Abstract
Description of the algorithms for the WinGPSAll module of the PEGASUS data processing and analysis system using the EGNOS System Test Bed ESTB, a prototype version of the European implementation of a Satellite Based Augmentation System SBAS.
## CHANGE RECORD

<table>
<thead>
<tr>
<th>Issue</th>
<th>Date</th>
<th>Chapter</th>
<th>Description of Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-F</td>
<td>All</td>
<td></td>
<td>Previous Issues for ESTB SIS versions V1.0 (RTCA DO 229 Change 1) and V1.1 (RTCA DO 229A) “PEGASUS Technical Notes, PEG-TN-01, Issue F”</td>
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<tr>
<td>G</td>
<td>17/01/2003</td>
<td>All</td>
<td>Issue associated with ESTB SIS version 1.2 (RTCA DO 229 A/B) Harmonisation of documentation associated in the PEGAS*Plus Project</td>
</tr>
<tr>
<td>H</td>
<td>09/04/2003</td>
<td>7, 10</td>
<td>minor correction of documentation</td>
</tr>
<tr>
<td>I</td>
<td>17/06/2003</td>
<td>All</td>
<td>minor editorial changes</td>
</tr>
</tbody>
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1 Introduction

1.1 Purpose of this document

The purpose of this document is to describe the messages broadcast by the ESTB and how the data are applied in the PEGASUS algorithms.

The ESTB is a test bed for the European Geostationary Navigation Overlay Service (EGNOS), the European component of a world-wide Satellite Based Augmentation System (SBAS) for the current generation of satellite navigation systems. The purpose of the SBAS is to enhance the available accuracy and integrity of the core GNSS-1 elements GPS and GLONASS.

The PEGASUS development is based on the prototypes of the SAPPHIRE phases I and II. The SAPPHIRE database collects GNSS raw measurements and computed data and implements algorithms for processing the data stored in the database. So far, SAPPHIRE has implemented a GPS position solution algorithm, several integrity algorithms and a constellation analysis tool. With PEGASUS, these algorithms are extended to process not only GPS, but also ESTB data.

1.2 Overview

This document is divided into several parts. The first part, chapter 2, describes the architecture of the ESTB and its implementation, although it should be noted that the description contained in this chapter is mainly based on the implementation of the ESTB during the phase between February 2000 and June 2001. In the meantime, the ESTB changed its signal format twice


and therefore the main parts of the signal generation and signal implementation. If current information on the signal content and the broadcast schedule is needed, the interested reader is referred to the appropriate web pages [16].

The second part is dedicated to the processing of the SBAS correction and integrity data (chapters 3 through 10). The third part is dedicated to the processing of the SBAS ranging data (chapters 11 through 13). The document will be extended and modified along with the evolution of the ESTB.

It is important to acknowledge that the currently transmitted ESTB “Signal In Space” version 1.2 (although labelled RTCA DO 229B) is specified mainly by the RTCA DO 229B [4] standard – but certain individual parts still conform to the RTCA DO 229A [3] standard.

It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications. This document is intended to describe the algorithmic implementation that is necessary to process the transmitted ESTB SIS in order to obtain a position solution and integrity information. Thus, in the individual chapters it has been decided to adhere to the RTCA DO 229B [4] standard and to identify the parts which still conform to the RTCA DO 229A [3] standard. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

1.3 Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI</td>
<td>Degradation Factor</td>
</tr>
<tr>
<td>AL</td>
<td>Alert Limit</td>
</tr>
<tr>
<td>AORE</td>
<td>INMARSAT satellite Atlantic Ocean Region East</td>
</tr>
</tbody>
</table>
CAT-I Precision Approach Flight Phase Category I
CPF Central Processing Facility

DOP Dilution Of Precision

E Eastern Longitude
ECAC European Civil Aviation Conference
EGNOS European Geostationary Navigation Overlay Service
ENR Flight Phase En-Route
ESTB EGNOS System Test Bed

