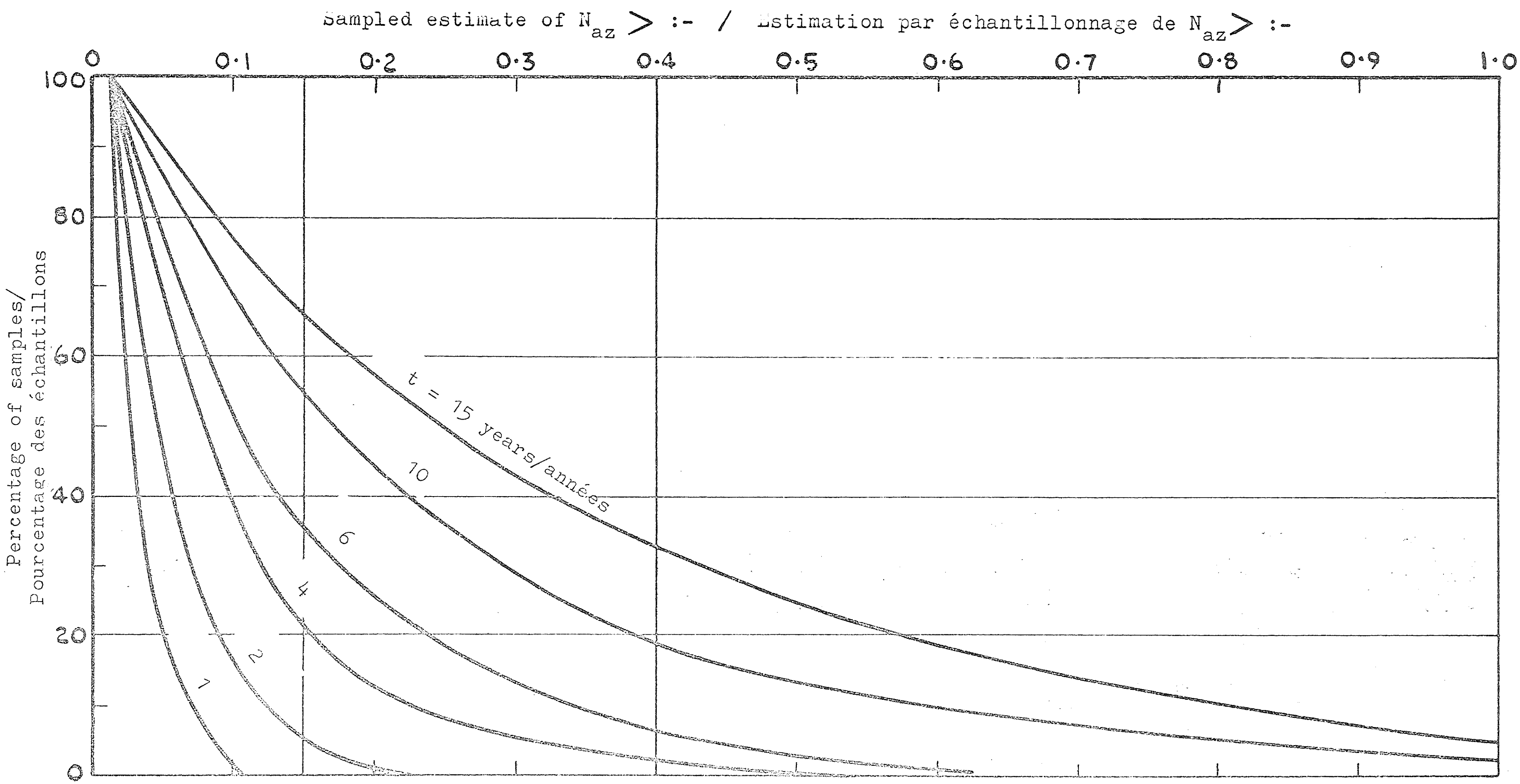

Fig. 2.4 APPROXIMATE D.S. COLLISION RISK ESTIMATES ASSUMING NO STATIC CALIBRATION ERRORS GREATER THAN 800 FEET/

ESTIMATIONS "DS" APPROXIMATIVES DU RISQUE D'ABORDAGE SI L'ERREUR D'ETALONNAGE STATIQUE NE DEPASSE PAS 800 PIEDS

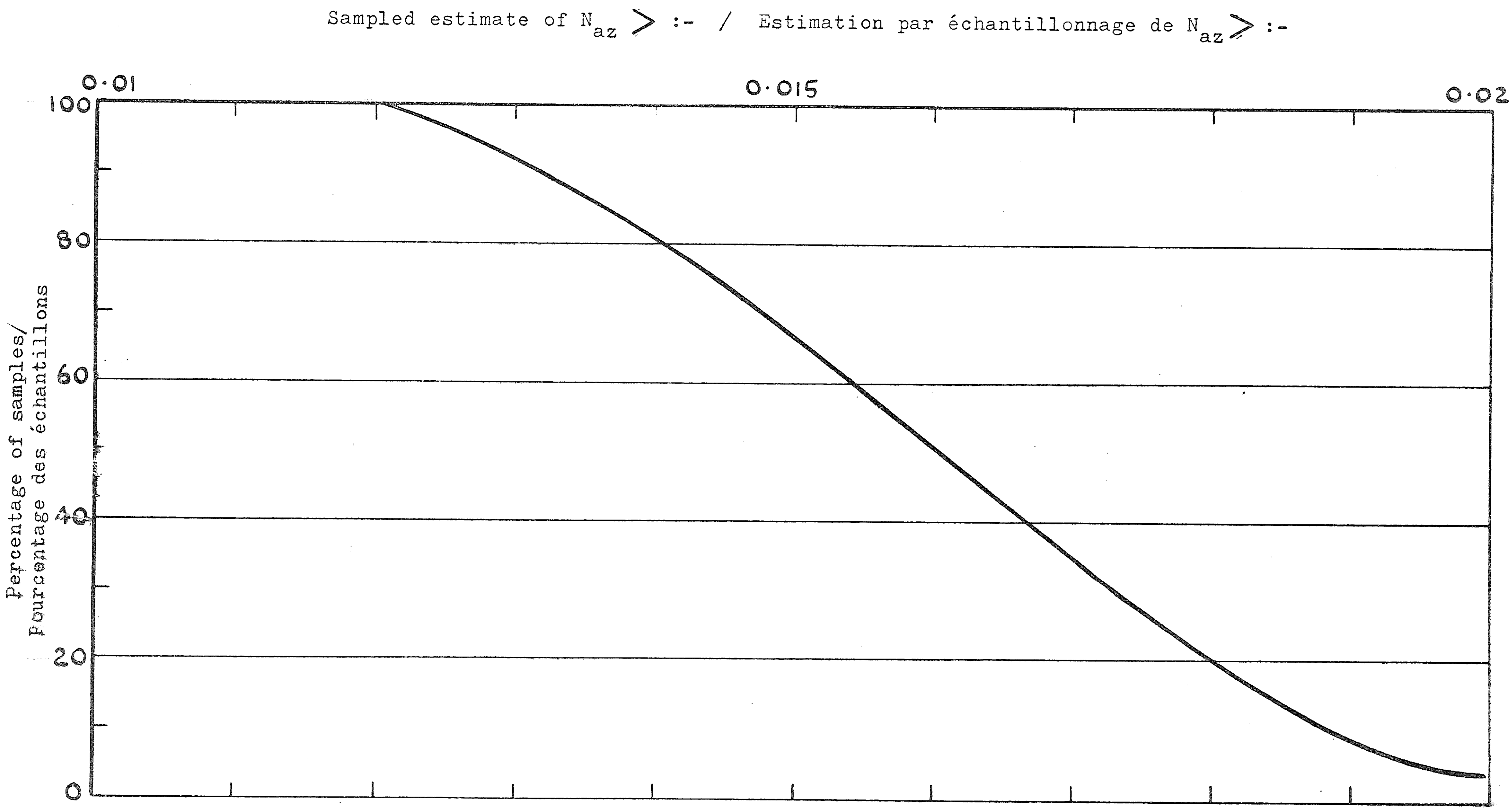

Fig. 2.5 APPROXIMATE D.S. COLLISION RISK ESTIMATES ASSUMING NO STATIC CALIBRATION ERRORS GREATER THAN 300 FEET/

ESTIMATIONS "DS" APPROXIMATIVES DU RISQUE D'ABORDAGE SI L'ERREUR D'ETALONNAGE STATIQUE NE DEPASSE PAS 300 PIEDS

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7		Analysis of Trailing-Cone Measurements of the Static Pressure Error of Large Transport Aircraft. NLR Memorandum VM-68-16
8		Summary of Discussions of the 3rd Meeting of the NAT Systems Planning Group. (Paris, 17-28 April 1967)
9		Report on Vertical Separation Study, North Atlantic Region, 15 July to 30 September 1963. IATA Doc Gen 1951 (1964)
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No.	Author	Title, etc.
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13	A. Pool	The Performance-Check Method for Monitoring the Static Pressure Errors of Transport Aircraft. NLR Report V-1961 (1967)
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APPENDIX 2-B

A heuristic explanation of the basic mathematical model of collision risk, with application to lateral, vertical and staggered separation

1. The probability that a collision will occur is simply the probability that separation is simultaneously lost in all three dimensions - longitudinal (x), lateral (y) and vertical (z):

$$P = P_x P_y P_z$$

where P_x = the probability of longitudinal overlap

P_y = the probability of lateral overlap

P_z = the probability of vertical overlap

Note that the overall probability is the product of three component probabilities, and is therefore a very small number. In accordance with the theory of random processes, such multiplication of probabilities is only valid if the component events are independent of one another, and this assumption is therefore built into the mathematical model. If, for example, aircraft which make large lateral blunders were also likely to make large vertical blunders, the resulting equation could drastically misestimate the collision risk.

2. One is, however, not interested in the probability of a collision, but in the number of aircraft involved in collisions during ten million flying hours. If this number is called N_a, then the basic collision equation as developed by the RAE and officially accepted by previous NAT SPG meetings is

$$N_a = 2 . \frac{10^7}{2} . \frac{P}{d} = 10^7 . \frac{P}{d}$$

where d = the average time during which in the model overlap exists in all 3 dimensions.

3. In order to understand the last equation it must be kept in mind that P_x, P_y and P_z are the probabilities (which can be interpreted as the number of hours per hour) that overlap will exist in each of the dimensions. In the model it is not taken into account that some of these periods of overlap may be shorter because the aircraft have collided due to the fact that at some time during that period, overlap started to

exist in the other 2 dimensions. P must therefore be understood as the number of hours per hour that the overlap is lost in all 3 dimensions if the aircraft were made such that they could pass through each other without being destroyed. The number of actual collisions of real aircraft will then be the number of hours per hour that overlap exists in 3 dimensions simultaneously (P), divided by the average time that such an overlap exists (d). Therefore, the number of collisions per hour will be P/d. The number of aircraft involved in such collisions will be twice as large, or 2P/d. During the hour considered the 2 aircraft considered will each fly for one hour, so the number of aircraft involved in collisions per flying hour will be $\frac{1}{2} \cdot 2P/d$, or P/d. Per 10^7 flying hours the number of aircraft involved in collisions will therefore be as shown in the equation shown above.

4. As a further elucidation of this equation, and also to give an intuitive feeling for the highly unlikely event which is considered, the following experiment can be conceived: An ordinary deck of cards is shuffled, and a bridge hand is dealt out (i.e. 13 cards are selected at random). If the hand includes the top 12 spades (i.e. all of the spades with the possible exception of the deuce), you die. (You should be willing to play this game, since you frequently do things much more dangerous: the probability that this will occur is $\frac{1}{15,875,388,990}$, which is about the same as the value of P in some of the separation cases considered here). Furthermore, as soon as the cards are examined, they will be reshuffled and dealt again, and the same game is played. The game is to be played over and over from the instant you enter the NAT region until you leave it. Naturally, you are now interested in the length of time it takes to shuffle, deal and examine; this is the duration of the event d. As the number of times per hour that the cards are dealt is 1/d, the number of times per hour that 12 spades will appear is P/d. In the game played over the NAT region, the pertinent length of time is the time it takes for one aircraft to pass through the space reserved by another, or - equivalently - the time required for a point (say the center of one aircraft) to pass through a box 300 feet long (=2 λ_x), 300 feet wide (=2 λ_y) and 80 feet high (=2 λ_z).

5. Going back to the equations given at the beginning of this Appendix,

substitution of the value of P in the equation of N_a gives

$$N_a = 10^7 . P_x P_y P_z . \frac{1}{d}$$

6. Values will now be derived for the parameters in this equation, for different cases of reduced separation. For the three types of separation considered (lateral, vertical and composite) a calculation of the numerical calue of P_x can be made. If the average distance between aircraft on one flight path be D, then another aircraft with length λ_x placed randomly on an adjacent flight path would have a probability of longitudinal overlap with one of the aircraft on the other flight path of λ_x/D. If the aircraft has a similarly occupied flight path on the other side, for which the aircraft are randomly placed with respect to the other two flight paths, then the probability of longitudinal overlap will be $2\lambda_x/D$. As some flight paths have other flight paths on both sides, and others have only a flight path on one side, the average value will be somewhere between these two values.

7. It should be noted that another basic assumption is built into the model at this point; namely, that aircraft are fed into adjacent tracks at random. If this assumption should be incorrect, the estimate could be too large or too small by thousands of percent.

8. In the actual calculation of P_x the RAE did not determine the average value of D and the coefficient which must be multiplied by λ_x/D, but used a slightly different approach. P_x can be interpreted as the average number of aircraft which are longitudinally displaced by less than $\pm \lambda_x$ on adjacent flight paths from a typical aircraft. This can be determined from the reported times of arrival at a given point over the ocean. As λ_x is a very short interval (1/40 mile, or about one fifth of one second's flying time), the actual count was made over a longer distance ($\pm S_x$, where S_x = 120NM) and the result later multiplied by λ_x/S_x. The average number of aircraft within $\pm S_x$ of a typical aircraft was called E. The theoretical maximum value for this parameter would be 4, because there are two lateral (or vertical) neighbouring flight paths and on each of these a maximum of two aircraft per 240 miles. Actually the outer tracks (or flight levels) have only one neighbour each and, even during

heavy traffic, the average spacing is less than 15 minutes. The RAE made actual counts of proximate pairs and found the value of E to be about 1.2* (after correction for the average increase in traffic density in the next five years). Therefore the probability of longitudinal overlap is found as

$$P_x = \frac{\lambda_x}{S_x} . E = \frac{1/40}{120} . 1.2 = \frac{1}{4000}$$

It may be noted in passing that it does not matter whether S_x is chosen to be 120NM or some other number. If S_x were larger, E would be larger, and only the ratio E/S_x matters.

