International Civil Aviation Organization

Twenty-Sixth Meeting of the Regional Airspace Safety Monitoring Advisory Group (RASMAG/26)

Video Teleconference, 20 – 23 September 2021

Agenda Item 3: Reports from Asia/Pacific RMAs and EMAs

BOBASMA HORIZONTAL SAFETY MONITORING REPORT

(Presented by BOBASMA/India)

SUMMARY

This paper presents the periodic safety assessment for the implementation of 50 NM Reduced Horizontal Separation in the Bay of Bengal Arabian Sea Indian Ocean airspace for the period January to December 2020.

1. INTRODUCTION

1.1 The annual Safety review for the continued safe use of 50 NM Reduced Horizontal Separation (RHS) on Sixteen routes in the Bay of Bengal Arabian Sea Indian Ocean airspace, for the year 2020 was conducted using the TSD of December 2020 of Chennai, Kolkata, Mumbai, Colombo, Kuala Lumpur and Male FIRs.

1.2 Owing to the drastic reduction in international overflying traffic on the sixteen routes on which 50 NM RHS is implemented, due to the pandemic, there was no report of LLD or LLE during the entire twelve month period.

2. DISCUSSION

2.1 The Report of the Safety Assessment conducted for monitoring the horizontal safety risk is attached as **Appendix A**.

Executive Summary

2.2 **Table 1** summarizes the Bay of Bengal Arabian Sea Indian Ocean airspace horizontal risk estimates. **Figure 1** presents the lateral and longitudinal collision risk estimate trends for BOBASIO airspace during the period January 2020 to December 2020.

Bay of Bengal Arabian Sea Indian Ocean Airspace – Estimated annual flying hours = 32084 Hours ***(Note: Estimated hours based on Dec 2020 Traffic Sample Data)***			
Source of Risk	**Risk Estimation**	**TLS**	**Remarks**
RASMAG 25 Lateral Risk	1.58813×10^{-9}	5.0×10^{-9}	Below TLS
RASMAG25 50NM Longitudinal Risk	4.96852×10^{-9}	5.0×10^{-9}	Below TLS
Lateral Risk	0.637515×10^{-9}	5.0×10^{-9}	Below TLS
50NM Longitudinal Risk	0.870503×10^{-9}	5.0×10^{-9}	Below TLS

Table 1: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

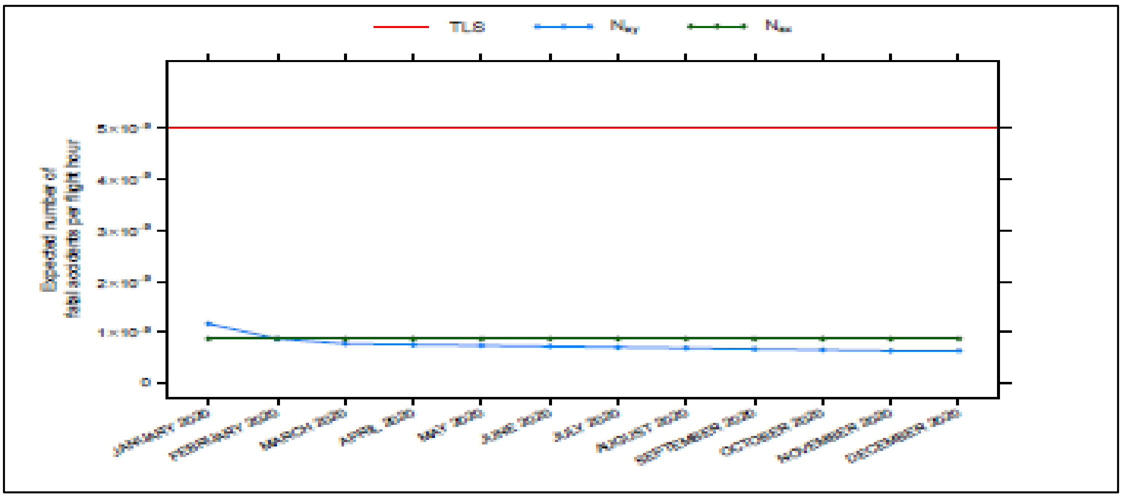

Figure 1: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

a) note the information contained in this paper; and

b) discuss any relevant matters as appropriate.

…………………………….

Appendix – A

ASSESSMENT OF LATERAL AND LONGITUDINAL COLLISION RISK OF BOBASIO AIRSPACE IN 2020

1. INTRODUCTION

In this article we reinvestigate the collision risk between two aircraft flying over the Bay of Bengal, Arabian Sea and Indian Ocean (BOBASIO) region. This safety assessment is under taken jointly by Airports Authority of India (AAI) and the Indian Statistical Institute, Delhi Centre under the MoA signed between the two organizations. The aim of this study is to confirm that Target Level of Safety (TLS) which is 5 X 10^{-9} fatal accidents per flight hour, is currently met.

We first observe that in the BOBASIO region, the Longitudinal separation standard currently is 50 NM on routes M300, N571, P574, P570, L301, L507, L510, L759, M770, N563, N877, N895, P628, P646, P761 and P762.

The lateral separation between two parallel routes is at least 50NM for all routes.

In this article we carry out the quantitative risk analysis based on two types of datasets supplied by six FIRs in the region.