FC Fast Corrections
FEC Forward Error Checking
FTE Flight Technical Error

GBAS Ground Based Augmentation System
GEO Geostationary Satellite
GIC Ground Integrity Channel
GIVE Grid Ionospheric Vertical Error
GIVEI GIVE Indicator

GLONASS Global Navigation Satellite System
GNSS Global Navigation System Systems
GPS Global Positioning System
GRS Geostationary Ranging Station

HAL Horizontal Alert Limit
HPL Horizontal Protection Level

ID Identifier
IGP Ionospheric Grid Point
IOD Issue Of Data
IODC Issue Of Data Clock (GPS)
IODE Issue Of Data Ephemeris (GPS)
IODF Issue Of Data Fast Corrections
IODI Issue Of Data Ionosphere
IODP Issue Of Data PRN Mask
IODS Issue Of Data Service
IOD Issue Of Data
IOR INMARSAT satellite Indian Ocean Region
IPP Ionospheric Pierce Point

MCC Mission Control Centre
MOPS Minimum Operational Performance Standards
MT Message Type

N Northern Latitude
N/A Not Applicable
NLES Navigation Land Earth Station
NMA Norwegian Mapping Agency
NPA Flight Phase Non-Precision Approach
NPV-I Flight Phase Non-Precision Approach with Vertical Guidance I
NPV-II Flight Phase Non-Precision Approach with Vertical Guidance II
NSE Navigation System Error

PEGASUS Prototype EGNOS GBAS Analysis System Using SAPPHIRE
PRN Pseudo Random Noise (satellite identifier)
PRC Pseudo Range Correction
1.4 Definitions

Flight Technical Error: The Flight Technical Error is defined to be the difference between the required flight path and the displayed position of the aircraft.

Horizontal Protection Level: The Horizontal Protection Level HPL is the radius of a circle in the horizontal plane (the plane tangent to the WGS84 ellipsoid), with the centre being at the true aircraft position, which describes the region which is assured to contain the indicated horizontal position with a specified probability. It is the horizontal region for which the missed alert requirements can be met.

Vertical Protection Level: The Vertical Protection Level VPL is half of the length of a segment on the vertical axis (perpendicular to the horizontal plane of WGS84 ellipsoid), with its centre being at the true position, which describes the region which is assured to contain the indicated vertical position specified probability. It defines the vertical region for which the missed alert requirements can be met.

Navigation System Error: The Navigation System Error is defined to be the difference between the aircraft's true position and its computed and displayed position.
Misleading Information: Misleading Information is defined to be any data which is output that has an error larger than the alert limit without any indication of the error within the Time-To-Alert for the applicable phase of flight.

Verification: Verification is a confirmation by examination and provision of objective evidence that the specified requirements have been fulfilled. A verification process is performed to show the system is built right (see [19]).

Validation: Validation is a confirmation by examination and provision of objective evidence that the particular requirements for a specific intended use are fulfilled. A validation process is performed to show the right system is built (see [19]).

Certification: Certification is a procedure by which a third party gives written assurance that a product, process or service conforms to specific requirements (see [19]).

1.5 References

[9] Mueller, K. Tysen; Dr. Loomis, Peter V. W.; Biester, Mike : Performance Comparison of Candidate U.S. Coast Guard WADGPS Network Architectures, ION National Technical Meeting 1994
[10] Pullen, Sam; Dr. Enge, Per; Dr. Parkinson, Bradford: Simulation-Based Evaluation of WAAS Performance: Risk and Integrity Factors, ION GPS Satellite Division 1994
[16] -: ESA EGNOS web pages http://www.esa.int/export/esaEG/estb.html
The PEGASUS*Plus project established an extension to the original PEGASUS project that decode, process and evaluate the GNSS / SBAS data. Recent developments have integrated all these modules into a PEGASUS frame and the necessary documentation has been modified accordingly – except for the URD where it has been decided not to generate a new issue.