9. The values of P_y and P_z depend on the type of separation considered. For a lateral separation of 90 NM, P_y becomes $P_y(90)$, the probability that two aircraft nominally 90 miles apart will overlap laterally. From data on deviations of aircraft from track over the North Atlantic Ocean the value of $P_y(90)$ has been determined to be $\frac{1}{1,000,000}$. The value of P_z for this case is $P_z(0)$, the probability that aircraft on the same flight level will overlap vertically. (i.e. within $2\lambda_z = 80$ feet). Here one meets the famous paradox that "the better the navigation, the greater the collision risk" because, if aircraft lose lateral separation, one cannot then rely on sloppy vertical station-keeping to cause a miss. In fact, $P_z(0) = 1/4$ because, on the average, one aircraft of such a pair will be between 160 feet above or 160 feet below the other, and the height of an aircraft is taken to be 40 feet.

Thus $\frac{2 \times 40}{2 \times 160} = \frac{1}{4}$.

10. The "duration" of a collision is the time which the aircraft would take to pass through each other if they were not destroyed in a collision. For a collision of 2 aircraft flying at exactly the same forward and vertical speed and colliding laterally this would be

$$d = \frac{2\lambda_y}{\dot{y}}$$

where $2\lambda_y$ is the total distance over which overlap occurs and $\dot{y}$ is the lateral speed with which the aircraft are moving relative to each other. For a lateral collision of aircraft nominally separated

*All numbers in this Appendix are approximate and for illustration only.

by 90 NM, one uses $\dot{y}(90)$ which is clearly a large number, since a plane cannot get 90 NM off track during a short flight without a high lateral velocity. Radar data indicate that $\dot{y}(90)$ is about 60 knots or 100 feet per second. As $2\lambda_y = 300$ feet, the time d is about $\frac{300}{100} = 3$ seconds or $\frac{1}{1200}$ hour.

11. With the numbers given above one can now calculate the value of N_a for collisions of aircraft on the same flight level with a nominal lateral separation of 90 NM. This is called $N_{ay}(90)$. It is

$$N_{ay}(90) = 10^7 . P_x . P_y(90) . P_z(0) . \frac{1}{d} =$$

$$= 10^7 . \frac{1}{4000} \cdot \frac{1}{1,000,000} \cdot \frac{1}{4} \cdot \frac{1}{1/1200} =$$

$$= 10^7 . \frac{1}{16,000,000,000} . 1200 = 0.75 \; *$$

which is above the target levels of safety proposed by the Group: 0.15 to 0.4.

12. The actual expression for d is a good deal more complicated than the above. To get average velocity, the "absolute value" must first be taken by disregarding direction (otherwise the average speed would always be zero, because half the aircraft are moving to the right and half to the left). This is indicated by vertical lines: $|\dot{y}|$. Next, the average of this quantity is taken, indicated by a horizontal line: $\overline{|\dot{y}|}$. According to the laws of statistics, the average relative velocity of a pair of aircraft is 41 % greater than the average absolute velocity of a single aircraft. And, finally, one will in general not pass through this box in a lateral direction only, so the movement in the longitudinal and vertical directions must also be taken into account. Thus, the actual equation is

$$\frac{1}{d} = \frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}$$

(For technical reasons, one writes ΔV instead of $\dot{x}$)

In the case of a lateral collision, the 41 % correction does not apply, because one aircraft is usually moving much more rapidly than

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* Note again that these numbers are illustrative only.

the other. Furthermore, the first and third terms are so small with respect to the second term in this case, that they add little to it. The value of 1/d is therefore about as given above: 1200.

13. In the case of vertical separation, $\overline{|\dot{y}|}$ becomes $\overline{|\dot{y}|}(0)$ = 20 kts, implying that the average lateral velocity of an aircraft which is close to track is about 14 kts (20 is 41 °/o more than 14). Hence

$$\frac{\overline{|\dot{y}|}}{2\lambda_y} = \frac{20}{1/20} = 400$$

to this must be added

$$\frac{\overline{|\dot{z}|}}{2\lambda_z} = \frac{1}{1/75} = 75 \quad \text{and} \quad \frac{\overline{|\Delta V|}}{2\lambda_x} = \frac{13}{1/20} = 260$$

This yields 1/d = 735. P_x is about the same as for the case of lateral separation, yielding $P_x = \frac{1}{4000}$. P_y becomes $P_y(0) = \frac{1}{800}$ because, on the average, one aircraft of such a pair (nominally 0 NM apart laterally) will be anywhere from 20 NM to the left to 20 NM to the right of the other (average deviation from track 14 NM plus the 41 °/o correction), and the wing span of an aircraft is taken to be 1/40 NM ($2\lambda_y$ = 1/20). Thus $P_y(0) = \frac{1/20}{40} = \frac{1}{800}$. P_z becomes $P_z(1000)$, the probability of vertical overlap of two aircraft nominally separated vertically by 1000 feet. This still is a controversial number, which we take here to be $\frac{1}{3000}$. Hence

$$N_{az} = 10^7 \cdot P_x \cdot P_y(0) \cdot P_z(1000) \cdot \frac{1}{d} =$$

$$= 10^7 \cdot \frac{1}{4000} \cdot \frac{1}{800} \cdot \frac{1}{3000} \cdot 735 = 0.75$$

again above the target levels of safety.

14. For staggered separation, E is about twice as large as before, because each aircraft now has four nearest neighbours instead of two. Hence $P_x = \frac{1}{2000}$. Now $P_y = P_y(60) = \frac{1}{90,000}$ and $P_z = P_z(1000) = \frac{1}{3000}$. The value of 1/d lies between the two given above — say 900. Then

$$N_{ayz} = 10^7 \cdot \frac{1}{2000} \cdot \frac{1}{90,000} \cdot \frac{1}{3000} \cdot 900 = 0.03$$

which is well below the target levels of safety proposed by the Group.

15. One further simplification has been made. The deduction of the equation for R tacitly assumed that both of the aircraft which might collide were travelling in the same direction. Hence, the E included in the overall equation is only E(same), the "same-direction occupancy". To this must be added a term involving E(opp), the opposite-direction occupancy, multiplied by a $\frac{1}{d}$ which has $\overline{V}$ in place

of $|\Delta V|$. With the bulk of eastbound and westbound flights occurring at different times of day, E(same) is much larger than E(opp). On the other hand, the $\frac{1}{d}$ by which it is multiplied includes a $\bar{\bar{V}}$ of approximately 1000 knots, so that opposite direction traffic, in spite of its small bulk, contributes appreciably to the collision risk.

16. The reader should now be prepared to look at the full-blown collision risk equation (for example, on page 2-A-1) with equanimity. But even with equanimity, he will probably ask: "Does anybody really believe all this?" The answer, even by a dedicated mathematician, must be "not entirely". But considerable confidence can be placed in some of the types of conclusions which may be drawn from it.

17. While only one of the numbers in the calculations above is highly controversial - namely $P_z(1000)$ - there are questions about almost every number which might raise or lower it by 50 % or so. When one combines so many uncertain numbers, one can hardly have much faith in the results. It is intended to express this by applying "confidence limits" (a technical term in statistics) to the conclusions, but the fact of the matter is that there is very little confidence even in these confidence limits. Thus, for example, N_{ay} is estimated (for a 90 NM separation) to be about 0.75; the actual conclusion of SPG IV was close to this, based on much more complicated considerations, including, for example, different errors in different parts of the ocean. But there is no assurance that N_{ay} is greater than 0.4. There is, however, pretty high confidence that it is larger than 0.15, but one cannot even be completely sure about that. Thus, absolute estimates from this mathematical model are fraught with uncertainty.

18. On the other hand, the mathematical model permits to draw certain types of relative conclusions with high confidence. For example, if N_{ay} is computed for 90 NM and 120 NM separations, the only differences in the two computations are in P_y and $\dot{y}$; all other quantities are the same for both. While the ratio, for example, of $\frac{P_y(90)}{P_y(120)}$ is not known exactly (this is related to the much-discussed

question of "tail shape"), it is clear from the radar data that that it is not a small number like 2 or 3 nor a large number like 50 or 100. One can be extremely confident in the assertion that a 90 NM separation is appreciably more dangerous than a 120 NM separation - the best estimate at present is that it is six times as dangerous. Similarly, one can be equally confident that loss of 60 NM-plus-1000 feet "composite" separation is considerably less likely than loss of 120 NM separation alone.

19. These statements could not have been made as recently as three years ago. What has been achieved through the data-collection effort of 1967-68, plus the mathematical and data reduction work at RAE, plus the deliberations of SPG, is the not-inconsiderable ability to make, with high confidence, several assertions, including the following: if lateral separation is reduced from 120 to 90 NM, capacity will be increased by about 50 % while the danger of collision is greatly increased; if lateral separation were to remain as 120 NM and 60 NM-plus-1000 feet 'composite' separation were introduced, capacity would be increased by about 100 % while collision danger would be changed by an insignificant amount. Finally the long-controversial questions as to the relative contributions to collision risk of same-direction and opposite-direction traffic can also be answered; opposite-direction traffic is so dangerous that safety could appreciably be increased if even the presently small amount of opposite-direction traffic could be segregated.

Glossary

x -	longitudinal distance or longitudinal
y -	lateral distance or lateral
z -	vertical distance or vertical
N_a -	number of accidents per ten million flying hours
N_{ay} -	number of accidents per ten million flying hours due to loss of lateral separation
N_{az} -	number of accidents per ten million flying hours due to loss of vertical separation
N_{ayz} -	number of accidents per ten million flying hours due to loss of staggered separation
P_x -	probability of longitudinal overlap of a pair of aircraft
P_y -	probability of lateral overlap of a pair of aircraft
P_z -	probability of vertical overlap of a pair of aircraft
P_z (0) -	probability of vertical overlap of a pair of aircraft nominally separated by zero feet of altitude
P_z(1000) -	probability of vertical overlap of a pair of aircraft nominally separated by 1000 feet of altitude
P_z(std) -	probability of vertical overlap of a pair of aircraft nominally separated by some standard distance (such as 1000 feet) in altitude
$\dot{z}$ -	vertical speed, or relative velocity in the vertical direction of a pair of aircraft
$\overline{\lvert\dot{z}\rvert}$ -	average (taken without regard to "toward" or "away") relative vertical velocity of a pair of aircraft
ΔV	$\dot{x}$, or relative longitudinal speed of a pair of aircraft
E -	occupancy
E_z (same) -	average number of aircraft flying in the same direction which a typical aircraft has vertically "proximate" to it, i.e. on an adjacent flight level, within $\pm$ 15 minutes flying time
E_z (opp) -	average occupancy of opposite direction traffic which is vertically proximate
λ -	dimension of an aircraft, assumed to be a rectangular box
λ_x -	length, 150 feet = $\frac{1}{40}$ mile $2\lambda_x = \frac{1}{20}$ mile
λ_y -	wingspan, 150 feet = $\frac{1}{40}$ mile $2\lambda_y = \frac{1}{20}$ mile
λ_z -	height, 40 feet = $\frac{1}{150}$ mile $2\lambda_z = \frac{1}{75}$ mile

APPENDIX 2-C

BIBLIOGRAPHY ON VERTICAL SEPARATION

1. Report on vertical separation study, North Atlantic Region, 15 July to 30 September 1963. IATA Doc. Gen. 1951 (1964).

2. Analysis of trailing-cone measurements of static pressure error of large transport aircraft. NLR Memorandum VM-68-19.

3. Progress report on the evaluation of the performance-check method for monitoring the static pressure error of commercial aircraft. NLR Memorandum VM-68-22.

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3. Summary of Agenda Item 3: Review of the possibility to apply "composite separation" (i.e. a form of separation composed of the simultaneous application of reduced lateral and vertical separation) to aircraft operating in the NAT Region.

3.1 Introduction

3.1.1 A proposition for a mathematical treatment of staggered separation was presented by the Netherlands. The method proposed is given in Appendix 3-A. The pattern of available flight paths is shown in figure 3.1. A discussion is given of the collision risk if aircraft nominally fly along these flight paths. In addition, the increase in risk has been calculated if aircraft are allowed to change tracks or flight levels. One difficulty was that no agreed values of the probability of vertical overlap for aircraft flying at 1 000 feet nominal vertical separation which must be used in the calculation are as yet available. These values were discussed under Agenda Item 2 of this meeting, but no firm conclusions were reached. A solution for this problem was found by giving wherever necessary not the collision risk due to composite separation N_{ayz} but its relation to the vertical collision risk: $N_{az}(1000)/N_{ayz}$. Thus it is possible to make use of the values for N_{az} based on different assumptions given in figures 2.2 to 2.5 of this report (see pages 2-A-41 to 2-A-45).

3.2 Assignment of Target Level

3.2.1 At previous meetings it was agreed* that the overall target level of safety from collision over the North Atlantic would be to keep the fatal accident rate in the range from 1.2 to 0.45 per 10 million aircraft flying hours or lower and that it would be convenient and reasonable to split this rate equally between vertical, lateral and longitudinal separation. In the system under present consideration, aircraft may collide with :

- either of the two aircraft nominally ahead and behind by 15 or more minutes;
- either of the two aircraft nominally 2 000 ft. above and below;
- either of the two aircraft nominally 120 NM on either side;
- any of the four aircraft nominally separated by 60 NM and 1 000 ft.

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*Note: - Summary NAT/SPG-2, para. 4.12, page 4-4
Summary NAT/SPG-3, para. 1a), page 1-M-1

It might therefore seem logical to redivide the collision risk into four equal parts. However, this was considered undesirable and inconvenient. Instead, 1/3 of the risk is now arbitrarily assigned to each of the 1st, 3rd and 4th categories above, and zero risk to the 2nd. This is not to assert that the collision risk for such aircraft vertically separated by 2 000 feet is indeed zero, but merely that present equations predict for this risk a number so much smaller than the others that it would have a trivial effect if added to them.

3.3 For a given amount of traffic the risk due to loss of lateral separation of 120 NM, for which one third of the total collision risk was allowed in para. 3.2, will be much lower than it is in the present system. This is due to the end effect: the number of flight paths, which do not have tracks at 120 NM on both sides but only can have aircraft at 120 NM on one side, is much larger than for the presently used system. The risk due to loss of lateral separation of 120 NM in a staggered system is, therefore, less than was calculated for the present system during the Fourth Meeting of the Group.

3.4 When comparing vertical separation of 1 000 feet with the staggered separation, the reason why the latter is much safer than the former can be explained as follows: if two aircraft in staggered separation are to collide, one or both aircraft must have a large lateral deviation and at the same instant one or both must have a large vertical deviation. This means that two or more improbable blunders have to be made by a pair of aircraft at the same moment. This is a very unlikely event if the occurrence of the blunders in the two directions is statistically independent. This is the basic assumption for the mathematical treatment given in Appendix 3-A. The Group agreed that this assumption of independence was applicable, amongst other things, because completely different sets of instruments measuring completely different quantities are being used for vertical and lateral separation.

3.4.1 On the other hand, if, for example, a large fraction of both types of blunders are made simultaneously by a small fraction of operators (including non-IATA, private, etc.) then this assumption is invalid and underestimates risk.