- Traffic Sample Data (TSD):

Traffic sample data from Chennai, Kolkata, Mumbai, Colombo, Kuala Lumpur, and Male FIRs for the month of December 2020 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

- Duplicate records were removed.
- Records with Exit time less than Entry time were removed.
- Records with flight level less than F280 were removed.
- Records with unusually high or low traversal times were removed.

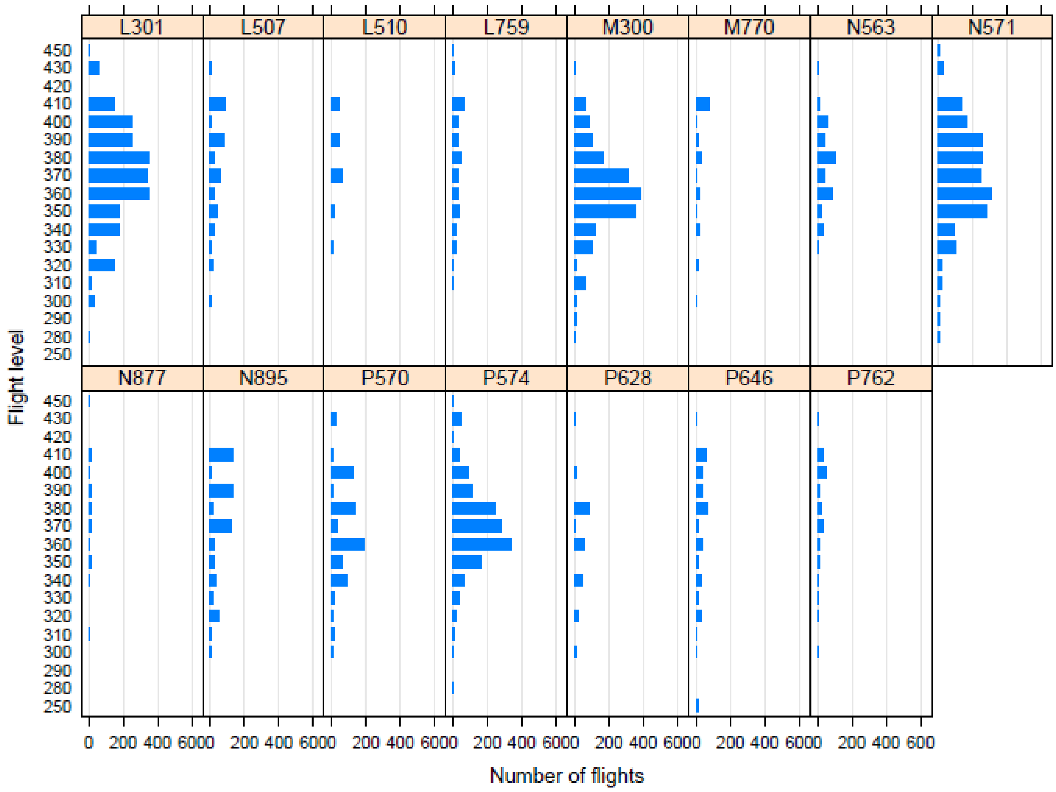

Figure 1: Number of fights by route and fight level based on December 2020 TSD.

17171 records that were retained after filtering were considered for the subsequent statistical analysis. Figure 1 provides a graphical summary of the number of flights by route and flight level for RHS routes.

- Gross Navigational Error (GNE) Data:

Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata, and Mumbai, as summarized in Table 1.

Year	Month	FIR	Flights	LLD	LLE
2020	JANUARY	KOLKATA	7495	0	0
2020	FEBRUARY	KOLKATA	6246	0	0
2020	MARCH	KOLKATA	4938	0	0
2020	APRIL	KOLKATA	1835	0	0
2020	MAY	KOLKATA	2386	0	0
2020	JUNE	KOLKATA	2566	0	0
2020	JULY	KOLKATA	2781	0	0
2020	AUGUST	KOLKATA	3080	0	0
2020	SEPTEMBER	KOLKATA	3341	0	0
2020	OCTOBER	KOLKATA	3822	0	0
2020	NOVEMBER	KOLKATA	3819	0	0
2020	DECEMBER	KOLKATA	3765	0	0
2020	JANUARY	CHENNAI	7287	0	0
2020	FEBRUARY	CHENNAI	6900	0	0
2020	MARCH	CHENNAI	4276	0	0
2020	APRIL	CHENNAI	934	0	0
2020	MAY	CHENNAI	1186	0	0

2020	JUNE	CHENNAI	1474	0	0
2020	JULY	CHENNAI	1673	0	0
2020	AUGUST	CHENNAI	1742	0	0
2020	SEPTEMBER	CHENNAI	1786	0	0
2020	OCTOBER	CHENNAI	1909	0	0
2020	NOVEMBER	CHENNAI	2012	0	0
2020	DECEMBER	CHENNAI	2162	0	0
2020	JANUARY	MUMBAI	23317	0	0
2020	FEBRUARY	MUMBAI	16701	0	0
2020	MARCH	MUMBAI	13824	0	0
2020	APRIL	MUMBAI	3332	0	0
2020	MAY	MUMBAI	4049	0	0
2020	JUNE	MUMBAI	6493	0	0
2020	JULY	MUMBAI	7325	0	0
2020	AUGUST	MUMBAI	7897	0	0
2020	SEPTEMBER	MUMBAI	7785	0	0
2020	OCTOBER	MUMBAI	11126	0	0
2020	NOVEMBER	MUMBAI	9650	0	0
2020	DECEMBER	MUMBAI	8560	0	0
		Total	199474	0	0

Table 1: Summary of reports of Gross Navigational Errors.