1.6 Constraints

Although the implementation of the SBAS correction and ranging information is based on documents which describe the intent quite clearly, several points in these documents are open to interpretation and application. These open questions have been marked inside this document with the following convention.

The problem is identified by the text following this question mark. This problem is discussed and a solution to the problem is suggested, which has then been implemented inside PEGASUS. Still, the interested reader is invited to provide feedback in order to clarify the points subject to discussion and to improve the processing of the data inside PEGASUS. The following check mark closes the discussion on this particular problem. ✓
2 The EGNOS Satellite Test Bed

The description of the EGNOS Satellite Test Bed is taken from the ESTB User Interface Document [13]. Further information can also be found on the ESTB web page [16].

It should be noted that the description contained in this chapter summarises the implementation of the ESTB during the phase between February 2000 and June 2001. In the meantime, the ESTB changed its signal format twice


and therefore the main parts of the signal generation and signal implementation. If current information on the signal content and the broadcast schedule is needed, the interested reader is referred to the appropriate web pages [16]. No updated description could be obtained at the current moment of updating this document.

2.1 Description of the ESTB General Functions

The ESTB is a complete prototype of the EGNOS system. It implements the main functions (ranging, GPS differential corrections and integrity) with real time elements enabling different user experiments or tests. Of course, the ESTB functions, complexity and performance are reduced by an order of magnitude with respect to the EGNOS system, principally concerning the availability, the robustness and redundancy of facilities.

2.2 Description of the ESTB Implementation

The ESTB system is illustrated in figure 1. It is composed of:

- Remote Integrity Monitoring Stations (RIMS) gathering data for the purpose of generating corrections and integrity messages deployed over Europe at hosting sites,
- Central Processing Facility (CPF) to compute on-line differential corrections and integrity data, based at Hönfoss in Norway (NMA premises),
- three EURIDIS Reference Stations implementing a wide triangular observation base for ranging purposes with the stations located in Aussaguel (France), Kourou (French Guyana) and Hartebeeshoeck (South Africa),
- a Mission Control Centre (MCC) located in Toulouse, France, (CNES premises), to compute ranging messages and prepare navigation messages for transmission through the AOR-E payload,
- two Navigation Land Earth Station (NLES) based on existing stations of the INMARSAT network, one located in Aussaguel (France) implementing the broadcast link to the AOR-E Navigation payload, the other located at Fucino (Italy) implementing the broadcast link to the IOR Navigation Payload. Each NLES is compliant with requirements from the INMARSAT SDM [18].

a ground network composed of sub-networks to transmit data from reference stations to central processing facilities, to the mission control centre and to the appropriate geostationary uplink stations.
2.3 User equipment requirements

To be able to receive the ESTB signal and process ESTB messages, the user has to acquire a GNSS receiver able to process a Signal-in-Space as defined in

- the ICAO SARPS [12]
- the MOPS [4], with the exception of the message type 0 (see warning hereafter)
- and the GPS SPS [14].

The receiver has to track the geostationary satellite with the Pseudo Range Number (PRN) set to ESTB signals :

- PRN 120 for AOR-E navigation payload
- PRN 131 for IOR navigation payload

in order to obtain lock on the ESTB satellites L1 signal, to measure pseudoranges, to acquire the ESTB broadcast messages and to compute user positions including GPS/SPS and ESTB services.

Warning about message type 0 : In case of reception of this type of message, the MOPS receiver is supposed to discard all the data messages from the geostationary satellite. If positions are delivered by the receiver, it is only for test purposes and using these positions for another matter is at the user’s own risk without any claim against the ESTB signal provider. The ESTB always broadcasts message type zero, so the use of the signal would not be possible if the receiver followed the requirement in the MOPS [4].
### 2.4 Modes and Provided Services

The ESTB Signal in Space is broadcast by the AOR-E and the IOR geostationary satellites. The ESTB has begun its broadcast via the AOR-E in February 2000. Current and updated information can be obtained by contacting the appropriate personnel through the ESA web pages [16].