3.5 Calculation of the Estimated Risk due to Loss of Staggered Separation

3.5.1 The main conclusion from the mathematical treatment in Appendix 3-A is that the value of N_{ayz} is about 1/30 of the value which N_{az} would have under the same assumptions, i.e., comparing simultaneous 60-mile and 1000-foot separation (each aircraft having four diagonal neighbours) with simple 1000-foot separation (each aircraft having two vertical neighbours), all other things being equal. The value of N_{ayz} for which no recalibrations of the static system are necessary should therefore be determined from the corresponding value of N_{az}. If it is assumed that the static pressure error of each aircraft is checked before it is delivered and if it is assumed that the average service life of an aircraft is 15 years, then the curves for a 15 year period between recalibrations given in figures 2.2.b and 2.4 can be used. Figure 2.5 already applies to aircraft which have not been recalibrated. Table 3.1 includes "90% confidence level values" for the collision risk in such a composite system, derived from figures 2.2.b, 2.4 and 2.5.

3.5.2 These values are only valid if the opposite direction traffic is completely segregated from the same direction traffic. The effect on risk due to composite separation through missing opposite-direction traffic would be an increase by 35%. The corresponding figures are shown in the column labelled "same and opposite direction traffic" in Table 3.1.

3.5.3 If 2000-feet changes of altitude did occur on the average once per flight, then the composite separation risk would be increased by 0.018 as shown in Appendix 3-A. The Group agreed that, on the basis of present practice in the track system over the North Atlantic, this number of changes of flight level would be a very cautious assumption. Though no figures were available on the average number of such changes, data shown on some days gave values very much lower than one change over 2 000 feet per flight. With the flight levels which will be passed at 60 NM distance both occupied, then 0.018 should be added to the collision risk. This has been done in the last two columns of Table 3.1.

3.5.4 In Appendix 3-A the risk of changes of track, both horizontally over 120 NM and over 60 NM with simultaneous change of 1 000 feet in altitude were calculated. The Group decided that it would not consider such manoeuvres which include crossings through the track system, unless the conditions are such that they would be possible under the present system.

Table 3.1

Values of the Collision Risk of Staggered Separation for Different Assumptions ("90% confidence level" values)

Boundary of tail	Same-direction traffic only	Same- and opposite-direction traffic	Including one change of 2 000 feet per flight	
			Same-direction traffic only	Same- and opposite-direction traffic
1 200 ft	0.12	0.16	0.14	0.18
800 ft	0.027	0.037	0.045	0.055
300 ft	0.0006	0.0008	0.019	0.019

3.5.5 The results indicate that the collision risk for same-direction traffic would be less than the lower target level of safety (N_{az} = 0.15) for all tail shapes considered even if on the average one change of flight level of 2 000 feet should be allowed per ocean crossing. For the other 2 cases the collision risks would be less than the lower target level if the assumptions on the tail shape are made a little less severe than a level tail to 1 200 feet (but still much more severe than a level tail to 800 feet). For all cases they were far below the higher target level of safety (N_{az} = 0.40).

3.6 Operational implications

3.6.1 The Group then considered the operational implications of such a system. These were of two kinds:

i) in respect of the oceanic area, and
ii) in transition from oceanic to domestic airspace.

3.7 Oceanic Area

3.7.1 Opposite direction traffic

3.7.1.1 While the level of safety of opposite direction traffic in staggered track systems had been shown to be lower than for same direction traffic it was agreed to investigate both systems. It was pointed out that the opposite direction flow at the time of the two major daily peaks of traffic flow was small (about 5 per hour requiring 2/3 flight paths in 1968). Moreover, the NAT meteorological situation ensured that on about 80% of occasions these opposite direction flows were separated laterally. This left 20% of occasions on which the minimum time track was near the "great circle" route for both the main flow and the smaller opposite direction flow. In these circumstances segregation would lead to route deployment as illustrated below which might be typical on a busy day in 1970.

Tracks

← 360 NM →

Flight levels	A	B	C	D	E	F	G
380	WB	-	WB	-	-	-	-
370	-	WB	-	WB	-	EB	-
360	WB	-	WB	-	-	-	-
350	-	WB	-	WB	-	EB	-
340	WB	-	WB	-	-	-	-
330	-	WB	-	WB	-	EB	-
320	WB	-	WB	-	-	-	-
310	-	WB	-	WB	-	-	-

WB = Westbound
EB = Eastbound

Fig. 3.1

Flight levels	A	B	C	D	E	F	G
380	WB	-	-	-	-	WB	-
370	-	-	EB	-	WB	-	WB
360	WB	-	-	-	-	WB	-
350	-	-	EB	-	WB	-	WB
340	WB	-	-	-	-	WB	-
330	-	-	EB	-	WB	-	WB
320	WB	-	-	-	-	WB	-
310	-	-	-	-	WB	-	WB

Tracks: ← 360 NM →

Fig. 3.2

Fig.3.1 represents a situation on which it would be marginally advantageous to group the eastbound traffic to the north of the westbound flow, whereas Fig.3.2 illustrates a possible solution where the main part of the westbound flow might be grouped to the north side of the eastbound flow. Other possibilities are apparent but both the administrative difficulties of implementation and the economic effect on airlines would need to be studied and a balance struck between administrative complication in the agreement between Gander and Shanwick of the oceanic track structure to be used and the economic implications. Fig.3.1 represents the simple case and Fig.3.2 the more complex variant. Clearly the ATS view was that, if feasible, staggered tracks for opposite direction traffic offered the simplest solution. Failing that, the Group recommend further analysis to determine whether the simple segregation represented by Fig. 3.1 would be acceptable.

3.7.2 Separation standards

3.7.2.1 The Group recognized that changes to Doc 7030 would be required to permit the use of a composite track structure but believed that this would present little difficulty in respect of clearance to enter the system, that is the application of "strategic" separation in accordance with the system. The Group however, recognized that although these "separations" were good for the crossing there was a need for the specification of separation standards or procedures for "tactical" use within the organized track structure and off it.

3.7.2.2 Separation standards on the organized track structure in respect of aircraft joining or crossing it

3.7.2.2.1 The present standards should apply.

3.7.2.3 Within the organized track structure - track changes

3.7.2.3.1 As the risk involved in a change from one track to another at the same level had been demonstrated to be unsafe in a fully-loaded system it was thought that the normal separation standards should be applied before reclearing the aircraft in question. It was pointed out that although restrictive, the possibility of track changes was also very limited even with the present system.

3.7.2.4 Within the organized track structure - level changes

3.7.2.4.1 The Group noted that with the present system, level changes on the same track were only feasible when the system was relatively lightly loaded. A level change might be requested by a pilot on grounds of fuel economy or to avoid turbulence, or be instituted by ATC if there were grounds for believing that longitudinal separation was infringed. The same general conditions would apply with the introduction of staggered tracks with, however, the additional complication that a flight level change of 2000 feet would involve flying through levels assigned to aircraft on adjacent tracks at 60 NM distance. Clearly the normal separation standards could be applied (2000 feet and 120 NM) for this maneouvre but would involve a search for aircraft on adjacent tracks which might be "traffic" to the climbing or descending aircraft. It was suggested that, since only a moderate increment of risk would result each time an aircraft should climb 2000 feet on track through the flight levels of adjacent tracks, a procedure could be developed permitting this practice but limited in application to once per aircraft per Atlantic crossing. It was pointed out however, that a procedure of this kind represented a radical change; procedures had never been limited in this way and to do so could lead to complexity and hence the possibility of error in application. This subject would need further study.

3.7.2.5 It has been thought necessary to describe in some detail the problem outlined in 3.7.2.3 and 3.7.2.4 because necessary procedures together with the separation standards to be applied would have to be developed. Nevertheless, the Group did not consider that this seriously affected the validity of the proposal because in many traffic conditions the possibility of level or track changes are minimal even with the present system.

3.7.3 Emergency procedures and unusual events

3.7.3.1 The Group did not consider that the introduction of staggered tracks would significantly effect the action which would be taken respectively by pilots and controllers in such situations. In one respect staggered tracks afforded an advantage since, for example, in the case of an aircraft having to descend for reasons, say, of loss of pressurization there might be fewer flight levels to penetrate than with the present system. On the other hand, there would be less room to turn around and return to base than with the present system with tracks 120 NM apart. The Group nevertheless thought that procedures could be devised but whether such procedures need be specified would require consideration. Provided that pilots were well briefed on the track structure in which they were flying it might be sufficient and more appropriate to leave the manoeuvre to the pilot with ATC affording a return to normal separation as soon as possible.

3.8 Effect on the Transition Area

3.8.1 A major difficulty that would be encountered by ATS in applying a composite system, or for that matter any system which implies reduced separation standards in the oceanic area, is associated with the area of transition from the oceanic track system to the domestic airways system, particularly in areas outside reliable radar cover. This difficulty arises as a result of:

1) The need to re-clear those flights operating at oceanic flight levels in the oceanic area to appropriate directional levels in the domestic system, and

2) the limitations of the present domestic airway systems, caused by the inadequate numbers of tracks in the transition area between the oceanic and domestic track structures.

3.8.2 For instance, the Group noted the difficulties at present being experienced by Ireland in relation to eastbound over-flying traffic. This traffic may present itself at the oceanic boundary to the west on three tracks occupying all levels, those corresponding to the direction of flight and those normally reserved for opposite direction traffic. A major problem is feeding this traffic into the limited United Kingdom route structure to the east over Strumble in which the use of levels corresponding to the direction of flight was obligatory, except during certain hours and under specified conditions.

3.8.3 The Group decided that with the inevitable growth in North Atlantic traffic, system capacity must by some means be increased. Whatever system is adopted, be it a composite track or some other system, its full capacity cannot be exploited unless routes and/or procedures were developed to increase the domestic acceptance rate.

3.8.4 In agreeing on the need to develop additional routes in domestic airspace to facilitate the integration of traffic fed into it from a composite track system, the Group considered the limited "area navigation" concept as a means to provide a better route structure in the transition area between the oceanic and domestic airspace. Navigation along these routes could be accomplished by employing, for example, computer systems updated by VOR/DME or DME/DME, or other ground-referenced aids, or Doppler into eventual radar cover where integration with the domestic airways structure could be accomplished. The simplicity of this technique would facilitate the early implementation of such a concept. A formal route facility structure would later be required.

3.8.5 The Group also considered that the provision of effective radar cover of the transition area in which integration with the domestic airspace occurs is a pre-requisite to the introduction of a composite separation system. Radar would be used by the controller to monitor navigation and ensure separation between aircraft.

3.9 Clearances

3.9.1 The Group agreed that, as far as practicable, clearance should contain the complete oceanic and domestic routing to the point of integration into the appropriate airways system.

3.10 Conclusion

3.10.1 The Group concluded that, from the ATC viewpoint, a type of composite separation described was practical in the NAT area provided radar cover of the transition area was available and appropriate track structures were established in the continental air-space. The Group would therefore hope that States would take the required action to facilitate its implementation on a trial basis.

3.10.1.1 Specifically the Group concluded that:

(a) The calculations indicate that a system of tracks separated laterally by 60 NM whereby adjacent tracks are staggered by 1000 feet would be likely to afford an acceptable level of safety. (For the exact statistical values obtained when applying this separation to one-way traffic as opposed to two-way traffic, see Table 3.1.)

(b) Such a system may be practicable in the NAT oceanic area.

(c) The assumption of para. 3.4.1 that very poor operators are not present in large numbers in the NAT Region is significant.

3.10.1.2 The Group also saw the following courses of action:

(a) Further information should be obtained on changes in static system error and their duration to establish whether altimeter accuracy is sufficient to support staggered tracks for opposite direction traffic.

(b) States concerned should examine ways and means of extending the NAT track structure into domestic airspace thus extending the time available to redeploy traffic from NAT to the separations appropriate in domestic air-space.

(c) Steps should continue to be taken to ensure that the great majority of operators can and do maintain track and flight levels within tolerances similar to or better than those indicated herein, or to develop procedures to segregate traffic that does not meet those standards.

(d) Those actions outlined in para. 2.7.2 would enhance even further the safety of a system of composite separation.

APPENDIX 3 - A

Increasing the potential traffic density over the North Atlantic Ocean by using composite separation.

1 Introduction.

1.1 The use of the present pattern of flight paths in the track system for jet aircraft over the North Atlantic leads to economic losses for aircraft which are assigned to the outer tracks and/or the extreme flight levels. A section through this present track system, perpendicular to the direction of flight, is given in figure 1*. In order to improve the economics of operations across the North Atlantic, several methods of increasing the traffic density in the track system have been considered: decrease of lateral separation to 90 NM . (figure 2), decrease of vertical separation to 1000 feet (figure 3) and decrease of longitudinal separation. Statistical studies were made of these possible ways of reducing separation but, mainly because of lack of sufficient data, it could not be proved that the safety from collisions was sufficient.

1.2 In this Appendix a different type of flight path pattern is considered (figure 4), which for a given traffic density will give a much smaller collision risk. Here, as in the present system, the vertical separation between aircraft on the same track is a multiple of 2000 feet and the lateral separation between aircraft on the same flight level is 120 NM .; with aircraft on adjacent tracks there is a simultaneous separation of 60 NM and 1000 feet. For a collision to occur between a pair of aircraft on adjacent tracks, 2 blunders are required, one in the lateral and one in the vertical direction.

1.3 An important advantage of this "composite" or "staggered" separation is, that any future increase in the accuracy of track keeping or height keeping will increase the safety from collisions, at least as far as the aircraft on adjacent tracks are concerned. This is not the case in the other systems, as an increase in the accuracy of track keeping will increase the collision risk of the system of figure 3, and an increase in height-keeping accuracy will increase the collision risk of the system of figure 2. It would seem likely that at least the track keeping accuracy will increase in the near future, due to increased use of Doppler and inertial navigation systems.

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*The Figures are shown on page 3-A-11.

1.4 The use of additional flight levels will provide extra problems for ATC in the transition areas, where the aircraft must return to the normal separation rules which require 4000 feet between same-direction aircraft on the same track. It would seem, however, that these problems will be less than those for a system with 1000 feet vertical separation as in figure 3, because aircraft on adjacent flight levels are already laterally displaced by 60 NM.

1.5 A disadvantage of composite separation is, that during a climb over 2000 feet on the same track, the aircraft passes 2 tracks at only 60 NM lateral distance. The highest possible safety can only be obtained if these 2 flight paths are cleared of traffic during such an operation. In section 2.2 the added risk of such transitions has been calculated for the case that these flight paths are occupied. A similar calculation for transitions from one track to another on the same flight level is given in section 2.3. As such transitions are very rare even in the present system, the final effect, even if they are allowed, will not be large.

1.6 In this Appendix an estimate is made of the collision risk in a system with composite separation. The calculations have been made for same-direction traffic only. This is because the data from Appendix 2A, which are used in the calculation, apply to this case only. With suitable values of the parameters, the method described in this report can also be used for a combination of same-direction and opposite-direction traffic.