It is to be noted that due to the COVID-19 pandemic and associated restrictions, the number of fights drastically reduced all over the world in 2020. This is reflected in Table 1 as well. Figure 2 represents the reduction in the number of fights as a proportion of the corresponding number of fights in the previous year (2019). In December 2020, the average number of fights in the three Indian FIRs were 32% of the corresponding total in 2019. Thus, one would expect a substantial reduction in the collision risk this time, and that is indeed what we observe.

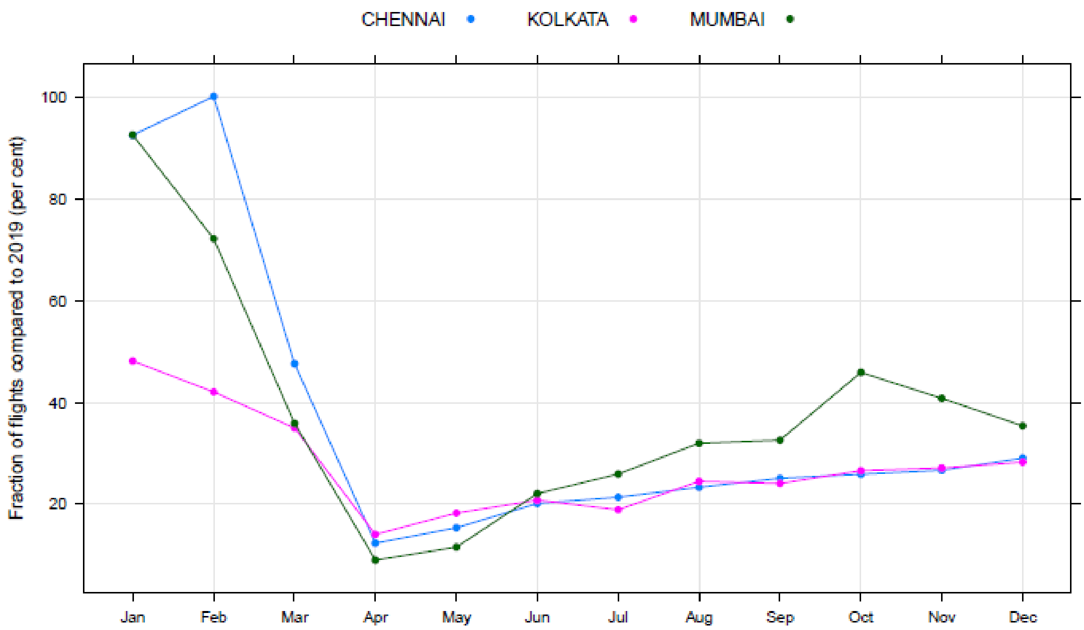

Figure 2: Proportion of total ights by month and FIR compared to previous year.

In Section 2 we discuss the risk assessment for the lateral direction and Section 3 gives the same for the longitudinal direction.

2. Lateral Collision Risk Assessment

2.1 Lateral Collision Risk Model

In order to compute the level of safety for lateral deviations of operations on the BOBASIO region we use the *Reich Lateral Collision Risk Model*. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y(S_y)P_z(0)\frac{\lambda_x}{S_x}\left\{E_y(same)\left[\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right] + E_y(opp)\left[\frac{2\overline{|\bar{V}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right]\right\}$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.

The parameters in the equation (1) are defined as follows:

- Nay: = Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned Sy NM lateral separation.
- Sy: =Minimum planned lateral separation.
- λx:= Average length of an aircraft flying in BOBASIO region.
- λy := Average wing span of an aircraft flying in BOBASIO region.
- λz := Average height of an aircraft flying in BOBASIO region.
- Py (Sy):= The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by Sy.
- Pz (0):=Probability of vertical overlap of two aircrafts assigned the same flight path at the same flight level.
- Sx: =Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- Ey (same):= Same direction lateral occupancy at same assigned flight level.
- Ey (opp):= Opposite direction lateral occupancy at same assigned flight level.
- |ΔV|:= Average relative speed of two aircraft flying on parallel routes in same direction.
- |V|:= Average ground speed on an aircraft.
- |y˙(Sy)|:= Average relative lateral speed of aircraft pair at loss of planned lateral separation of Sy.
- |z˙|:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, Pz (0) is the probability of vertical overlap; Py (Sy) is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x}$ Ey (same) and $\frac{\lambda_x}{S_x}$ Ey (opp) relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\overline{|V|}}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\bar{V}$ and the average overlap time is $\frac{2\overline{|V|}}{2\lambda_x}$

The model is based on the following hypothesis:

- All routes are parallel.
- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.
- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.04117171 NM	Estimated from TSD (see Section 2.3).
λ_y	0.04308855 NM	Estimated from TSD (see Section 2.3).
λ_z	0.01301296 NM	Estimated from TSD (see Section 2.3).
$P_y(50)$	4.09412×10^{-8}	Estimated using a mixture model (see Section 2.4).
$P_Z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).