A set of modes has been defined for the ESTB according to the status of some ESTB components. In each mode, some information about test receiver deliveries are given regarding the augmentation service relative to the basic GPS SPS. The list of possible ESTB modes is:

- ESTB mode 0 (all corrections broadcast)
- ESTB mode 2 (fast corrections and ionospheric corrections broadcast)
- ESTB mode 3 (fast corrections broadcast)
- ESTB mode ranging only (no corrections broadcast)

The broadcast by the geostationary satellite AOR-E (i.e. the PRN 120) in all modes can include the ranging functionality for this satellite. At the time being, the geostationary satellite IOR (i.e. the PRN 131) does not include the ranging functionality and thus transmits only in modes 0, 2 and 3.

The different types of messages broadcast by the ESTB are summarised in the following table. The content and use of each type of message is described in the relevant MOPS [4].

<table>
<thead>
<tr>
<th>ESTB mode</th>
<th>Types of broadcast messages</th>
<th>Availability through</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>AORE (PRN 120)</td>
<td>IOR (PRN 131)</td>
</tr>
<tr>
<td>Mode 0</td>
<td>0, 1, 2-5(1), 7(2), 9, 10(3), 12, 18, 25, 26, 27(3)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0, 1, 2-5(1), 7(2), 9, 10(2), 12, 18, 25(4), 26, 27(3)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Mode 3</td>
<td>0, 1, 2-5(1), 7(2), 9, 10(2), 12, 25(4), 27(3)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Ranging Only</td>
<td>0, 1, 9, 12, 17, 24, 63(5)</td>
<td>yes</td>
<td>No</td>
</tr>
</tbody>
</table>

(1) The PRN 120 is set to “unmonitored” in the fast correction message type broadcast (message types 2-5)
(2) The values contained in the broadcast are fixed to a certain configuration of the ESTB service management approach.
(3) Message type MT 27 is broadcast only when testing the ESTB outside the nominal area
(4) In modes 2 and 3, message type MT 25 (slow corrections) do contain only the time information
(5) The definition of MT 24 in the ranging only mode is not quite clear

**Table 1**: ESTB modes and message types

*Warning*: There is no standard ESTB sequence(s) of messages except in “only ranging” mode. The ESTB messages are only constrained to the maximum update interval defined for each type of message in the MOPS [4].

*Note*: The IOR transmits no message type MT 10 and the transmissions for message types MT 9 and MT 12 are filled with zeros. Further information can also be found on the ESTB web page [16].
3 Preliminary

The next two sections of this document describe the processing of the SBAS correction and ranging information.

This first section covers the processing of the SBAS differential correction and integrity data. Chapter 4 describes the general layout and approach to the organisation and processing of the differential corrections. In chapter 5, the fast correction data is introduced, while in chapter 6 the long-term (or slow) correction data is described. In chapter 7, the ionospheric correction model is explained and in chapter 8 the tropospheric error correction model is described.

In chapter 9, the application of the differential corrections is shown and the logic behind the satellite selection for the positioning solution is described. In chapter 10, a brief description of the navigation solution algorithm and the protection level calculation is provided. The application of the message type MT 27 (service region indication) is described in chapter 11. However, the ESTB implements the message type MT 27 broadcast only in an “expansion mode” approach. Further information on the “expansion mode” could not be obtained at the moment of updating this document, but it is expected that this information will be available in the future.

The second section covers the processing of the SBAS ranging data. In chapter 12, a brief description of the SBAS ranging data and its processing is provided. Chapter 13 explains the processing of the SBAS ephemeris data, after which the chapter 14 describes the processing of the SBAS almanac data.
SBAS Differential Correction and Integrity
4 General

4.1 Message Types and Format

The raw navigation message of the SBAS SIS contains 500 bits, transmitted in each second. This raw navigation message is ½ convolutional encoded with a Forward Error Correcting (FEC) Code. Thus, the baseline data rate of the SBAS SIS will be 250 bits per second.