2. The method for estimating the collision risk.

2.1 The collision risk for level flight only.

2.1.1 The collision risk in a system with composite separation as shown in figure 4 will be equal to the sum of 3 risks:

i) the risk $N_{ay}(120)$ that an aircraft will collide with another aircraft which is nominally flying at the same flight level with a nominal lateral separation of 120 NM

ii) the risk $N_{az}(2000)$ that an aircraft will collide with an aircraft which is nominally flying on the same track with a nominal vertical separation of 2000 feet.

iii) the risk N_{ayz} that an aircraft will collide with an aircraft which is nominally flying on a flight path which has a lateral separation of 60 NM and also a vertical separation of 1000 feet with respect to the flight path of the first aircraft.

The first risk has been estimated during the fourth meeting of the NAT Systems Planning Group. The figure which should be used for $N_{ay}(120)$ in a composite separation system will be somewhat less than was calculated there, because the value of the lateral occupancy E_y will be less in a composite system. This is because, for a given number of available flight paths, there will be less flight paths which have other flight paths on both sides. The risk due to loss of vertical separation is discussed under agenda item 2 of the present Summary. Although the value of $N_{az}(2000)$ will probably be very small, it is impossible even to make a rough guess of its value. This is due to the fact that the number of data available did not even suffice to make a reasonable guess of $N_{az}(1000)$ and that extrapolation to 2000 feet is utterly impossible. It seems sufficient to state that the sum of these 2 risks will be somewhat less than the risk in the present system, because of the smaller values of the lateral and vertical occupancies.

2.1.2 In the remainder of this Appendix only the third risk will be discussed. The derivation of the equation for N_{ayz} is similar to that of the other risk equations, as for instance eq. (1) in Appendix 2-A of the present Summary. For same-direction traffic the equation becomes:

$$N_{ayz} = 10^7 . P_z(1000) . \frac{\lambda_x}{S_x} . E_{yz}(\text{same}) . P_y(60) . \left\{ \frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}(60)}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right\} \quad (1)$$

where

N_{ayz} = the collision risk due to loss of composite separation, i.e. the estimated number of aircraft which will become involved in fatal collisions due to loss of composite separation with aircraft on adjacent tracks during 10^7 flying hours

$P_z(1000)$ = the probability of vertical overlap (i.e. a vertical separation of less than λ_z) for aircraft nominally having a vertical separation of 1000 feet. This quantity is equal to $P_z(\text{std})$ discussed in Appendix 2A of the present Summary

$P_y(60)$ = the probability of lateral overlap (i.e. a lateral separation of less than λ_y) for aircraft flying on tracks nominally spaced by a lateral distance of 60 NM. This quantity can be calculated from the distribution of the across-track errors of single aircraft given in the

Summary of Discussions of the fourth meeting. Its value is 11×10^{-6}

E_{yz}(same) = the composite occupancy, i.e. the average number of same-direction aircraft occupying segments of length $2\ S_x$ on the 4 flight paths nearest to the flight path of a typical aircraft. A slightly cautious estimate for this parameter is $E_{yz}(\text{same}) = 2\ E_z(\text{same})$. In Appendix 2A of the present Summary a distribution function of E_z(same) is specified, which has a central value of 0.73

S_x = the along-track criterion for proximity. As in previous cases a value of 120 NM is used

$\overline{|\Delta V|}$ = the average of the absolute values of the relative along-track speed between a pair of same-direction aircraft about to collide. As in previous cases a value of 13 kts is used

$\overline{|\dot{y}|}(60)$ = the average of the absolute values of the relative across-track speed between a pair of aircraft nominally separated by a lateral distance of 60 NM, at the moment they are about to collide. For this parameter a value of 47 kts can be read from figure 1.16 of the Summary of discussions of the fourth meeting.

$\overline{|\dot{z}|}$ = the average of the absolute values of the relative vertical speed between a pair of aircraft about to collide. As in previous cases a value of 1 kt is used.

λ_x = the average effective aircraft length. As in previous cases a value of 150 feet = 0.025 NM is used

λ_y = the average effective wing span. As in previous cases a value of 150 feet = 0.025 NM is used.

λ_z = the average effective aircraft height. As in previous cases a value of 40 feet = 0.0066 NM is used.

2.1.3 The only parameter for which no agreed value exists is $P_z(1000)$. A discussion on this parameter is given under Agenda item 2 in this Summary. Because of the way in which the information is presented in Appendix 2A, it is more convenient to present eq. (1) in a somewhat different way:

$$\frac{N_{az}(1000)}{N_{ayz}} = \frac{P_y(0)\left\{\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}\ (0)}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right\}}{2P_y(60)\left\{\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}\ (60)}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right\}} \quad \ldots\ldots\ldots (2)$$

where

$N_{az}(1000)$ = the collision risk due to loss of vertical separation in a system using 1000 feet vertical separation for same-direction traffic. This quantity is discussed under Agenda item 2 of this Summary.

$P_y(0)$ = the probability of lateral overlap for aircraft nominally flying on the same track. A value of 0.0012 is used in Appendix 2A

$\overline{|\dot{y}|}\ (0)$ = the average relative across-track speed between a pair of aircraft nominally flying on the same track. A value of 20 kts is used in Appendix 2A.

Substitution of the values given above in eq.(2) gives:

$$\frac{N_{az}(1000)}{N_{ayz}} = 31 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

2.1.4 Equation (3) states, that a system with composite separation with same-direction traffic only is 31 times safer than the corresponding system with a vertical separation of 1000 feet. When interpreting this equation, a few points must be kept in mind:

i) in the calculation it has been assumed that all aircraft flying in the track system will remain on the same track and flight level during their complete flight in the Ocean areas. In sections 2.2 and 2.3 the added risk of changing levels and/or tracks is calculated.

ii) the value of N_{az} as calculated in Appendix 2A depends on the period between recalibrations of the static pressure error. The ratio of 31 only applies for comparing risks for which the same period between calibrations is used. All aircraft receive a check, if not a calibration, of their static pressure error before delivery by the manufacturer. If the service life of an aircraft is estimated to be 15 years, then aircraft which are not recalibrated during their service life should fall in the category of aircraft with a period between recalibrations of 15 years. The figures for that period between recalibrations will be used in the analysis below.

iii) in previous collision risk estimates all parameters in eq. (2) have been represented by single figures, not by distributions. The value of 31 can therefore be regarded as a fixed value.

2.1.5 The confidence distributions for N_{ayz} can therefore be derived from the lines labeled t = 15 years in figures 2.2b, 2.4 and 2.5 of Appendix 2A by dividing the figures on the horizontal axes by 31. If the 90 % confidence level is used, as in the analysis of lateral separation in the Summary of Discussions of the fourth meeting, then the estimated value of N_{ayz} will be:

- 0.12 if the very pessimistic assumption of a level tail to 1200 feet is used for the static pressure error
- 0.027 if the assumption of a level tail to 800 feet is used for the static pressure error
- 0.0006 if the optimistic assumption is made that the static pressure error will never exceed 300 feet.

2.2 The additional risk of changing flight levels on the same track.

2.2.1 If an aircraft in a composite track system climbs 2000 feet to the next flight level on its track, while the flight paths which are passed within 60 NM on the adjacent tracks are unoccupied, then there is no additional risk. If the flight paths on the adjacent tracks are, however, occupied then there will be some additional risk. This will be calculated in this section.

2.2.2 If the average rate of climb of the aircraft during the transition from one flight level to the other is $\dot{z}$, then there will be a period of $\frac{2\lambda_z}{\dot{z}}$ during each climb of 2000 feet in which the climbing aircraft has vertical overlap with an aircraft flying on an adjacent track. During this period the value of P_z will be 1. If the average number of climbs over 2000 feet per ocean crossing is p and if the average duration of a flight in the oceanic area is 3.25 hours, then the average proportion of the total flight time that this vertical overlap exists with an aircraft on an adjacent flight path will be

$$\frac{2\lambda_z \cdot p}{3.25\,\dot{z}}$$

The added risk per 10^7 flying hours due to these periods of vertical overlap will then be

$$\Delta N'_{ayz} = \frac{2\lambda_z \cdot p}{3.25\ \dot{z}} \cdot 10^7 \cdot \frac{\lambda_x}{S_x} \cdot E_y(\text{same}) \cdot P_y(60) \cdot \left\{ \frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}(60)}{2\lambda_y} + \frac{\dot{z}}{2\lambda_z} \right\} \quad ..(4)$$

where

E_y(same) = the lateral occupancy. A cautious estimate for the most probable value for this parameter can be calculated from figure 1.19 of the Summary of Discussions of the fourth meeting. This value is 0.61.

This added risk replaces the normal risk during the total duration of the climb. This total duration of the climbs over 2000 feet (= 0.33 NM) takes a proportion of the total flying time of

$$\frac{0.33\ p}{3.25\ \dot{z}}$$

The total added collision risk for a system with composite separation, in which the average number of climbs per ocean crossing is p, will be

$$N'_{ayz} = \Delta N'_{ayz} - \frac{0.33\ p}{3.25\ \dot{z}}\ N_{ayz} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where N_{ayz} is given by eq. (1) and $\Delta N'_{ayz}$ is given by eq. (4).

2.2.2 If the average rate of climb during transitions from one flight level to another at cruising altitudes is taken as 400 feet/min (about 4 kts), then the added collision risk per 10^7 flying hours due to transitions to other flight levels will become

$$N'_{ayz} = (0.018 - 0.025\ N_{ayz}) \cdot p \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

If it is assumed that the average number of climbs over 2000 feet will be 1 per ocean crossing (which is much more than practice shows at present), then the 90% confidence level of the total collision risk ($N_{ayz} + N'_{ayz}$) will be:

- 0.14 if the very pessimistic assumption of a level tail to 1200 feet is used for the static pressure error
- 0.044 if the assumption of a level tail to 800 feet is used for the static pressure error
- 0.019 if the optimistic assumption is made that the static pressure error will never exceed 300 feet.

2.3 The additional risk of changing tracks on the same flight level.

2.3.1 The changes of track over 120 NM at the same flight level can be treated in the same way as was done for the changes of flight level on the same track in the previous section. The added risk during periods of lateral overlap will be

$$\Delta N''_{ayz} = \frac{2\lambda_y \cdot q}{3.25\ \dot{y}}\ 10^7 \cdot \frac{\lambda_x}{S_x} \cdot E_z(\text{same}) \cdot P_z(1000) \cdot \left\{ \frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\dot{y}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right\} \quad \ldots (7)$$

where q = the average number of changes of track over 120 NM per ocean crossing.

Elimination of $P_z(1000)$ in eq.(7) can be done, as in section 2.1, by dividing by $N_{az}(1000)$. This gives

$$\frac{\Delta N''_{ayz}}{N_{az}(1000)} = \frac{2\lambda_y \cdot q}{3.25\ \dot{y}} \cdot \frac{1}{P_y(0)} \cdot \frac{\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\dot{y}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}}{\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}|}\ (0)}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}} \quad \ldots\ldots\ldots (8)$$

2.3.2 This risk replaces the normal risk during the total duration of the transition. This total duration of the lateral transitions over 120 NM takes a proportion of the total flying time

$$\frac{120\ q}{3.25\ \dot{y}}$$

The total added risk due to transitions over 120 NM at the same flight level therefore is

$$N''_{ayz} = \left(\frac{\Delta N''_{ayz}}{N_{az}(1000)} \cdot \frac{N_{az}(1000)}{N_{ayz}} - \frac{120\ q}{3.25\ \dot{y}}\right) N_{ayz} \quad \ldots\ldots\ldots\ldots (9)$$

where $\Delta N''_{ayz} / N_{az}(1000)$ is given by eq. (8) and $N_{az}(1000)/N_{ayz}$ is given by eq. (2).

2.3.3 A cautious value for $\dot{y}$ can be found if it is assumed that the transition from one track to another will be effected just within the period between 2 reporting points, i.e. 10 degrees in longitude or about 400 NM at the normal latitudes of North Atlantic traffic. Then

$$\dot{y} = \frac{120}{400} \cdot 480 = 144 \text{ kts}$$

Substitution of the values for all parameters in eq. (9) gives as the value for the added risk due to transitions to other tracks at the same flight level

$$N''_{ayz} = (0.38 \times 31 - 0.26)\ N_{ayz} \cdot q = 12 \times N_{ayz} \times q \quad \ldots\ldots\ldots (10)$$

2.3.4 This means that, if on the average 1 such transition should occur per flight, the collision risk would be increased by a factor of 13. As such transitions will, however, only occur very seldom in normal ocean traffic, the value of q will be much lower than one. The added risk will therefore still be small.

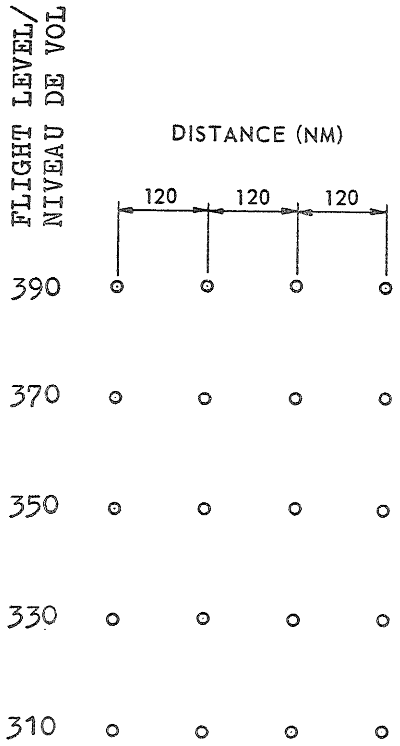


FIG. 1 THE PRESENT PATTERN OF FLIGHT PATHS IN THE TRACK SYSTEM.

DISPOSITION DES TRAJECTOIRES DANS LE SYSTEME ACTUEL.

FIG. 2 A PATTERN OF FLIGHT PATHS WITH LATERAL SEPARATION REDUCED TO 90 NM.

ESPACEMENT LATERAL RAMENE A 90 NM.

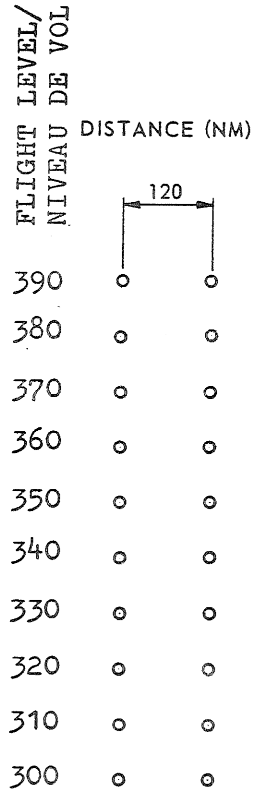


FIG. 3 A PATTERN OF FLIGHT PATHS WITH VERTICAL SEPARATION REDUCED TO 1000 ft.

ESPACEMENT VERTICAL RAMENE A 1 OOO PIEDS.