S_y	50 NM	Conservative value, taken to be the current longitudinal separation in all but four routes.
Ey (same)	0.02798901	Estimated from the TSD (see Section 2.6).
Ey (opp)	0	No opposite directional lateral occupancy at same assigned ight level.
$\|\nabla V\|$	27 knots	Value obtained from TSD (see Section 2.7).
$\|\dot{y}(50)\|$	75 knots	Conservative value taken from EMA Handbook (see Section 2.8).
$\overline{\|\dot{z}\|}$	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk Nay.

$$Nay = 0.637515 \ \text{X} \ 10^{-9}$$

2.1 Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

2.2 Estimating Probability of Lateral Overlap: Py (Sy)

The probability of lateral overlap of aircraft nominally flying on adjacent fight paths, separated by Sy, is denoted by Py (Sy) and is defined as

$$Py\,(Sy) = P(\ |\,Sy + Y_1 - Y_2\,| \leq \lambda y\,), \qquad (2)$$

where Y_1 and Y_2 are assumed to be the lateral deviations of two aircraft which are nominally separated by Sy. We assume that Y_1 and Y_2 are identically distributed but statistically independent with a distribution Fy.

We model Fy as mixture distribution having a core distribution Gy and a non-core distribution Hy.

- The core distribution Gy, represents errors that derive from standard navigation system deviations.These errors are always present, as navigation systems are not perfect and they have a certain precision.
- The non-core distribution Hy, represents Gross Navigation Errors (GNE), that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$Fy(y) = (1 - \alpha)\ Gy(y)\ +\ \alpha\ Hy(y). \quad (3)$$

Type	Length	Wingspan	Height	Flights
B77W	73.9	64.8	18.5	1635
A359	66.8	64.75	17.05	1086
B789	62.8	60.1	16.9	667
A333	63.6	60.3	16.85	463
B77L	63.7	64.8	18.3	402
B788	56.7	60.1	16.9	338
B748	76.25	68.45	19.35	227
B738	39.2	34.4	12.57	154
B744	70.6	64.8	19.4	153
A320	37.57	34.1	11.76	151
A332	58.8	60.3	17.4	130
A343	63.6	60.3	16.85	46
B737	33.6	34.3	12.6	43
B763	54.9	47.6	15.9	36
GLEX	30.3	26.9	7.6	23
B772	63.7	60.9	18.4	23
B739	41.94	34.3	12.6	19
A319	33.84	34.1	11.76	18
GL5T	28.69	28.65	7.7	7
A321	44.51	34.1	11.76	7
GLF4	26.9	23.7	7.4	6
CL60	20.85	19.6	6.3	6

Table 2: Dimensions of aircraft types in meters, along with number of records in the TSD

The mixing parameter α is the probability of a gross navigational error.

The core lateral deviation distribution Gy is modeled by a Double Exponential distribution with a parameter $\beta y > 0$ as the rate, that is, if $Y_1 \sim Gy$ then

$$P(\ |Y1|\ >\ y)\ =\ e^{-\beta_y y};$$

where $y \geq 0$. In other words we assume that the core distribution has a density of the form

$$g_y(y) = \frac{\beta_y}{2}\ e^{-\beta_y y}$$

Finally the non-core distribution Hy is modeled by a “Separated Double Exponential" distribution with parameters $\mu_y > 0$, representing the “separation and $\gamma_y > 0$ the rate parameter, that is, if $Y_2 \sim Hy$ then

$$P(\ Y_2 > \mu_y\ + y\ ) = \frac{1}{2}\, e^{-\gamma_y y} \quad \text{and}$$

$$P(\ Y_2 > -\ \mu_y\ -\ y\ ) = \frac{1}{2}\, e^{\gamma_y y}$$

where $y \geq 0$. This really means that the non-core distribution Hy gives no mass in [- μ_y ; μ_y] and outside it decays as a Double Exponential distribution with rate parameter γ_y. The density of this distribution is given by

$$HyY = \begin{cases} \frac{\gamma_y}{2}\ e^{\gamma_y(y+\mu_y)} & if\ y < -\mu_y \\ 0 & if\ -\mu_y \leq y \leq \mu_y \\ \frac{\gamma_y}{2}\ e^{-\gamma_y(y-\mu_y)} & if\ y > \mu_y \end{cases}$$

This modelling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors. This in turn gives a better understanding of the mixing parameter $\boldsymbol{\alpha}$ which we estimate by taking the 95% upper confidence limit from the available GNE data. This confidence limit does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods. The value comes out to be

$$\hat{\alpha} = 1.6\ \text{X}\ 10^{-5}.$$

Here we would like to note that even though the non-core distribution Hy has a discontinuous density hy, it does not create difficulty in this risk assessment.