The block format for the 250 bits per second data rate is shown in table 2 and figure 2. The 8-bit preamble starts at bit 0 of the 250-bit message, followed by the 6-bit message type identifier at bit 8. The data field then starts at bit 14, followed by parity field that starts at bit 226.

<table>
<thead>
<tr>
<th>Bit Position</th>
<th>Number of Bits</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>preamble</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>message type identifier</td>
</tr>
<tr>
<td>14</td>
<td>212</td>
<td>binary message</td>
</tr>
<tr>
<td>226</td>
<td>24</td>
<td>parity</td>
</tr>
</tbody>
</table>

Table 2: SBAS SIS Message Format

The preamble is a 24-bit unique word, distributed over three successive blocks. These three 8-bit words are made up of the sequence of bits (01010011, 10011010 or 11000110). The receiver must successively lock on different preambles in order to avoid decoding the same preamble and message twice, but not necessarily in successive blocks. This assures frame synchronisation while allowing for occasional block errors without repeating a complete synchronisation.

The message type identifier is a binary coded integer value (range 0-63), thus resulting in 64 different possible message types for the SBAS SIS. The following table 3 contains a summary of the message types that are contained in the current ESTB SIS version 1.2. This version is transmitted starting from end of August 2002.

It is important to acknowledge that the currently transmitted ESTB SIS V1.2 (although labelled RTCA DO 229B) is specified mainly by the RTCA DO 229B [4] standard – but certain individual parts still conform to the RTCA DO 229A [3] standard. The table includes as well information on which of the standards is used in the current ESTB SIS V1.2.
The transmitted message types can be grouped into several applications. The message type MT 0 broadcasts a “do not use this satellite” message, whereas the message type MT 1 is used for the organisation of the fast and slow corrections and is discussed in this chapter. The message types MT 2 – MT 5, MT 6 and MT 7 are used for the fast corrections. They are discussed in chapter 5. Message type 25 contains the slow corrections and is described in chapter 6. The ionospheric correction parameters are contained in the message types MT 18 and MT 26. A description of these parameters can be found in chapter 7. The message type MT 10 describes the degradation of the upper bounds on the residual errors of the fast corrections, slow corrections and ionospheric corrections over time. A description of the content of the message type MT 10 can be found inside this chapter, whereas the application of the information contained in the MT 10 will be found with the relevant chapters on fast corrections (chapter 5), slow corrections (chapter 6) and ionospheric corrections (chapter 7). Message types MT 9 and MT 17 contain the ephemeris or the almanac data for the SBAS satellites. These messages are detailed in the chapters 13 and 14. The application of message type MT 27 is discussed in chapter 11.

<table>
<thead>
<tr>
<th>Message Type</th>
<th>Purpose</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Do Not Use The GEO For Anything</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>1</td>
<td>PRN Mask Assignments</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>2-5</td>
<td>Fast Corrections</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>6</td>
<td>Integrity Information</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>7</td>
<td>Integrity Degradation Information</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>9</td>
<td>SBAS Ephemeris</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>10</td>
<td>Degradation of Fast, Slow and Ionospheric Corrections</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>12</td>
<td>SBAS Network Time/UTC Offset</td>
<td>This message type is transmitted by the current ESTB version 1.2.</td>
</tr>
<tr>
<td>17</td>
<td>SBAS Almanac</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>18</td>
<td>Ionospheric Grid Point Mask</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229A [3]</td>
</tr>
</tbody>
</table>
### Table 3: ESTB SIS Message Types