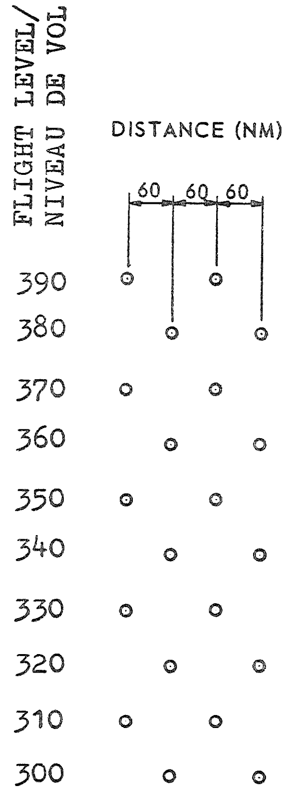


FIG. 4 A SYSTEM WITH STAGGERED SEPARATION.

TRAJECTOIRES EN QUINCONCE.

APPENDIX 3-B

List of References

1. Report on vertical separation study, North Atlantic Region, July 15 to September 30, 1963 - IATA Doc Gen. 1951 (1964).

2. Analysis of trailing-cone measurements of static pressure error of large transport aircraft. NLR Memorandum VM-68-19.

3. Progress report on the evaluation of the performance-check method for monitoring the static pressure error of commercial aircraft. NLR Memorandum VM-68-22.

4. Summary of Agenda Item 4: Review of progress made in the assessment of longitudinal separation in certain parts of the Region, including the possible use of DME for the assessment of such separation

4.1 Introduction

4.1.1 The Group had before it working papers on the estimate of collision risk in the longitudinal case: these covered the case where aircraft had monitoring devices, and the case where they did not. Papers were also presented concerning the ATC mechanics of implementing a monitoring system.

4.2 Reduction of Longitudinal Separation Standard without Monitoring Devices

4.2.1 The Group considered the question of procedural longitudinal separation without monitoring. The UK presented the results of RAE work on this subject and this is included in Appendix 4-A. Because the time available for discussion was limited, the assumptions contained in this Appendix were not exhaustively examined, but the Group thought that the basic assumptions appeared reasonable.

4.2.2 More significantly, it was apparent from the RAE work that the data sample used (obtained from the 1967/68 NAT data collection exercise) was insufficient to draw firm conclusions. Thus, although Table 6.1 of Appendix 4-A indicated that no pair of aircraft which were planned to be at intervals up to thirty minutes apart on the same track in fact lost more than ten minutes of separation, the sample size was limited to 132 pairs. If data on pairs up to a nominal sixty minutes spacing were included, and if losses as well as gains were taken as representative of changes in separation, and some additional data which is available were examined, then it might be possible to get a total sample size of about 1000 pairs.

4.2.3 However, it was pointed out, for example, that in order to ensure matching up to the target level of safety which had been set by the Group for the purpose of these studies, the probability of catching up 14.5 minutes would have to be less than 1 in 10 000. It followed that this number of proximate pairs would need to be examined, or more for a high level of confidence. The Group estimated that this might require data collection for a period exceeding three years, by which time the changes in aircraft equipment might make the data obsolete. The amount of data required would be about ten times that obtained during the lateral exercise.

4.2.4 The limitations on availability of relevant data in sufficient quantities made longitudinal separation less amenable to scientific analysis than vertical separation, which for the same reason was more difficult than the lateral problem

4.2.5 In these circumstances the Group decided that it was not desirable to proceed with data collection at this time to validate a change in the procedural longitudinal separation standard in the NAT Region.

4.3 Reduction in Errors in ETAs at the Oceanic Boundary

4.3.1 At present the air traffic controllers tend to allow for errors in the oceanic boundary ETA given by pilots before their entry in the oceanic area. This is done because such errors occur rather frequently. A more precise estimate will permit such allowances to be reduced, and therefore can help to reduce the effective spacing. It was apparent in discussion that the problem was less at the western oceanic boundary than in the east, probably because of the shorter distance from European terminals to the oceanic boundary. The U.K. informed the Group of procedures that were to be instituted by Shanwick in 1969 which, it was hoped, would improve the situation. These procedures are as follows: Aircraft when calling Shanwick for clearance will be asked to advise the time they passed their last reporting point. Shanwick will then calculate ETAs for the relevant oceanic entry points. It was hoped this practice would produce more accurate ETAs and lead to more efficient use of the airspace in the North Atlantic Region.

4.4 Reduction of Longitudinal Standard with Monitoring Devices

4.4.1 The Group then reviewed a working paper presented by the U.K. which reflected work done by RAE in estimating the effects of using DME to monitor longitudinal separation. Although only a limited amount of time was available for discussion the U.K. Member pointed out that the entire study was based on the following assumptions: that pilots would take action if separation was reduced to 40 NM (approximately 5 minutes flying time) and that it was reasonable for such action to be required on 1% of crossings. The effect of these assumptions is that a minimum separation of 9 minutes between reported entry times would be possible instead of the present 15 minutes (plus 3 minutes per 0.1 Mach in both cases). The system would provide a useful increase in capacity without requiring an unacceptable amount of throttle adjustment. The RAE study is attached in Appendix 4-B.

4.4.2 As it is necessary that the monitoring system (DME) has a reliability of almost 100%, a study of the reliability of DME modified for operation in an air to air mode would have to be carried out before finalizing any plan for implementation.

4.4.3 The Group then decided to discuss the general philosophy of monitoring longitudinal station keeping as a means of reducing the separation standard. It was pointed out during these discussions that in view of the efforts presently under way in the area of Collision Avoidance Systems (CAS) and Proximity Warning Indicators (PWI), satellite communications and surveillance systems, the the relative time scales should be examined closely and the cost/benefit aspects derived accordingly.

4.4.4 Although the Group did not feel that it could recommend any specific action at this time, IATA and IFALPA were both anxious that the door be left open to further study of the subject. IFALPA and IANC held as a matter of major importance in maintaining standards of safety the development and service testing of any such device which showed promise of reliably monitoring separation between aircraft. The Group felt that some of this further study might be correlated with analytical studies of other methods of monitoring longitudinal separation or with work on CAS/PWI systems. It would also be necessary to study the technical problems of aircraft equipment fit in order to ensure that the cost of the system could be justified.

APPENDIX 4-A

Methods of Estimating Collision Risks associated with Procedural Longitudinal Separation Standards

1 INTRODUCTION

This Appendix develops a theory of collision risk for longitudinal separation standards and presents the results of some preliminary analysis of data from the 1967 exercise. At first sight it seems unlikely that sufficient data could be analysed to assess collision risks accurately. It also seems unlikely that it would be safe to reduce the standard from the present value of 15 minutes to 10 minutes.

2 DESCRIPTION OF THE CONTROL SYSTEM

The method of calculating collision risk described in this paper is applicable to a system in which the longitudinal separation standard is applied to times as reported by aircraft at the start of each track (by re-clearing if necessary after receiving the aircraft reported times) but subsequent times reported by aircraft are ignored and reliance is placed on aircraft maintaining filed Mach numbers to avoid total loss of longitudinal separation. It is assumed that where the second aircraft of a pair is cleared at a higher Mach number the separation applied at the entry point is increased by 3 minutes per 0.01 Mach in accordance with current practice.

The system as described above differs from current Mach number techniques in that times reported by aircraft in mid-ocean are ignored.

3 THE COLLISION RISK EQUATION

3.1 Collision risks for the longitudinal case are given by an equation similar to those for the lateral and vertical cases, i.e.

$$N_{ax} = 2 \times 10^7 \pi_x(\text{std}) P_y(0) P_z(0) \left[\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

3.2 N_{ax} is the expected number of accidents per 10 million flying hours, due to failure of longitudinal separation. Each of the terms on the right-hand side of equation (3.1) is discussed briefly below.

3.3 $P_y(0)$: Probability of lateral overlap

3.3.1 $P_y(0)$ is the probability of lateral overlap (i.e. lateral separation less than a wingspan) for a pair of aircraft assigned to the same track. This quantity was calculated from a distribution of track-keeping errors calculated at the Fourth NAT/SPG Meeting as representing the navigation performance over the area. The distribution, calculated as described in sections 1.4.3.1.2 and 1.4.3.1.3 of Reference 1 is a weighted average of distributions observed by different radars. From this distribution the value $P_y(0) = 0.0012$ was calculated.

3.3.2 The fact that this value is so small shows that aircraft derive considerable protection from "longitudinal collisions" because of their poor track-keeping ability in the NAT region. The risks would be larger if navigation were more precise. The increasing use of Doppler would increase the risk only slightly, but the effects of introducing inertial navigation may be significantly large. Some thought should be given to the end of the oceanic track where aircraft coming off the ocean will navigate more accurately over a VOR, although this will be outside the coeanic control area.

3.4 $P_z(0)$: Probability of vertical overlap

3.4.1 $P_z(0)$ is the probability of vertical overlap (i.e. vertical separation less than aircraft thickness) for a pair of aircraft assigned to the same flight level. The Group agreed at its Second Meeting to retain a value of 0.25 for this parameter.

3.5 π_x(std): Probability of longitudinal overlap

π_x(std) is the probability that a typical aircraft has another aircraft, assigned to the same track and flight level, with longitudinal overlap (i.e. longitudinal separation less than an aircraft length). Methods of estimating π_x(std) will be discussed later in this paper.

3.6 λ_x: Average aircraft length

The collision risk equation in Reference 2 used a parameter Λ_x which was intended to represent the length of an average aircraft, adjusted if necessary to allow for fatal accidents caused by vortex penetration. The evidence from results of flight tests conducted by the United States and Canada did not reveal significant hazards. Further investigation is required before firm conclusions can be reached. The NAT/SPG agreed at its Fourth Meeting to assume that the vortex hazard is negligible and take Λ_x as equal to λ_x the metallic length of an average aircraft which is taken as 0.025 NM (150 feet) as specified in paragraph 4.15.13.1 of Reference 2.

3.7 λ_y: Average effective wingspan

λ_y is taken as 0.025 NM (150 feet) as laid down in 4.15.13.1 of Reference 2.

3.8 λ_z: Average effective height of an aircraft

λ_z is taken as 0.0066 NM (40 feet) as laid down in 4.15.13.1 of Reference 2.

3.9 $\overline{|\dot{y}|}$: Average relative across-track speed

$\overline{|\dot{y}|}$ is the average relative across-track speed of two aircraft, assigned to the same track, about to collide. A value of 20 knots is taken, as laid down in 4.4.2.1 of Reference 1.

3.10 $\overline{|\dot{z}|}$: Average relative vertical speed

$\overline{|\dot{z}|}$ is the average relative vertical speed of two aircraft about to collide. $\overline{|\dot{z}|}$ is taken as 1 knot, as specified in Section 4.4.2.1.3 of Reference 1.

3.11 $\overline{|\dot{x}|}$: Average relative along-track speed

$\overline{|\dot{x}|}$ is the average relative along-track speed of a pair of aircraft which collide due to loss of longitudinal separation. The group agreed (4.4.2.3.1 of Reference 1) that the value of 13 knots used for lateral and vertical collision risk calculations was not appropriate to longitudinal separation since $\dot{x}$ must be larger if longitudinal separation is to be lost during the period taken to cross the NAT OCA. Methods of estimating $\overline{|\dot{x}|}$ will be discussed later in this paper.

3.12 Collision Risk

Substituting the numerical values above into equation (3.1) one has:

$$N_{ax} = 2 \times 10^7 \, \pi_x \, 0.0012 \times 0.25 \left[\frac{\overline{|\dot{x}|}}{0.05} + \frac{20}{0.05} + \frac{1}{0.0132} \right] \quad \ldots\ldots\ldots\ldots (3.2)$$

Thus:

$$N_{ax} = 120\,000 \, \pi_x \, (\overline{|\dot{x}|} + 23.8) \quad \ldots\ldots\ldots\ldots (3.3)$$

4 PROBABILITY OF LONGITUDINAL OVERLAP

4.1 The problem of estimating the probability of loss of longitudinal separation is different from the corresponding problems for lateral or vertical separation. In the vertical case for example, if the separation standard were 1000 feet, any pair of aircraft would be assigned flight levels which differed by zero, or 1000 feet, or 2000 feet, and so on, and the "vertical collision risk" would be due almost entirely to pairs with a nominal separation of 1000 feet. The longitudinal case is different because if for example the minimum standard were 15 minutes aircraft could be put into the system with nominal separations of 15, 16, 17 or 18 minutes and so on, with significant and different contributions to collision risk from each of these separations. Thus it is necessary to calculate how likely each of these separations is (i.e. a set of numbers, or a continuous function) instead of a single number (occupancy or proximity) associated with a single possible separation.

4.2 Thus instead of a single number E_x one has a set of numbers $E_x(m)$ which denote the probabilities that the time reported by a typical aircraft at the beginning of the track differs from that of the next following aircraft on the same path, after adjusting for Mach number difference as described in Section 2, by m minutes.

4.3 The distribution of separation loss, after adjusting for Mach number difference has a probability distribution function denoted by $P_x(u)$.

4.4 These two distributions have to be convolved to give the probability distribution of the actual resultant separation. It is likely that the separation loss will be correlated with initial separation and thus that the estimates obtained will be pessimistic.

4.5 Formally

$$f(x) = \sum_m E_x(m)\, P_x\,(m-x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

where $f(x)$ is the probability distribution function of the resultant separation. π_x is the probability that the separation of a typical aircraft from the following aircraft has shrunk to less than λ_x, plus an equal quantity for the probability of overlap with the preceding aircraft.

4.6 Thus apparently

$$\pi_x \ (std) = 2\int_{-\lambda_x}^{\lambda_x} f(x)dx$$

$$\doteqdot 4\lambda_x \ f(0)' \text{ assuming } f(x) \text{ is a reasonably smooth function.}$$

4.7 However it must be remembered to allow for the fact that the data on $f(x)$ will be in minutes and so λ_x must be expressed in minutes of flying time. An average speed of 8 miles per minute is assumed, so

$$\lambda_x = \frac{0.025}{8} = 0.003125 \text{ minutes}$$

$$\text{and } \pi_x(std) = 0.0125\ f(0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

4.8 Using equation (4.1) to substitute for $f(x)$ in equation (4.2) one obtains:

$$\pi_x(std) = 0.0125 \sum_m E_x(m)\, P_x(m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.3)$$

4.9 <u>Collision risk</u>

Substituting this expression for $\pi_x(std)$ in equation (3.3) gives:

$$N_{ax} = 1500\ (|\overline{\dot{x}}| + 23.8) \sum_m E_x(m)\, P_x(m) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.4)$$

This expresses the collision risk in terms of the relative speed $\dot{x}$ and the convolution of two functions:

(i) E_x the "longitudinal occupancy distribution"

(ii) P_x the "loss of separation" distribution.