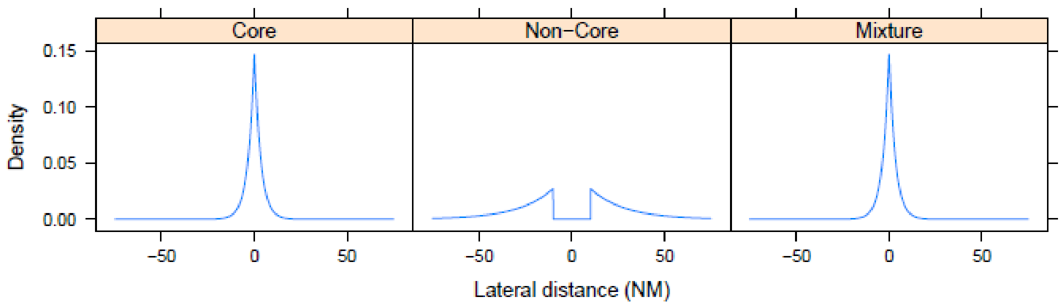

Figure 3: Modeling of lateral deviation.

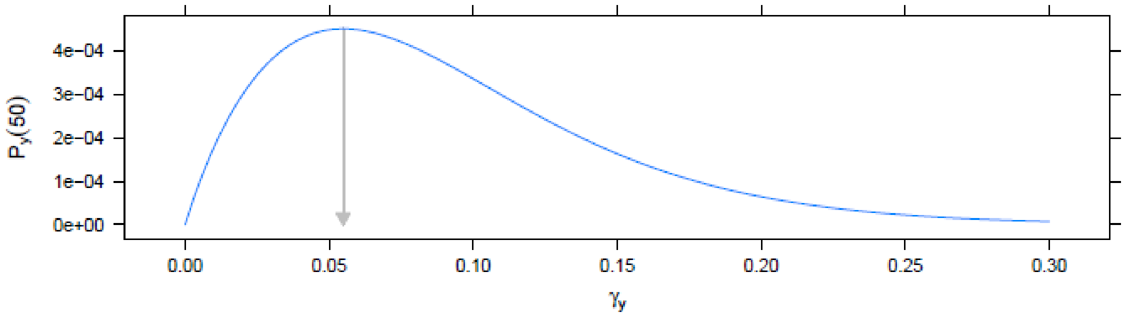

Figure 4: Wingspan overlap probability as a function of $_y$ with S_y = 50 NM initial separation.

The parameter βy is estimated under the RNP10 assumption of ± 10 NM deviation with 95% confidence, this leads to the estimate

$$\widehat{\beta_y} = -\frac{\log 0.05}{10} = 0.2995732$$

The parameter μy is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with Sy = 50 NM initial separation (see Figure 3). This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γ_y is 0.0548971 leading to the estimated value of Py (50) as 4.09412 X 10^{-8}.

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of P_y (50) and N_{ay} had we observed more GNEs. The results, given below, are still below the TLS. Note that the actual number of GNEs observed was 0.

No. of GNEs	**P_y (50)**	**N_{ay}**
0	4.09412 X10^{-8}	0.637515 X10^{-9}
5	6,05453 X10^{-8}	0.94278 X10^{-9}
10	7.90601 X10^{-8}	1.23108 X10^{-9}
15	9.53965 X 10^{-8}	1.48547 X 10^{-9}
20	1.11733 X 10^{-8}	1.73984 X10^{-9}

2.5 Estimating Probability of Vertical Overlap: Pz (0)

The probability of vertical overlap of aircraft nominally ying at the same ight level on laterally adjacent flight paths is denoted by Pz (0). It is defined by

$$Pz\ (0) = P(\ |\ Z_1 - Z_2\ | \leq \lambda z\ ),$$

where Z_1 and Z_2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z_1 and Z_2 are statistically independent with distribution Fz. Unlike in the computation of Py (Sy) where we assume the lateral deviations follow a mixture distribution here we may assume that Fz is a Double Exponential distribution with parameter βz > 0, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z|z|}$$

One can then estimate $\beta z > 0$ by

$$\widehat{\beta_z} = -\frac{\log 0.05}{0.032915} = 91.0142$$

This is under assumption that a typical aircraft stays within ± 200 ft = ± 0.032915 NM of its assignedflight level 95% of the time. This leads to an estimated value 0.4230008 for Pz (0).

Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

2.6 Estimating the Lateral Occupancy Parameters: E_y (same) and E_y (opp)

In equation (1) there are two occupancy terms, one for same direction occupancy Ey (same) and another one for opposite direction occupancy Ey (opp).

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, 2Sx, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between Sx and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing “flying in the same direction" by “flying in the opposite direction". Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

where Ty represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator ny is the number of proximate pairs and the denominator, n, is the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

WP1	WP2	Proximate	Total
BIDEX	ORARA	0	332
IGOGU	IGREX	40	1288
NOPEK	IGOGU	88	1224
GIRNA	IDASO	10	1290
VATLA	ORARA	0	384
IGOGU	EMRAN	0	914
LIBDI	MEPEL	0	184
RINDA	SAGOD	42	2322
MEPEL	IBITA	24	600
IBITA	TEBOV	14	630
SAGOD	IBITA	4	712
POMAN	IGAMA	0	1182
KITAL	LOTAV	54	2312
LOTAV	REXOD	56	2276
TOTOX	REXOD	18	1504
TOTOX	PARAR	228	3296
ADPOP	SUGID	0	1602
RASKI	PARAR	294	3968
NOBAT	SUGID	0	1602
POMAN	ODOLI	0	1182
KITAL	ASPUX	0	274
SULTO	DUGOS	0	170
UDULO	KAKIB	0	584
BUBKO	DOPID	0	354
VATLA	MABUR	0	384
SAGOD	MEPEL	4	480
MIPAK	LALAT	0	248

Table 3: Number of laterally proximate ights per route pair, based on TSD.