<table>
<thead>
<tr>
<th>Message Type</th>
<th>Purpose</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>Mixed Fast Correction/Slow Corrections</td>
<td>Although specified in the MOPS [4], the current ESTB version 1.2 SIS does not contain this message type 24. Further details on the implementation could not be obtained at the moment of updating this document.</td>
</tr>
<tr>
<td>25</td>
<td>Slow Corrections</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
<tr>
<td>26</td>
<td>Ionospheric Delay Corrections</td>
<td>This message type is transmitted by the current ESTB version 1.2. Signal Standard is 229A [3]</td>
</tr>
<tr>
<td>27</td>
<td>SBAS Service Region</td>
<td>This message type is transmitted by the current ESTB version 1.2, although implemented with a different intention (1) Signal Standard is 229A [3]</td>
</tr>
<tr>
<td>62</td>
<td>Internal Test Message</td>
<td>Although this message has been intended to be used for internal test procedures, it is not known at the moment whether the current ESTB version 1.2 uses this message type. It is not specified in the broadcast schedule.</td>
</tr>
<tr>
<td>63</td>
<td>Null Message</td>
<td>This message type is transmitted by current ESTB version 1.2. Signal Standard is 229B [4]</td>
</tr>
</tbody>
</table>

In contrast to the original intent of the MOPS [4], the ESTB version 1.2 uses this message type only when it is in an “expansion mode”, i.e. supports in addition to the required service region another area with ESTB applications. Further information on the “expansion mode” could not be obtained at the moment of updating this document, but it is expected that this information will be available in the future.

A description of the message types MT 24 is not given in this document, since in the ESTB version 1.2 no broadcast of these message types has been planned. Furthermore, the chapter on the content and application of the message type MT 27 is still evolving and not considered to be mature. No current information on this subject could be obtained.

Although the MOPS [4] require that, at least for some flight phases, the data from two different SBAS satellites must be processed, the currently available versions of the PEGASUS tools do not support this functionality. Only the data from one SBAS satellite can be processed.

This is primarily due to the fact that the current ESTB architecture supports only the AOR-E satellite (PRN 120) in the day-to-day transmissions. Nevertheless, ESA started transmissions the broadcast of the ESTB signal as well on the IOR satellite (PRN 131) on a regular basis starting from October 2002. However, the SIS transmitted by the IOR satellite contains currently no message type MT 10 (i.e. general degradation parameters) and an empty message content for the message types MT 9 (SBAS ephemeris) and MT 12 (time transformation). Updated information on this issue might be found at the ESA web pages [16].
Additionally, the majority of receivers incorporated in the PEGASUS tools so far support only the reception of one SBAS satellite.

During normal service operations, the sequence of transmissions of the several message types is in the responsibility of the service provider, who is responsible that the provisioned operational service can be performed within the specific requirements. However, a requirement is included in the MOPS [4] about the timing of the transmissions of the SBAS broadcast with particular attention to alarm conditions. Each message type that will result in an alarm condition (broadcast in a Type 2-5, Type 24, Type 6, Type 26 or Type 27) must be repeated three times after the initial notification of the alarm condition (for a total of four times in four seconds). Subsequent messages can be broadcast at the normal update rate.

### 4.2 Message Timeouts and Update Intervals

The SBAS equipment should only use data if it is current, i.e. before it has timed out. The time-out intervals are defined from the end of the reception of a message until the end of the reception of a new message which contains replacement information. The time-out intervals are different for each message type and, for the fast corrections, are a function of the flight phase (i.e. type of operation selected). The time-out intervals are given in the following table 4.

<table>
<thead>
<tr>
<th>Message Types</th>
<th>Maximum Update Interval</th>
<th>Time-Out Interval for En-Route, through Non-Precision Approach</th>
<th>Time-Out Interval for Precision Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6 seconds</td>
<td>reception of MT 0 results in de-selection of the SBAS satellite for 60 seconds (^{(1)})</td>
<td>reception of MT 0 results in de-selection of the SBAS satellite for 60 seconds (^{(1)})</td>
</tr>
<tr>
<td>1</td>
<td>120 seconds (^{(2)})</td>
<td>600 seconds</td>
<td>600 seconds</td>
</tr>
<tr>
<td>2-5</td>
<td>60 seconds</td>
<td>Variable</td>
<td>variable</td>
</tr>
<tr>
<td>6</td>
<td>6 seconds</td>
<td>18 seconds</td>
<td>12 seconds</td>
</tr>
<tr>
<td>25</td>
<td>120 seconds</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
<tr>
<td>10</td>
<td>120 seconds</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
<tr>
<td>9</td>
<td>120 seconds</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
<tr>
<td>7</td>
<td>120 seconds</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
<tr>
<td>18</td>
<td>300 seconds (^{(2)})</td>
<td>1200 seconds</td>
<td>1200 seconds</td>
</tr>
<tr>
<td>26</td>
<td>300 seconds</td>
<td>600 seconds</td>
<td>600 seconds</td>
</tr>
<tr>
<td>27</td>
<td>300 seconds (^{(3)})</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>12</td>
<td>300 seconds</td>
<td>86400</td>
<td>86400</td>
</tr>
<tr>
<td>17</td>
<td>300 seconds</td>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