4.10 Heuristic explanation

For any given value of m, $E_x(m)$ is the probability that a pair of successive aircraft start off with m minutes nominal separation, and $P_x(m)$ is the probability that they lose (m $\pm$0.5) minutes separation. Thus if these two events are independent the probability that both occur and the aircraft are abreast (within 0.5 minutes) is $E_x(m)P_x(m)$. To find the probability of the aircraft being side-by-side one must add up the probabilities for all permissible values of m, i.e. for all values greater than or equal to the separation standard. In this way one obtains the probability

$$\sum_m E_x(m)P_x(m)$$

and the collision risk is proportional to this expression.

5 LONGITUDINAL OCCUPANCY FUNCTION $E_x(m)$

5.1 $E_x(m)$ is a set of numbers which denote the probabilities that the reported time of a typical aircraft at the beginning of the track differs from that of the next following aircraft on the same path, after adjusting for Mach number difference as described in Section 2, by exactly m minutes.

5.2 A reasonable approximation for $E_x(m)$ can be deduced from Tables 7 and 8 of Reference 3. From Table 7 (Eastbound aircraft) the original distribution of separations is shown in Table 5.1. (The total number of intervals can be estimated from Table 1 of Reference 2 but was in fact checked by re-processing the data.)

Table 5.1

Approximate distribution of initial separations for Eastbound traffic

Separation	Number	Proportion
15	49	0.018
16	29	0.010
17	28	0.010
18	45	0.016
19	46	0.016
20	44	0.016
21	44	0.016
22	43	0.015
23	43	0.015
24	43	0.015

Table 5.2

Approximate distribution of proximities for Westbound traffic

Separation	Number	Proportion
15	37	0.014
16	23	0.008
17	23	0.008
18	26	0.010
19	26	0.010
20	30	0.011
21	31	0.011
22	31	0.011
23	31	0.011
24	31	0.011

5.3 The proportions listed in the tables are not quite the same as the required values $E_x(m)$ since they are not quite at the beginning of the track and no adjustment has been made for difference in filed Mach number. Thus all subsequent calculations will be rough approximations.

5.4 As a convenient approximation to the values in tables 5.1 and 5.2 one takes $E_x(15) = E_x(16) = E_x(17) = \ldots\ldots = 0.014$ .... (5.1)

For a different separation standard it is assumed that the distribution is shifted sideways. For example if the minimum separation standard were 10 minutes one should have

$$E_x(10) = E_x(11) = E_x(11) = \ldots\ldots = 0.014 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.2)$$

5.5 Collision Risk for 1967 Traffic

Substituting the value from equation (5.1) for $E_x(m)$ in equation (4.4) one obtains

$$N_{ax} = 1500\,(|\bar{\dot{x}}| + 23.8) \sum_{m=15}^{\infty} 0.014\ P_x(m) \quad \ldots\ldots\ldots\ldots\ldots \quad (5.3)$$

and thus

$$N_{ax} = 21\,(|\bar{\dot{x}}| + 23.8) \sum_{m=15}^{\infty} P_x(m)$$

so that

$$N_{ax} = 21\,(|\bar{\dot{x}}| + 23.8)\ P_x\,(>14.5) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.4)$$

where $P_x\,(>14.5)$ denotes the probability that a pair of aircraft lose more than 14.5 minutes separation (after allowing for differences in filed Mach number). Equation (5.4) may be regarded as an approximation to equation (4.4).

5.6 Collision risks for 1969-74

To allow for the increased traffic expected in 1969-74 (compared with 1967) the occupancy function in equation 5.1 should be increased to:

$$E_x(15) = E_x(16) = E_x(17) = \ldots = 0.02 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.5)$$

and the collision risk will be increased in the same proportion so equation 5.4 is replaced by

$$N_{ax} = 30\,(|\bar{x}| + 23.8)\,P_x\,(>14.5) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.6)$$

6 LOSS OF PLANNED SEPARATION

6.1 Introduction

In paras. 4 and 5 it was shown (not surprisingly) that the collision probability depends on how likely aircraft are to lose their planned separation. More formally a distribution of loss of separation $P_x(m)$ is required for equation (4.4) or a value of $P_x(>14.5)$ for the approximate equation (5.4).

6.2 Amount of data required

In discussing the amount of data required, and the adequacy of existing data, it is convenient to use equation (5.4). Making a reasonable guess at $|\bar{x}|$, N_{ax} is approximately 1500 times $P_x(>14.5)$ so for N_{ax} to be less than 0.15 (the lower boundary of the NAT/SPG target band), the probability of catching up 14.5 minutes would have to be less than one in 10 000. To demonstrate that such a level had been achieved in a given area it would seem to be necessary to observe 10 000 proximate pairs of aircraft (or more, if a high confidence level were required). However it might be possible to deduce that the average risk over the NAT Region as a whole was within the limit on the basis of a smaller sample taken at the end of the oceanic crossing, plus an agreed theoretical model of the way in which the loss of separation builds up during a crossing.

6.3 Data available

Some data are available from that collected during the 1967/68 data collection exercise. A search was made of the Gander radar data for pairs of westbound aircraft on the same track and initial flight level, close together in time. No aircraft was counted as part of more than one pair. 301 pairs were found with less than 60 minutes initial separation, and a second search found 132 pairs with less than 30 minutes. Larger samples should be available later when azimuth errors which occurred between 28 February and 31 March 1968 have been corrected. For each pair the final along-track separation was calculated from the radar record. The time interval between the sightings had to be adjusted to allow for the fact that the aircraft were not seen in exactly the same place.

6.4 The initial separation was found by comparing reported times at the boundary, which would be used by ATC as the basis for planned separation.

6.5 The gain of separation is found by subtracting the initial separation from the final separation, negative results denoting loss of separation.

6.6 ATC also has available and uses the Mach number which each aircraft is trying to maintain. Therefore the "unexpected gain of separation" has been calculated which remains after subtracting from or adding to the gain of separation an amount of 3 minutes for each 0.01 Mach number difference between the two aircraft concerned. Table 6.1 shows the results of these calculations.

Table 6.1

Distribution of unexpected loss or gain of separation
(Westbound flights)

Unexpected gain of separation (minutes)	Criterion of longitudinal proximity	
	30 minutes	60 minutes
-9 to -8	0	2
-8 to -7	2	2
-7 to -6	1	5
-6 to -5	4	14
-5 to -4	7	12
-4 to -3	15	18
-3 to -2	15	29
-2 to -1	15	37
-1 to 0	16	36
0 to 1	13	33
1 to 2	9	17
2 to 3	13	34
3 to 4	5	19
4 to 5	7	18
5 to 6	3	11
6 to 7	3	5
7 to 8	3	5
8 to 9	1	3
9 to 10	0	1
Total	132	301

6.7 Are gains typical of losses?

At the Fourth Meeting of the Group there was some discussion of the possibility of regarding the distribution of gains of separation as equal to the distribution of losses, thus doubling the effective sample size.

The causes of gains of separation are identical with the causes of losses, but it is possible that pilots might not notify changes of Mach number so strictly if they were sure they had no other aircraft in front of them, and this would make gains more likely than losses. The differences shown in Table 6.1 are such as might reasonably be expected to occur by chance, and accordingly the best way of using this limited data seems to be to take gains as typical of losses, i.e. to assume the values given are samples from a symmetrical distribution. This assumption could be reviewed when a larger sample is available. Figure 4.1 on page 4-A-13 is based on the third column of Table 6.1, and shows the proportion of pairs which lost or gained more than m minutes, after correcting for differences in Mach number.

6.8 Checking of large values

All available details of the pairs which lost or gained more than seven minutes were examined carefully. One pair which had lost 12.8 minutes was rejected because they had not remained at the same flight level throughout (but if the change had been a consequence of the unexpected loss of separation it should have been retained, to avoid a "selective bias" against large losses).

6.9 Apparent causes of large gains and losses

6.9.1 Inaccurate reports at entry

From comparisons between all the position reports of the aircraft-pair it would appear in some cases that an incredibily large change of separation has taken place shortly after entering the OCA. For example aircraft reporting 43 minutes apart at 15°W report 35 minutes apart at 20°W which would require a difference in Mach number of about 0.2. From examination of the way in which separation (between reported times) varies across the ocean it seems that the boundary reported times do not reflect the true positions of the aircraft. The ultimate cause or causes of these errors are not yet known. Four of the largest 13 values in Column 5 of Table 6.1 arose in this way.

6.9.2 Persistent Mach number errors

The ATC data provided by Shanwick OAC includes airframe registration numbers for about 90% of the flights, and this information was coded and put into the computer by RAE. While investigating loss of longitudinal separation it was therefore possible to collect together information on all aircraft-pairs of which a particular airframe was a member. Thus it would eventually be possible to distinguish those aircraft which consistently fly faster (or slower), relative to nearby aircraft, than the cleared Mach numbers would lead ATC to believe. A preliminary list has been drawn up confined to those 62 airframes which appeared in 20 or more pairs. For each of these the "average discrepancy" is calculated. An average time discrepancy of 3 minutes corresponds to a Mach number error of 0.01. The distribution of average discrepancies is shown in table 6.2. and it is understood that, in that table, positive discrepancies indicate that aircraft flew faster than expected. The large values in this table have not yet been checked. In this analysis a single flight was allowed to form part of more than one pair and a pair was defined as within 60 minutes on the same or adjacent flight levels.

Table 6.2

Distribution of average time-discrepancies of frequently observed aircraft

Average discrepancy (minutes)	Number of airframes
-5 to -4.1	1
-4 to -3.1	2
-3 to -2.1	4
-2 to -1.1	10
-1 to -0.1	17
0 to 0.9	10
1 to 1.9	10
2 to 2.9	7
3 to 3.9	1
TOTAL	62

6.10 Eastbound flights

In some cases times recorded manually at Kilkee are incompatible with times reported by aircraft at 15°W. It is suspected that the times recorded by Kilkee may be in error, as the recording of times was a secondary task (compared with recording positions) and the clock was at a distance from the staff and unconventionally arranged. (All 24 hours indicated sequentially on the dial.)

Table 6.3 was compiled using automatically recorded data from Kilkee in the same way as Table 6.1 and shows corresponding results for Eastbound flights.

Table 6.3

Distribution of unexpected loss or gain of separation

(Eastbound flights)

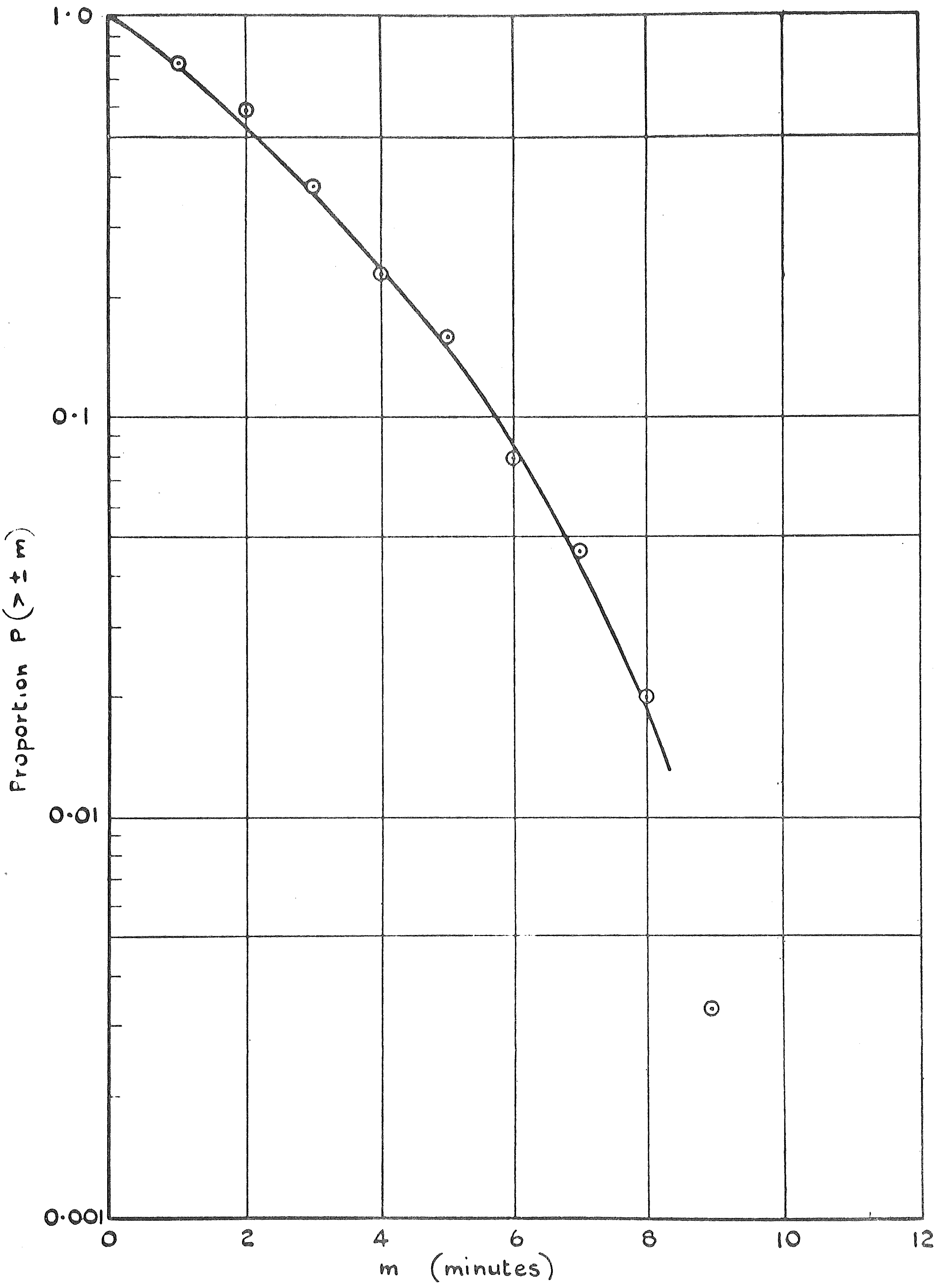
Unexpected gain of separation (minutes)	Criterion of longitudinal proximity	
	30 minutes	60 minutes
-9 to -8	0	2
-8 to -7	2	2
-7 to -6	3	7
-6 to -5	5	9
-5 to -4	11	15
-4 to -3	5	7
-3 to -2	11	17
-2 to -1	15	27
-1 to 0	20	30
0 to 1	14	35
1 to 2	13	27
2 to 3	11	23
3 to 4	9	15
4 to 5	4	4
5 to 6	2	4
6 to 7	2	7
7 to 8	3	3
8 to 9	2	1
9 to 10	1	2
TOTAL	133	237

7 RELATIVE ALONG-TRACK SPEED

7.1 $\dot{x}$ represents the relative along-track speed of a pair of aircraft about to collide due to loss of longitudinal separation. It is clear that such a pair must have a large average relative speed in order to lose separation, but the relative speed may be reduced as the pair close up and ambient conditions become more nearly equal.