2.7 Estimate of Average Relative Longitudinal Speed: $\overline{|V|}$

$\overline{|V|}$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction. $\overline{|V|}$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 26.16376. We use the conservative value 27. Here we note that the lateral proximate pairs are already determined while estimating the parameter Ey (same).

2.8 Estimate of Average Relative Lateral Speed: $\overline{|\dot{y}\, S(y)|}$

$\overline{|\dot{y}\, S(y)|}$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by Sy NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.9 Estimate of Average Relative Vertical Speed: $|\dot{z}|$

$|\dot{z}|$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $|\dot{z}|$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $|\dot{z}|$ with the vertical separation between the aircraft. As noted by various agencies data on $|\dot{z}|$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

3 Longitudinal Collision Risk Assessment

In order to compute the level of safety for longitudinal deviations of operations on the BOBASIO region we use the Longitudinal Collision Risk Model. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{\overline{|\dot{x}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(0)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right) X \left[\sum_{k=m}^{M} 2Q(k)P(K > k\right] \qquad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- Nax := Expected number of fatal accidents (two for every collision) per fight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum m NM longitudinal separation.
- m := Minimum longitudinal separation in NM.
- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λx := Average length of an aircraft flying on BOBASIO region.
- λy := Average wingspan of an aircraft flying on BOBASIO region.
- λz := Average height of an aircraft flying on BOBASIO region.
- Py (0) := Probability of lateral overlap of two aircrafts assigned same flight route and flight level.
- Pz (0) := Probability of vertical overlap of two aircrafts assigned the same flight path at the same flight level.

- $\overline{|\dot{x}\,|}$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point.

- $\overline{|\dot{y}(0)|}$:= Relative across-track speed of same route aircraft pair.

- $\overline{|\dot{z}\,|}$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

- Q(k) := Proportion of aircrafts for which the following aircraft has initial longitudinal separation k.
- P(K > k) := Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions.

The equation is derived under similar assumption as done in the case of lateral collision risk assessment. We should note here the the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events.

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk for Routes with 50 NM Longitudinal Separation

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	50 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes separation.
λx	0.04117171 NM	Estimated using flights on 50NM routes only.
λy	0.04308855 NM	Estimated using flights on 50NM routes only.
λz	0.01301296 NM	Estimated using flights on 50NM routes only.
Py (0)	0.2	Conservative estimate (see Section 3.1.1).
Pz (0)	0:538	Conservative value used in previous safety assessments (see Section 2.5).
$\overline{|\dot{x}\,|}$	22 knots	Conservative estimate using speed and distance between way points (see Section 3.1.2).
$\overline{|\dot{y}(0)|}$	1 knot	RASMAG/9 safety assessment (see Section 3.1.3).
$\overline{|\dot{z}\,|}$	1:5	Conservative value as per EMA Handbook

		(see Section 2.9).
Q(k)	See Table 4	Obtained from TSD (see Section 3.1.4).
P(K > k)	See Table 4	Computed using normal model on speed (see Section 3.1.5).

Finally this leads to the following estimate for the longitudinal collision risk Nax.

$$N_{ax} = 4.96852 \text{ X } 10^{-9}$$

3.1.1 Estimating Probability of Lateral Overlap: Py (0)

Py (0) is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting Sy = 0 in the equation (2). This leads to an estimate of Py (0) as 0.2.

However as noted earlier in the EUR/SAM report, this factor Py (0) has a significant effect on the risk estimate. Therefore, it should not be underestimated. Py (0) will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM,

k (mins)	k (NM)	Q(k)	P(K > k)
1	8	0.0000000	1.78853×10^{-1}
2	16	0.0000000	5.9126×10^{-2}
3	24	0.0000000	1.56552×10^{-2}
4	32	0.0000000	4.8587×10^{-4}
5	40	0.0000000	6.59038×10^{-5}
6	48	0.0000000	6.59038×10^{-5}
7	56	0.0000000	9.94155×10^{-6}
8	64	0.0000000	1.89067×10^{-6}
9	72	0.0000000	4.09161×10^{-7}
10	80	0.0415704	9.14734×10^{-8}
11	88	0.1108545	2.05464×10^{-8}
12	96	0.0519631	4.61707×10^{-9}
13	104	0.0773672	1.03755×10^{-9}
14	112	0.0854504	2.33158×10^{-10}
15	120	0.0715935	5.23953×10^{-11}
16	128	0.0623557	1.17743×10^{-11}
17	136	0.0704388	2.64592×10^{-12}
18	144	0.0704388	5.94591×10^{-13}
19	152	0.0831409	1.33617×10^{-13}
20	160	0.0669746	3.00263×10^{-14}

Table 4: Estimated values of Q(k) and P(K > k)

the RGCSP adopted a value of 0:059 as the value of Py (0) for the global system performance.