\(^{(1)}\) Strictly speaking, the time specified for the time-out interval of message type MT 0 should not be interpreted as a time-out interval. It is a waiting time until the user is allowed to re-use the transmissions of the geostationary satellite which transmits that message type MT 0.

\(^{(2)}\) When the PRN mask or the ionospheric mask is changed, it should be repeated several times before the new masks are used. This will ensure that all users receive the new mask before it is applied, maintaining high continuity.

\(^{(3)}\) The onset of a reduced service level is an alarm condition. Therefore a MT 27 which reduces the level of service in a region should be repeated 3 times (for a total of four times in four seconds).

*Table 4: Time-Out Intervals and Update Intervals*
The time-out interval of the fast corrections PRC provided in the message types MT 2-5 is variable and depends on the values of the degradation factors transmitted in the message type MT 7. The corresponding time-out intervals are also given in table 11.

Those data items that are not timed out shall be used until they are replaced. In the case of the PRN mask and the ionospheric masks, the equipment must retain the old and the new version of the mask to accomplish a smooth mask transition.

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the service region indication using the message type MT 27. The implementation these corrections still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the service region determination scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

The message type MT 0 is used in the MOPS [3] for transmitting alarm conditions and for testing a new SBAS satellite. If the user receives the message type MT 0 from one SBAS satellite, the data transmitted from that satellite must be disregarded for 60 seconds. Thus, no correction information and no ranging information that is transmitted by that satellite can be used.

The SIS of the ESTB version 1.2 transmits the message type MT 0 on a regular basis, due to the fact that the SIS can not and shall not be used for any official positioning service. Receiver software which is designed to the MOPS [4] thus will not be able to evaluate the actual and current performance of the ESTB.

In order to use the ESTB SIS for the PEGASUS development, the result of the message type MT 0 is disregarded (in a mode called EGNOSTESTMODE). Nevertheless, the message type MT 0 will be read in and processed. Once the EGNOS transmits the message type MT 0 for its original purpose, the message type MT 0 can be easily re-activated in the PEGASUS development.

The US american version of a Satellite Based Augmentation System SBAS, the Wide Area Augmentation System WAAS, uses a slightly different scheme to broadcast the message type 0. In order to save bandwidth for other transmissions, the WAAS broadcasts the contents of a message type 2 (i.e. fast corrections) in each message type 0. This is called by ESA the “SBAS mode 0/2”.

The message type MT 0 that is not empty may be used for ranging and correction purposes. In order to use the PEGASUS prototypes in such a mode, the prototypes will decode a non-empty message type 0 and interprete the data as message type MT 2 data (in a mode called WAASTESTMODE). Nevertheless, the message type MT 0 will be read in and processed. Once the EGNOS transmits the message type MT 0 for its original purpose, the message type MT 0 can be easily re-activated in the PEGASUS development.

4.3 Satellite Mask and Organisation of Correction Information

The fast and slow correction data are not provided with a PRN number attached to every item of correction information. For the organisation of the fast and slow correction data, the PRN mask will be used to designate which PRN belongs to which correction slot. The use of this mask instead of labelling fast corrections and integrity information continually with PRN numbers improves the efficiency of the broadcast.

