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Agenda Item 4: Airspace Safety Monitoring Documentation and Regional Guidance Material

ESTIMATION OF VERTICAL OVERLAP PROBABILITY (P_z) UPDATE

(Presented by MAAR)

SUMMARY

This paper presents the estimate of vertical overlap probability (P_z) using the more recent Altimetry System Error (ASE) from MAAR's ADS-B Height Monitoring System (AHMS). MAAR has been using $P_z(1000)$ of $\mathbf{1.7 \times 10^{-8}}$ in its risk calculation, which is the conservative value based on the Global System Performance Specification (GSPS). The estimation yields the result of $\mathbf{1.164 \times 10^{-11}}$, which is much smaller than the GSPS value. As for $P_z(0)$, the new estimate resulted in a value of **0.5254**, which is slightly smaller than the currently used value of **0.538**. However, P_z re-estimation will be discussed further within the Monitoring Systems Standards Subgroup (MSSS) of the RMA Coordination Group (RMACG) before MAAR actually replaces the existing values.

1. INTRODUCTION

1.1 Probabilities of vertical overlap are important parameters that affect mid-air collision risk. For RVSM airspace, the applicable P_z values consist of:

- $P_z(1000)$, which mostly affects the technical risk, which is the risk inherent in the system as designed, and
- $P_z(0)$, which mostly affects the operational risk, resulting from LHDs (errors/deviations from the design) occurring at whole flight levels.

1.2 In RASMAG/21, MAAR presented an update of vertical overlap probability (P_z) estimates in WP10. The estimation uses the same mathematical model and methodology introduced in "Fitting Aircraft Height-keeping Data to a Family of Symmetric Statistical Distribution Models and Estimating $P_z(1000)$," developed by the Federal Aviation Authority (FAA) Technical Center.

1.3 This paper presents a re-estimation of P_z using the identical method based on more recent data, including recent ASE data from MAAR's AHMS.

2. DISCUSSION

<u>Modelling the Probability of Vertical Overlap (P_z)</u>

2.1 It is impossible to find the probability of vertical overlap using empirical data. Therefore, statistical model is used to obtain this value.

2.2 **Probability of Vertical Overlap (P_z):** P_z is a significant parameter used to estimate the risk of mid-air collisions. Two aircraft are considered overlapped if the separation between them (measured from each aircraft's center) is less than average aircraft's height. P_z depends on the Total Vertical Error (TVE) of each aircraft. To obtain P_z, TVE distribution is self-convolved and integrated over twice average aircraft height.

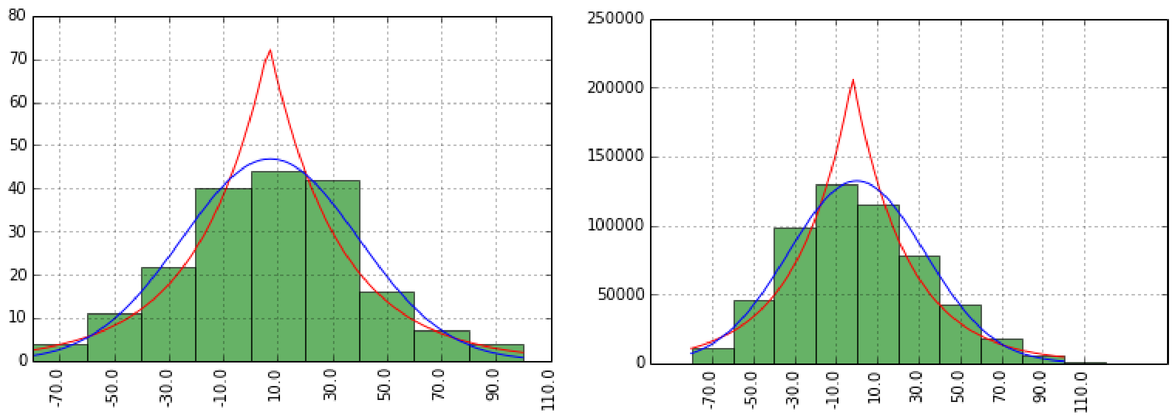
2.3 By design, most errors of large magnitude relative to the planned separation standard are rare. Therefore, distribution fitting is needed to fill in the need to extrapolate the frequency of large TVEs without having to wait for the errors to occur in the real world and get collected for the analysis.

2.4 **Probability Distribution of Total Vertical Error (TVE):** TVE can be broken down into two components:

- Altitude keeping error, which is composed of assigned altitude deviation (AAD) and technical large height deviation (LHD). Technical LHDs are deviations that are non-operational errors in nature such as those caused by equipment malfunction, aircraft contingencies, TCAS RAs or other environment factors such as weather turbulence and are not protected by ATC.
- Altimetry system error (ASE) for each Minimum Monitoring Requirement (MMR) group, which is modelled as a sum of two random variables - ASE mean and within-airframe ASE difference (representing fluctuation of each ASE sample from the ASE mean of the aircraft). After the distribution of ASE for each MMR group is determined, ASE distributions are mixed according to proportion of time aircraft in the group fly in RVSM airspace.

TVE is a sum of two random variables mentioned above - altitude keeping error (AAD + technical LHDs) and ASE. Each component is modelled separately, then combined using a mathematical operation called "convolution". Therefore, TVE distribution is obtained by convolving altitude keeping error distribution and ASE distribution.