As observed by many monitoring agencies and pointed out to us by AAMA, the RGCSP value of Py(0) does not acknowledge the close track-keeping observed with RNP 4 or GNSS-equipped RNAV 10/RNP 10 aircraft. Further the EMA Handbook recommends to take a conservative value as 0.2. So we take this conservative value for our analysis as well.

3.1.2 Estimation of the Parameter $\overline{|\dot{x}|}$

$\overline{|\dot{x}|}$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $\overline{|\dot{x}|}$ can be computed by the equation

$$\frac{d-m}{v_o} = \frac{d}{v_o-\overline{|\dot{x}|}}$$

leading to

$$\overline{|\dot{x}|} = \frac{mv_o}{d-m}$$

We conservatively estimate it by taking v_0 as the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is d = 842 NM. With m = 50 NM the final estimate turns out to be $\overline{|\dot{x}|}$ = 22.73 knots. We use a conservative value of 22 knots.

3.1.3 Estimation of the Parameter $\overline{|\dot{y}\ (0)|}$

$\overline{|\dot{y}\ (0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of 1 knot as given in the EMA Handbook.

3.1.4 Estimation of the Parameter Q(k)

Q(k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different fight levels were considered separately (see Figure 4), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q(k) as

$$Q(k) = \frac{\text{number of ight pairs with inter−arrival distance 8k}}{\text{total number of ight pairs with at least 80 NM separation}}$$

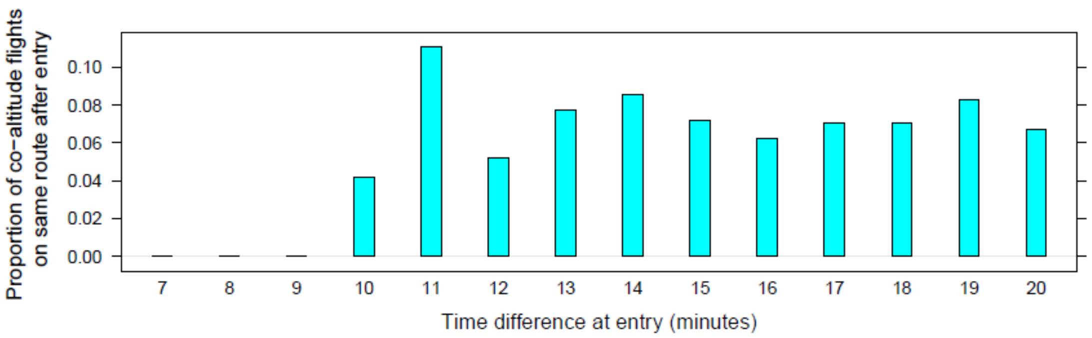

Figure 5: Values of Q(k) estimated from TSD. For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 7; 8; 9; : : : ; 20 minutes.

The final estimated values of Q(k) for k ranging between 1 and 20 minutes is given in the Table 4.

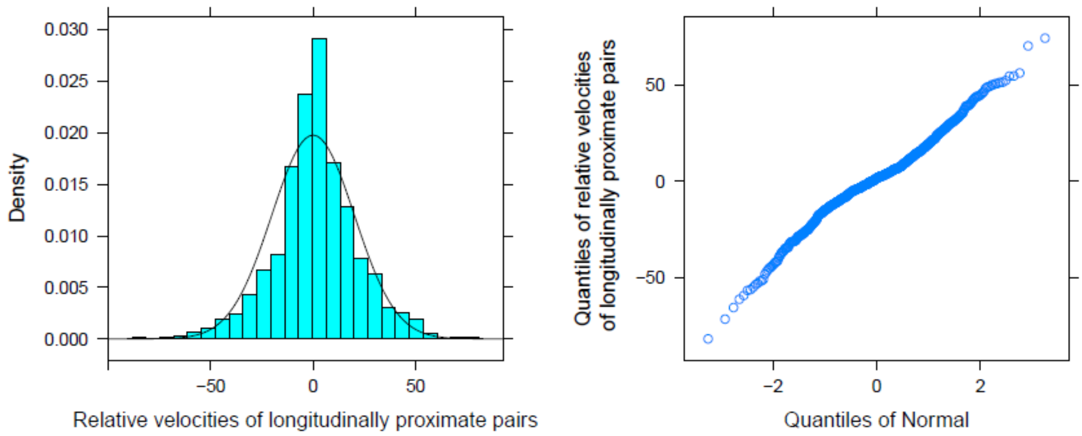

Figure 6: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does not necessarily seem to be a reasonable fit.

3.1.5 Estimation of the Parameter P(K > k)

To estimate P(K > k) we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and V^I be their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T_0 be the maximum duration of time before ATC intervenes. Then

$$P(K > k) = P\left(0 < \frac{k}{V^I - V} < T_O\right) = P(V^I - V > \frac{k}{T_O}$$

We note here that the value of T_0 is conservatively taken to be 0:5 hours.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 5) it is not clear that these differences necessarily

Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\,\frac{\beta_v}{2}\,e^{-\beta_v|v|} + (1-p)\,\frac{1}{\sqrt{2\pi}\,\sigma_v}\,e^{-\frac{v^2}{2\sigma_v{}^2}}$$

which is a mixture of Double Exponential and Normal densities with mixing proportion p.