7.2 Estimates of $\dot{x}$ were made for the pairs giving the largest values (7 minutes or more) in Table 6.1. Some of these cases occurred essentially because an expected change of separation had not taken place, so the $\dot{x}$ values were not relevant. In the other cases the average value of $\dot{x}$ was only 16 knots. This is not an accurate figure as it is difficult to estimate speeds accurately from Gander radar records, because aircraft switch to a domestic SSR code soon after entering radar cover.

FIG. 4.1



PROPORTION OF PAIRS SHOWING A LOSS OR GAIN OF SEPARATION OF MORE THAN m MINUTES, AFTER CORRECTING FOR DIFFERENCES IN CLEARED MACH NUMBER (See para. 6.7, page 4-A-9)

PROPORTION DES CAS (PAIRES D'AVIONS) OU A ETE OBSERVE UNE PERTE/ UN GAIN DE PLUS DE m MINUTES D'ESPACEMENT, CORRECTION FAITE POUR L'ECART DES MACH AUTORISES (par. 6.7, page 4-A-9)

LIST OF REFERENCES

1 Summary of Discussions of the Fourth Meeting of the NAT Systems Planning Group. Paris, 17 - 28 June 1968.

2 Summary of Discussions of the Second Meeting of the NAT Systems Planning Group. Paris, 21 November - 5 December 1966.

3 Studies of traffic packing for estimating mid-air collision risks over the North Atlantic. (By P.P. Scott) RAE Technical Report 68097.

APPENDIX 4-B

Estimation of the Effects of using DME to monitor Longitudinal Separation

1 INTRODUCTION AND SUMMARY

1.1 In this Appendix the data collected during the 1967/68 exercise is used to assess some aspects of a possible system using DME to monitor longitudinal separation over the North Atlantic.

1.2 It is estimated that, if the minimum separation between reported entry times were 9 minutes (plus 3 minutes per .01 Mach as at present), the proportion of aircraft closing up to within 5 minutes and requiring to take action to maintain separation will be about 1 per cent. This is thought to indicate that the system will provide a useful increase in capacity without requiring an unacceptable amount of throttle adjustment.

1.3 One requirement for the system to be safe is that a loss of separation requiring action should be conspicuously displayed to the pilot.

1.4 The study is equally valid for other methods of monitoring longitudinal separation, for example by satellites or ocean platform radars.

2 DESCRIPTION OF THE SYSTEM STUDIED

2.1 This Appendix describes the assumptions concerning the nature of the system which are necessary to the study.

2.2 It is assumed that the proposed system together with the proposed DME fit would enable a pilot to continuously monitor his distance from the preceding aircraft on the same track, unless the aircraft concerned were so far apart that he could never catch up.

2.3 One of the parameters of the system would be an "action threshold", and whenever separation fell below this level the pilot would be required to reduce speed to maintain this separation, and probably to inform ATC. This threshold value would probably be determined by the requirements of the transition area at the end of the oceanic area. Provisionally a value of 5 minutes flying time (40 NM) is assumed. The threshold value must be large enough to allow the pilot to avoid a collision, but at the relative speeds likely to occur this is a less exacting requirement. Even with a speed difference of .06 Mach (36 knots) he would have over an hour to avoid a collision.

2.4 It is uneconomic to fly an aircraft below its optimum speed, and distracting to have to continuously adjust the aircraft throttle. It is therefore assumed that aircraft entry times will be separated by at least an "entry minimum" value, which will be larger than the "action threshold" and will be chosen so that the action threshold will be reached on only about 1% of flights. Since the entry minimum is not determined by safety requirements it would be possible to find the optimum value by a process of trial and error, if a record were kept of occasions when the action threshold were reached. However it is very desirable to estimate the "entry minimum" at the present time, to aid decisions on whether the system is worthwhile, and this will be discussed in this Appendix.

2.5 It is assumed that ATC will continue to allow for differences in cleared Mach number by adding 3 minutes per 0.01 mach to the "entry minimum" when a faster aircraft follows a slower one.

2.6 Since a pilot will only be required to react to the DME about once per 100 crossings, it seems quite likely that he could overlook an indication of danger unless the system is carefully designed to ensure that the equipment is correctly used, and infringement of the action threshold conspicuously displayed in the cockpit.

3 PROBABILITY THAT ACTION IS REQUIRED

3.1 In this Appendix it is first assumed that the "entry minimum" standard is 10 minutes and the proportion of aircraft which will reach the "action threshold" and need to take corrective action is calcultated therefrom. By repeating the calculations for other values of "entry"minimum" the smallest value meeting the 1% criterion can be found.

3.2 Distribution of initial proximities

In Appendix 4-A it was shown that the distribution of intervals between aircraft entering the ocean can be approximated by

$$E_x(15) = E_x(16) = E_x(17) = 0.014 \qquad (3.1)$$

If the minimum entry interval were 10 minutes instead of 15 one might assume that

$$E_x(10) = E_x(11) = E_x(13) = 0.014 \qquad (3.2)$$

but this may be optimistic and it seems more prudent to assume that the reduced standard will enable more aircraft to be put on each path so that

$$E_x(10) = E_x(11) = E_x(12) = \ldots\ldots = 0.021 \qquad (3.3)$$

and that the expected increase in traffic between the data collection period (1967) and the DME implementation period (1969-74) will increase these values in proportion to the traffic to

$$E_x(10) = E_x(11) = E_x(12) = \ldots\ldots = 0.03 \qquad (3.4)$$

3.3 A proportion 0.03 of westbound aircraft will start with a ten-minute gap ahead of them (after allowing for the Mach number difference) and of these the proportion reaching the "action threshold" will be the proportion catching up more than 5 minutes before reaching Gander. From Fig. 4.1 of Appendix 4-A this proportion is 0.5 x 0.14 = 0.07. Similarly a proportion 0.04 of those aircraft starting with 11 minutes separation will reach the "threshold". The total proportion of aircraft requiring to take action will be

$$0.03\ (0.07 + 0.04 + 0.02 + 0.01)$$

or about 0.4%.

3.4 A similar calculation for an entry minimum of 9 minutes gives the initial distribution

$$E_x(9) = E_x(10) = E_x(11) \ldots. = 0.033 \qquad (3.5)$$

and the proportion reaching the "action threshold" as

$$0.033\ (0.12 + 0.07 + 0.04 + 0.02 + 0.01)$$

or about 0.9%.

4. <u>CHAIN REACTIONS</u>

4.1 It has been suggested by several people that a situation might occur in which a chain of ten or twelve aircraft were spaced along a path at five minute intervals, each pilot constantly adjusting speed to maintain separation. This idea is probably suggested by experience of car-driving. It will be shown that such an event cannot occur, because the entry minimum is larger than the "action threshold".

4.2 To reach the "action threshold" the second aircraft must catch up 4 minutes on the first (assuming an entry minimum of 9 minutes and an "action threshold" of 5 minutes). A third aircraft would have to catch up 4 minutes on the second aircraft, so for a chain of three aircraft, each one of which operating at the "action threshold", the third aircraft would have to gain 8 minutes relative to the first. Similarly if four aircraft were cleared onto a path at 9 minute intervals (an extremely unlikely event) they would not form a chain unless the fourth aircraft gained 12 minutes relative to the first. For a chain of 10 aircraft to form, the 10th aircraft would have to catch up 36 minutes on the first aircraft, requiring a difference in speed of about 0.12 Mach. It is therefore concluded that the formation of long chains of aircraft, each operating on the "action threshold" is impossible.

5. Summary of Agenda Item 5: Review of the criteria for the assessment of system performance

5.1 Introduction

5.1.1 Under this agenda item, the Group reviewed a paper presented by the United Kingdom member which describes the method used and the results obtained from an analysis of data obtained during the NAT Data Collection Programme of 1967/68 as regards deviation penalties imposed on individual flights in the NAT Region by air traffic control.

5.1.2 The method used for the analysis by the United Kingdom is briefly described in Appendix 5-A and the results of the analysis as well as the major points raised in the discussion of this subject are given below.

5.2 Results of analysis

5.2.1 The United Kingdom developed a computer programme which completes all the steps of the analysis described in Appendix 5-A. From the raw data obtained from forms 1-D of the NAT Data Collection Programme, the Programme estimates the ATC penalty for each flight in terms of extra distance flown and average height deviation. It constructs a histogram showing the proportion of flights which suffer each particular combination of horizontal and vertical deviation. It then estimates the fuel penalty for each flight. From the distribution of fuel penalties, the Programme computes the ATC fuel reserve increment. For this part of the Programme the data processing used was that contained in an RAE study of transatlantic fuel reserves (see No.3 in Appendix 5-B) and which computed the reserve requirements for Boeing-707.

5.2.2 From the output of this Programme, the 'LOW', 'MID' and 'HIGH' cost estimates were computed, as defined in Appendix 5-A.

5.2.3 During the development of this Programme, it was applied successively to samples of 200, 400, 700 and 1600 flights, chosen at random from those flights on which data regarding the "ideal" or "requested" track or level had been supplied on the backs of forms 1-D. The results of all these runs, whether expressed in terms of cost, fuel reserve or general pattern of deviations are very similar. This suggests that the sampling error is small. For this reason, although a somewhat larger sample could have been taken, it was thought that the sample size of 1600 flights was sufficient, and the following results refer to this sample.

5.2.4 The mean value of the penalty was 4 .17 minutes and that of fuel penalty 1182 lb. The corresponding 'LOW' cost penalty per flight is $36.90. The fuel reserve increment required to cater for ATC needs is 3188 lb. The cost of extra fuel burnt because of the weight penalty was estimated as $20.10. The 'MID' cost estimate is thus $57 per flight.

5.2.5 The method of estimating the 'HIGH' cost is slightly different from that described in the report mentioned under No.1 in Appendix 5- B. This is because in this regard reference is made to fuel reserve increments above those carried in the present system, while, when estimating the 'HIGH' cost, one is concerned with a fuel quantity already included in the current fuel reserve level. In effect, one considers what average <u>increase</u> in revenue could be obtained if the fuel reserve could be reduced by 3188 lb (say by a hypothetical reduction of oceanic separation standards to zero). The resulting estimate of revenue penalty is $16.50. The total of 'HIGH' cost is thus $73.50 per flight.

5.3 Comparison with Theoretical Model

5.3.1 The simplest theoretical model of the Atlantic system is one where it is assumed that all traffic plies between a single pair of terminals and moves only in one direction at a given time. The resulting distribution of aircraft among tracks and flight levels can then be estimated by means of formulae derived from the queuing theory. The cost penalties are derived from the distribution. To apply these formulae an estimate of the flow rate is also required. A figure of 20 aircraft/hour has been assumed as a reasonable estimate of the busy-hour rate during the period in question (No.4 in Appendix 5-B refers). The simple model then gives the following penalties.

'LOW' cost	-	$17.90
'Fuel reserve increment	-	2424 lb
'MID' cost	-	$33.20
'HIGH' cost	-	$45.80

5.3.2 The results from the NAT data study in terms of 'HIGH' cost penalty are thus 60% greater than those arising from the simple model. Possible reasons for this discrepancy are discussed below.

5.4 Opposite direction traffic flows

5.4.1 The east-and-westbound transatlantic flows are not completely separate in time but there is an appreciable overlap. On some occasions the two flow streams will be separated in space by wind effects, but not when the wind speed is small or when it varies little with latitude. Studies of the traffic flow when eastbound and westbound flows coincide have been made and it was found that the resulting distribution of aircraft among paths is less optimal than when the traffic flows are clearly separated.

5.4.2 It has been found that when the traffic flow is divided in a ratio of 20% to 80% for each direction along the same track, the cost penalty is about 20% greater than that when the two traffic flows follow two different tracks. Since the situation where the two flows of traffic follow the same track occur for only part of the time, the overall increase in penalty for this reason will however be less than stated above.

5.5 Multiplicity of terminals

5.5.1 Because of the large number of terminals on each side of the Atlantic, the preferred tracks for the transatlantic flights on any given day will normally not be parallel but will intersect each other at a variety of angles, usually small. It is the practice of ATC to smooth out these tracks into an organized track system, representing 'the greatest good for the greatest number'. For any flight whose planned track is not coincident with one of those of the track system, there will be a built-in penalty, even though it may be assigned the best possible track of the system. A simple geometrical treatment shows that this penalty is proportional to the square of the angle of intersection between the planned track and those in the organized track system. Where this angle is 5°, the study shows an average penalty of 2.7 minutes extra flying time. This will correspond to the following penalties:-

'LOW' cost estimate	- $18.50
Increment of fuel uplift	- 475 lb.
'MID' cost estimate	- $19.60
'HIGH' cost estimate	- $20.50

5.5.2 Bearing in mind that angles of intersection exceeding 10° are possible, it appears that this particular penalty may account for much of the difference between the results obtained from the NAT Data Collection Programme and the simple model. Two points which support this contention are:

i) Detailed study shows that there is a greater spread in distance flown in actual operations than was obtained from the theoretical model. The vertical distributions of aircraft are more nearly similar.

ii) Analysis of aircraft deviations show that the number of deviations resulting from the inability of ATC to comply with pilot's request after take-off is much smaller as regards horizontal deviations and therefore result in a lower cost penalty. This is to be expected since pilots are aware of the track system and their requests are made in relation to it. The study based on the NAT Data Collection Programme was however based on filed flight plans and willtherefore include the penalty for any divergence between the planned track and that eventually assigned in the organized track system.

5.5.3 To obtain further information on this aspect the programme has recently been extended to cover primarily "fringe routes", i.e. flights which follow a track which, because of its orientation cannot be integrated in the organized track system and intersect the main traffic flow on this system at appreciable angles. Initial results obtained so far seem however to indicate that the penalties imposed on those flights are <u>less</u> than those for flights following the main flow.

5.6 Summary of the study

5.6.1 The results of the study of deviation penalties show that the simple theoretical model developed by RAE somewhat under-estimates the deviation penalties in the system. However a logical extension of this model taking account of opposite direction traffic streams and the multiplicity of terminals should adequately represent the system for the purpose of estimating penalties.

5.7 Discussion of the study

5.7.1 When discussing the study presented by the United Kingdom member, a number of points were made and these are briefly listed below:

5.7.1.1 It was pointed out that the study had been strictly limited to the penalties incurred by flights while operating in the NAT Region. This was believed to be essential if any results at all were to be obtained since the analysis of penalties suffered by NAT flights while operating within the domestic airspace at either side of the North Atlantic would involve consideration of such a number of complex and complicated factors that it was unlikely that any assessment at all could be made which would prove to be generally acceptable.

5.7.1.2 It was also indicated that the present study assessed as "ATC penalties" those deviations which resulted from requests made by pilots while operating in the NAT area since insufficient data was available to eliminate these from the analysis. It was however stated that the United Kingdom intended to look further into the matter in order to see whether it had a significant effect on the results so far obtained.

5.7.1.3 When considering whether, due to the difference between forecast and actual meteorological conditions, it might not be possible that, in certain cases, the ATC imposed deviations resulted in fact in an improvement rather than in a penalty for the flight concerned, it was believed that, since this was a random factor and because of this tended to cancel itself out, it could be neglected.