2.5 **Probability Distribution of ASE means and within-airframe ASE differences:** the distribution of ASE means and within-airframe ASE differences for each aircraft group is assumed to follow either a Gaussian (normal) or 1st Laplace distribution. **Figure 1** shows an example of A380 group's ASE means and within-airframe ASE differences fitted with a Gaussian (blue line) and a 1st Laplace distribution (red line).



(a) ASE mean (b) within-airframe ASE difference

Figure 1: A380 group's ASE distribution

2.6 **Figure 1** Gaussian distributions (red lines) fit the data better. However, what influences $P_z(1000)$ more is the tail of the distribution. It can also be shown on **Figure 1** that the 1st Laplace distribution (red line) has a thicker tail, which would finally yield a more conservative estimate. Therefore, MAAR also estimate $P_z(1000)$ assuming 1st Laplace distributions for all groups' ASE means and within-airframe ASE differences. Please, note that MAAR does not use this assumption for $P_z(0)$ estimation since there is a good amount of data for the core portion of the distribution.

Tool and Data

2.7 MAAR has developed a program in Python employing the same model and methodology developed by the FAA to replace the legacy Fortran program.

2.8 The procedures to model altitude keeping error, ASE for each MMR group and TVE distributions are exactly the same as explained in MAAR's previous paper (RASMAG/21 – WP/10), with more recent data samples as follows:

- 1 month of AAD data from MAAR's ADS-B Height Monitoring System (AHMS) over Bangkok FIR during December 2019
- 2 years of technical LHD data from 2018-2019 reports submitted by SA/IO, SEA and Mongolian States
- 12 months of ASE data from MAAR's AHMS from January to December 2019
- 1 month of Traffic Sample Data (TSD) during December 2019 to calculate the proportion of time for each MMR group

The data is also scaled to the same time proportion.

Result: $P_z(1000)$

2.9 **Table 1** shows the estimated values of the probability of vertical overlap between two aircraft flying at adjacent RVSM flight level ($P_z(1000)$) for different aircraft heights from 2015 data (RASMAG/21 – WP/10), 2019 data, and 2019 data assuming all 1st Laplace distributions.

Type of Aircraft Height	**Average Aircraft Height (ft)**	**$P_z(1000)$**		
		2015 Data	**2019 Data (Gaussian & 1st Laplace)**	**2019 Data (all 1st Laplace)**
Average height of aircraft flying in SA/IO, SEA and Mongolian	50	1.443×10^{-10}	0.891×10^{-11}	$\mathbf{1.164 \times 10^{-11}}$
B747	60	1.965×10^{-10}	1.189×10^{-11}	1.558×10^{-11}
A380	79	3.076×10^{-10}	1.997×10^{-11}	2.630×10^{-11}

Table 1: Estimated $P_z(1000)$

2.10 Following the exact method developed by the FAA, the average height of the aircraft flying in the assessed airspace will be chosen to evaluate $P_z(1000)$, which yields the value of 1.164×10^{-11}. Even the more conservative value of $P_z(1000)$ calculated based on A380's height is 2.630×10^{-11}, which is still much smaller than the GSPS-based value of 1.7×10^{-8} used in MAAR's current overall risk estimate. This may be caused by these factors:

- Height keeping performance of aircraft and its measurements became more accurate. This would result in less fluctuate ASE means and within-airframe ASE differences.
- Technical LHDs may be under-reported. Technical LHDs, which are deviations that are non-operational errors and unprotected by ATC, are typically of larger magnitude than ASEs and usually influence the 'tail' of the distribution. Fewer LHDs may result in an under-estimation of $P_z(1000)$.
- There might actually be smaller number of technical LHDs in surveillance airspace. If ATC provides clearances to the flight crew in aircraft contingency events, bad weather conditions, then these are not considered LHDs.

2.11 MAAR plans to collect more technical LHDs from other sources to verify its effect on $P_z(1000)$ before actually using it in the overall risk estimation.

Result: $P_z(0)$

2.12 On the contrary, the probability of vertical overlap between two aircraft flying at the same flight level ($P_z(0)$) is a product of core-core interaction of the distribution and is the main driver of the operational risk portion. This means that the better the height keeping performance of an aircraft, the higher risk of collision for each minute of LHD at a whole flight level.

2.13 **Table 2** shows the estimated values of $P_z(0)$ for different aircraft heights from 2015 data (RASMAG/21 – WP/10) and 2019 data.

Type of Aircraft Height	**Average Aircraft Height (ft)**	**$P_z(0)$**	
		2015 Data	**2019 Data**
Average height of aircraft flying in SA/IO, SEA and Mongolian	50	**-**	**0.5254**
B747	60	0.5264	0.6068
A380	79	0.6486	0.7345

Table 2: Estimated $P_z(0)$

2.14 **Table 2** shows that $P_z(0)$ calculated based on the average height of aircraft flying in the assessed airspace from 2019 data is 0.5254, which is slightly smaller than the currently used value of 0.538 in MAAR's overall risk estimation. This suggests that the current value of $P_z(0)$ is still suitable for MAAR's assessed airspace. However, since P_z varies greatly with the choice of aircraft height, future discussions could focus on the most suitable aircraft height to be used.

2.15 The P_z re-estimation will be discussed further within the Monitoring Systems Standards Subgroup (MSSS).

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

a) note the information contained in this paper;

b) acknowledge the importance of LHD reporting as explained in 2.10, especially Category G, H, I, J, and K; and

c) discuss any relevant matters as appropriate.

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