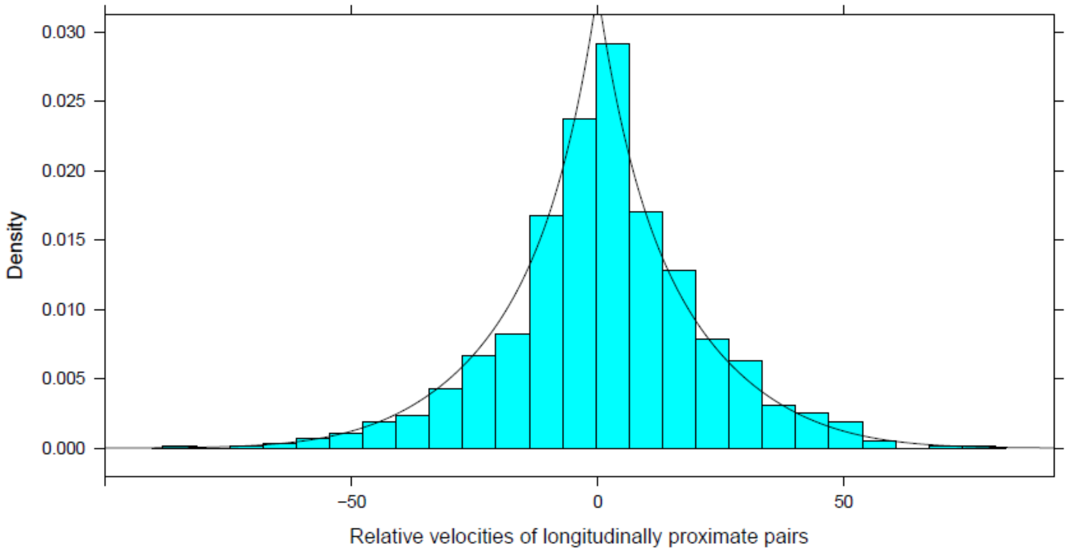

Figure 7: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density (estimated using the EM algorithm).

We then estimate the parameters of this mixture model by their maximum likelihood estimates (MLEs). Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs.The algorithm converged rapidly to give the following estimates:

$$\hat{p} = 0.5567919\ , \hat{\beta}_v = 0.09330622\ , \text{and}\ \hat{\sigma}_v = 25.15696\,.$$

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with these value of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

With these we estimate the values of P(K > k) for k ranging between 7 and 20. These are presented in Table 4.

4 Summary of the Safety Assessment

The following table gives a summary of the safety assessment of the BOBASIO region for the month of December 2020 .

Type of Risk	Estimated Risk	TLS	Remarks
Lateral Risk	0.637515 X 10^{-9}	5 X 10^{-9}	Below TLS
Longitudinal Risk	0.870503 X 10^{-9}	5 X 10^{-9}	Below TLS

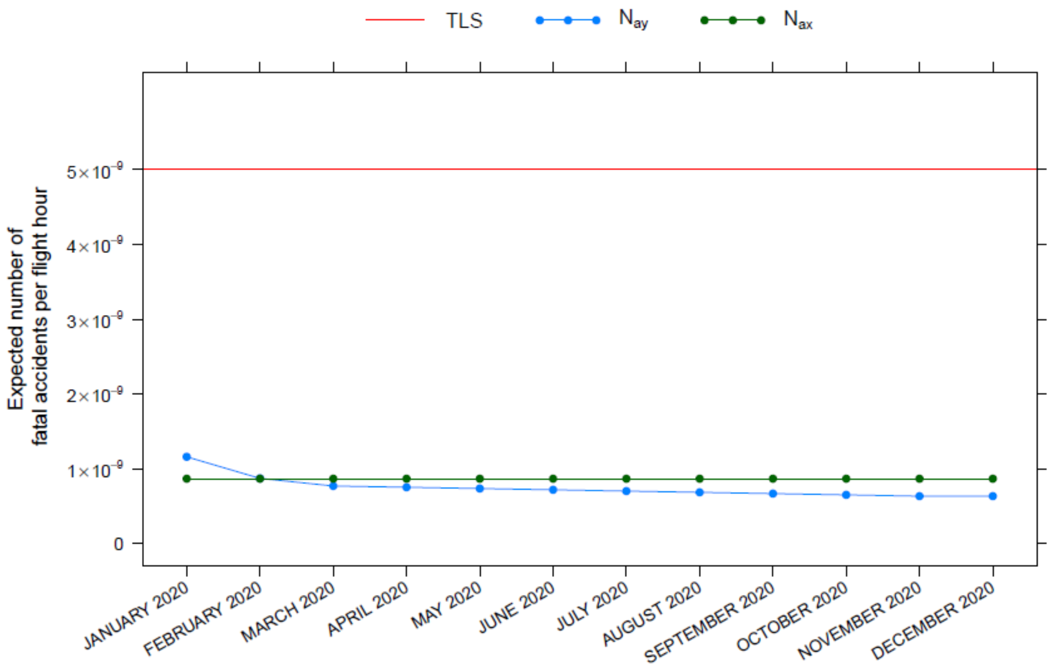

Figure 8: Assessment of Compliance with Lateral and Longitudinal TLS Values.

Figure 8 presents the results of the collision risk estimates for each month using the cumulative 12-month LLD reports since January 2020