5.7.1.4 It was also stated that the study had revealed that there existed a relationship between penalties and the lateral separation standards employed in the NAT Region, in the sense that any reduction in lateral separation would result in a reduction of penalties which was directly proportionate to the amount which lateral separation was reduced. (For instance, a reduction from the present value of 120 NM lateral separation to 90 NM would result in a 25% reduction of the cost of penalties).

5.7.1.5 As regards the question of fuel reserves required in general and those required to cater for ATC imposed deviations in the Atlantic portion of a flight it was stated that fuel reserves could be related to regularity of operations taking into account normal aircraft performance and a regularity figure of 99%. This established the general fuel reserves. By this process it is possible to isolate the element of fuel reserves required for ATC imposed deviations alone; it is found by comparison that these extra fuel reserves amount to approximately 15% of the total fuel reserves required. It was however stated that the above brief summary of the process used in obtaining information on this matter was extremely complicated in application since it required the evaluation of a rather large mass of data. In addition, it was stated that although there is always a cost involved in the carriage of reserve fuel, a considerable economic difference will only occur when the elimination of this reserve enables the flight to carry additional payload.

5.7.1.6 Finally, it was stated that the study had revealed that the present ATC system operating in the NAT Region was capable of exploiting approximately 70% of the theoretical capacity offered by the system and that, with further refinements applied, especially to the technique of assignment of tracks to flights and their proper sequencing at the entry points of the system, there was still room for improvement.

5.8 Conclusions

5.8.1 The Group noted that, in view of the circumstances of its collection, the information recorded on the backs of form 1-D should be treated with some reserve. Nevertheless it believed, that the method used by the United Kingdom in arriving at an assessment of system performance had considerable merit since it provided a practical tool for obtaining meaningful values in terms of cost in order to demonstrate the degree of efficiency of a system.

5.8.2 It therefore noted with satisfaction that the United Kingdom was prepared to continue its studies on this subject with a view to achieving further improvements in those areas where this appeared possible. It also noted that the United Kingdom was prepared to make available any further results in a comprehensive document which would be made available to the Group well in advance of its next meeting and it was stated that this documentation would also cover the effects of a fixed ATS route structure for sub-sonic air traffic in the NAT Region on the cost penalty situation.

5.8.3 In view of this, and fully aware of the need to achieve further progress in this important matter, the Group agreed that the subject of assessment of system performance should be included in the agenda of its next meeting.

APPENDIX 5-A

Description of the method used in an analysis of deviation penalties based on the NAT Data Collection Programme of 1967-68

1. INTRODUCTION

1.1 The Royal Aircraft Establishment (RAE) of the United Kingdom has conducted a study of the cost penalties currently incurred by aircraft due to oceanic ATC deviations on the North Atlantic. This study is based on information supplied by IATA on data sheets of transatlantic flights giving:

i) the track and flight level requested by the pilot in his filed flight plan;

ii) the track and flight level which would have been requested by the pilot in a completely free environment;

iii) the track and flight level assigned by ATC in the oceanic clearance.

1.2 The flights in question took place during the months of July and August 1967. The cost penalties arising from these deviations have been estimated by a method described in RAE Technical Report 66148 of 1966 (see Appendix 5-B, No. 1.) This study was to allow the comparison of the results of the study of actual deviations with those which were obtained from a theoretical model of the transatlantic traffic system, in order to show how far the theoretical model is valid and can be applied to hypothetical situations, e.g. an increase in the flow of air traffic, or the effect of reducing separation standards.

2. METHOD OF ASSESSING DEVIATIONS

2.1 The basic information for each flight is given on the NAT POST-FLIGHT ANALYSIS Form 1-D. The form itself contained information on the ATC cleared track of the flight concerned and that in terms of the latitudes at which the ATC assigned track crossed certain longitudes and the assigned flight levels at these points. Some airlines have added data on the back of the form on the flight track they requested and that which they would have regarded as ideal, or both (whenever both the 'ideal' and the 'requested' tracks and levels are given, and they differ, the deviation was assessed from the 'ideal' track and level). This data formed the basis of the study.

2.2 An estimate will have to be made of the increase in air distance flown arising from ATC deviation. This will be affected by the wind structure in the Atlantic which may distort the shape of the least-time track from the great circle between origin and destination. In order to simplify the study, the extra distance to be flown was estimated by the following method:-

i) the difference in latitude between the actual and the ideal or requested tracks at each reporting meridian is noted;

ii) a hypothetical track is constructed whose differences in latitude from the great circle at each reporting meridian correspond to those in i).

iii) the difference in distance is taken between the length of this hypothetical track and that of the great circle, assuming still air conditions.

2.3 This method will, on average, slightly underestimate the extra distance flown because of ATC deviation. The error is small, and the method employed avoids the necessity of examining meteorological data for each flight.

2.4 For vertical deviation, the average of the differences is taken between requested and allocated flight-levels at the reporting meridians. In most cases the difference is constant over all meridians, i.e. flight at a constant flight level was requested and the same or another constant level was allocated.

3. ANALYSIS OF DEVIATIONS

3.1 The study of the distribution of deviations among aircraft falls into several stages:-

i) Construction of a histogram showing the proportion of aircraft suffering various combinations of vertical deviation and increase in track length;

ii) Estimation of the increments of fuel used and flight time corresponding to the various combinations of vertical and horizontal deviation in the histogram;

iii) Estimation of fuel and time penalties averaged over all aircraft in the sample. From this the 'LOW' cost estimate of traffic penalty is derived;

iv) Estimation of the element of fuel reserve carried because of ATC oceanic deviations. This is calculated by the statistical process of convolution. The method employed determines the fuel uplift required for a given level of regularity taking into account all sources of fuel variability such as wind, temperature, holding at destination and aircraft performance variation. This process is performed twice, firstly ignoring the fuel required to cater for compliance with oceanic ATC clearances and then including them. The two amounts of fuel uplift which correspond to the required regularity of service are then compared. The difference between them is the "ATC fuel reserve". It is assumed that this fuel reserve must be carried by all aircraft in the system, irrespective of the deviations they actually encounter, since they do not know before take-off what these deviations will be;

v) Estimation of the fuel burnt to carry the "ATC fuel reserve". Since the aircraft takes off heavier than it would without the ATC fuel reserve, the fuel consumption for the flight will be somewhat greater. The cost of this extra fuel, added to the 'LOW' cost, gives the 'MID' cost estimate of traffic penalty;

vi) Estimating the revenue loss on the occasions when the "ATC fuel reserve" reduces payload capacity. Another way of expressing this is to establish the revenue from the extra payload that could be carried if ATC deviations did not occur. The lost revenue cost added to the 'MID' cost gives the 'HIGH' cost estimate of traffic penalty.

APPENDIX 5-B

Bibliography on cost analysis of NAT operations

No.	Author	Titles etc.
1.	V.W. Attwooll	Methods of costing deviations from the optimum cruise flight path, with examples from the London-New York route. R.A.E. Technical Report 66148 (1966)
2.	V.W. Attwooll	The optimum design of a parallel route structure for air traffic from London to New York for various separation standards. R.A.E. Technical Report 64034 (1964)
3.	Working Party on Supersonic Transport Fuel Reserves	A statistical method for assessing the sector reserve fuel for Concorde. R.A.E. Technical Report 68028 (1968)
4.	Miss M. May	An analysis of counts by Prestwick of jet aircraft crossing 10° West, September 1966-August 1967. Aviation Operational Research Branch. A.T.C. Information Memo. No.5, September 1967

6. *Summary of Agenda Item 6: Review of work programme of the Group, including arrangements for the next meeting

6.1 Review of the work programme

6.1.1 The Group reviewed its work programme, as reflected in para. 6 of Summary/4 and agreed that there was no need for any change at this time. It therefore concluded that it should continue to be considered valid until the next meeting of the Group.

6.2 Arrangements for the next meeting

6.2.1 Date, site and duration of the next meeting

6.2.1.1 The Group agreed that its next meeting should be held sometime in early September 1969 for a duration of approximately two weeks with the understanding that the exact date would be determined in good time between the members of the Group. (The period from 1 to 13 September 1969 was tentatively retained.) It was also agreed that this meeting should be held in the Paris Office of ICAO.

6.2.2 Tentative Agenda

6.2.2.1 Taking account of the decisions when considering Agenda Items 3, 5 and 7, the Group agreed that, while the detailed Agenda would be subject to consultation between members at a later date, it should tentatively provide for at least the following items:

i) Establishment of methods for the assessment of system performance (see para. 5.8 on page 5-4 of this Summary).

ii) Review of progress of studies towards the application of composite separation (see para. 3.10 on page 3-7 of this Summary).

iii) Review of the work so far accomplished by the Group and possible resultant action (see para. 7.4 on page 7-3 of this Summary).

iv) Preparation of appropriate documentation for the Fifth NAT RAN Meeting, mainly in respect of its Item 3.

6.2.3 Attendance

6.2.3.1 The Group agreed that IANC, IATA and IFALPA should again be invited to participate in those parts of the meeting which were of direct interest to them.

6.2.4 Meeting arrangements

6.2.4.1 The Group hoped that it would be possible for ICAO to make the usual arrangements for this meeting.

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* This item was considered by members of the Group only.

7. Summary of Agenda Item 7: Any other business

7.1 Introduction

7.1.1 Under this agenda item, the Group considered three subjects as follows:

i) Comments made by the Federal Republic of Germany and Sweden on Summary/4.

ii) Comments made by IFALPA on Summary/4 and on the work of the Group in general.

iii) Review of the work so far accomplished by the Group.

7.2 Comments made by the Federal Republic of Germany and Sweden on Summary/4

7.2.1 Following the distribution of Summary/4 to all NAT States comments on a number of points had been received by the Paris Regional Office of ICAO from the Federal Republic of Germany and Sweden. They had been passed to the Group for consideration and reply.

7.2.2 After detailed discussion of the subjects raised in these comments the Group appointed a small drafting group for the preparation of replies to them and agreed that the replies, after having been approved by the Group, should be forwarded by the Chairman to the ICAO Representative of the European Office for onward transmission to the States concerned.

7.2.3 Some of the comments by the two States dealt with the recommendation of the Group that States and operators should review all cases where lateral deviations from track of more than 20 NM had occured and examine the need for corrective action (para. 1.3.2.7 on page 1-10 of Summary/4 refers) and it had been suggested by Sweden that monitoring of track keeping should be carried out in a standardized manner. The Group felt that, even though this might not be possible, it might nevertheless be useful to all NAT States if they were to receive at approximately 6-monthly intervals, summaries of all deviations as observed by suitable located radar stations. It was therefore agreed that Canada, France and Ireland (when possible) and the United Kingdom would provide the Paris Regional Office on request with data collected by radar stations operated by these States and that this data should be circulated in suitable form to all NAT States, as well as to IATA, IFALPA and IANC. It was further agreed that any details required to apply this procedure should be worked out directly between the Paris Office and the four States concerned.

7.3 <u>Comments made by IFALPA on Summary/4 and on the work of the Group in general</u>

7.3.1 The Group first dealt with the comments made by Mr. Masland of IFALPA in a letter addressed to the Chairman of the Group prior to the Meeting. After a long discussion the Group decided, and IFALPA agreed to this, to refrain from amending Summary/4 as this was likely to cause more confusion than was worth the possible benefit which might have been obtained from clarifying a number of points which, as has showed the general distribution of Summary/4 to all interested parties, did not seem to have given rise to the misinterpretations of which IFALPA had been afraid. It was however agreed that, should the points referred to by IFALPA in Summary/4 come up again for consideration at future meetings of the Group, the remarks of IFALPA should then be considered also.

7.3.2 In addition to the above comments, IFALPA had submitted a paper which dealt with the work of the NAT/SPG in general and in which they had made a number of points which, in their opinion were essentials in order to achieve further progress in the NAT Region. These points, as revised by the Group, were briefly the following:

i) Dead-reckoning devices, such as Doppler computers, should be up-dated at more frequent intervals than at present.

ii) Navigation Systems used in the NAT Region must be simple to use and must be free from invitations to blunder.

iii) Back-up should be provided by the ability to use two different navigation systems aboard the aircraft.

iv) Any station-referenced area cover navigation system used for up-dating self-contained navigation systems should provide reliable cover within the area where this was required.

v) Doppler or inertial navigation systems must be in constant use if tracking is to be accomplished with precision.

vi) The Navigation/ATC system must be so simple and self-evident that it will still be used effectively and correectly even at the end of a long flight or a flight that is taxing or fatiguing for whatever reason.

7.3.3 In the ensuing discussion it appeared that there were differences in appreciation as to the time-scale and the technical and economical possibilities for achieving the necessary improvements. It was also pointed out that a continuous process of evolution was under way both, as far as the technical and procedural aspects of NAT operations were concerned, which contributed to the improvement of the system. In addition, both the United Kingdom and the United States members gave brief accounts of the work undertaken by their administrations in order to further air navigation in the NAT Region.

7.3.4 The paper also stated IFALPA's disappointment in the lack of returns of requests for information on the causes of gross errors detected in the course of the NAT Data Collection Programme 1967/68. As regards the investigation into gross errors, it was pointed out that, as far as could be ascertained, all administrations had made serious efforts in this field. In addition, it was noted that IATA was in the process of studying navigation equipment fit and the serviceability of such equipment throughout its concerned Member airlines; and of offering advice to operators where this seemed desirable. It was intended that this work should continue.

7.4 Review of the work so far accomplished by the Group

7.4.1 Based on the discussions recorded in para. 7.3 above and as a result of a suggestion made by the Netherlands member, the Group discussed briefly whether it should prepare at this meeting a brief review of the work so far accomplished by it to serve as a sort of "inventory" against which it would be possible to assign priorities to the future work of the Group.

7.4.2 It became however, immediately apparent that it would not be possible to achieve this at this meeting since no time had been provided for this matter. In addition it was also felt that since the progress achieved so far made itself felt in many fields where the relation to the Group's work was not immediately obvious, it would be unwise to embark on such an undertaking without proper preparation.

7.4.3 Finally, it was believed that the Group would be required to undertake such a task prior to the forthcoming NAT RAN Meeting in 1970. It would therefore, only result in an unnecessary effort if this was done now and had to be repeated in any case at the next meeting.

7.4.4 In view of this situation the Group agreed that this matter should be put on the agenda of the next meeting and that the Secretary should prepare a draft paper on this subject and circulate it to all participants of the next meeting well prior to it so that it could serve as a basis for discussion.

8. *Summary of Agenda Item 8: Election of the next Chairman

8.1 On a proposal by Mr. D. A. Blake, member of the United Kingdom, and seconded by Mr. R. Huard, member of the United States of America, the Group re-elected unanimously Mr. J. F. Sapin, the member from France as the Chairman of the Group until the end of the next meeting.

*This item was considered by members of the Group only.

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