The PRN mask is given in message type MT 1. It consists of 210 ordered slots, each of which indicates if data is provided for the corresponding satellite as defined in table 5. The mask can have up to 51 bits set to “1” in the 210 corrections slots.
The PRN mask is followed by a 2-bit issue of data PRN (IODP) to indicate the mask’s applicability to the corrections and integrity data contained in the following messages. The data content of MT 1 is shown in the following table 6.

<table>
<thead>
<tr>
<th>Bit Position</th>
<th>Number of Bits</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>210</td>
<td>PRN mask</td>
</tr>
<tr>
<td>210</td>
<td>2</td>
<td>IODP PRN</td>
</tr>
</tbody>
</table>

**Table 6: Message Type 1**

Each correction set in the MT 2-5, 6, 7, 25, 24 is characterised by its PRN Mask Number which can have the maximum value of 51 slots set to “1”. The assignment from the PRN Mask Number to the real PRN with the use of the mask in message type MT 1 is shown in the following table 7.

<table>
<thead>
<tr>
<th>Bit No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>…</th>
<th>38</th>
<th>……</th>
<th>120</th>
<th>……</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRN Mask</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>PRN Code No</td>
<td>GPS PRN 3</td>
<td>GPS PRN 5</td>
<td>GPS PRN 7</td>
<td>GLONASS Slot 1</td>
<td>ESTB AOR-E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 7: PRN Mask Assignments**

The time-out interval of message type MT 1 does not depend on the flight phase and given in the following table 8.

<table>
<thead>
<tr>
<th>message type</th>
<th>time-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 1</td>
<td>600 seconds</td>
</tr>
</tbody>
</table>

**Table 8: Time-Out Intervals for MT 1**

A transition of the PRN mask would – according to the MOPS [4] - probably only occur when a new satellite is launched or when a satellite fails and is permanently taken out of service. The transition of the PRN mask to a new PRN mask will be controlled with the 2-bit IODP, which will sequence to a number between 0 and 3. The same IODP will appear in the applicable message types 2-5, 7, 24 and 25.

If the IODP of the mask does not agree with the IODP in the applicable message types 2-5, 7, 24 and 25, the user will not use the applicable message until the IODPs agree. However, if the user can resolve the satellite PRN numbers during a change-over of the IODP in the PRN mask, the appropriate message types can be
applied by making the appropriate mapping of the PRN numbers across the change-over. Thus, the user must be able to store and use two PRN masks. The appropriate mask matching the IODP in the message types 2-5, 7 and 25 can be used during a mask-transition. In order to avoid interruptions during the PRN mask transition, the user must be able to use portions of both stored masks simultaneously.

However, if the IODP changes in those message types before a reception of the new PRN mask in MT 1, these message types must not be used until the receipt of the new mask. The change of IODP in the PRN mask message should always occur before the IODP changes in all other messages.

4.4 Degradation of Correction Information

The fast, slow and ionospheric correction data are provided only at certain instances of time (due the bandwidth of the transmission of the information) and are valid throughout an interval until they experience a time-out. In order to be able to use the corrections and upper bounds on the residual errors in the whole interval in which these corrections are valid, a degradation of the correction information is assumed. The models for the degradation of the correction and the upper bounds of the residual errors are provided in the MOPS [4] and are updated through the transmission of parameters that are applicable for these models. The data of these parameters are transmitted in the message types MT 7 and MT 10.

There is a time gap of several seconds between the update of pseudorange corrections and upper bounds on the residual error for each satellite. To take the degradation of the correction values and error bound due to the presence of the Selective Availability of the GPS over time into account, the message type MT 7 transmits degradation factors for the User Differential Range Error UDRE. The content of the message type MT 7 is listed in the table 9. The application of the transmitted information on the degradation of the fast corrections is provided in the relevant chapter 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Latency TL</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0...15</td>
<td>s</td>
</tr>
<tr>
<td>IODP</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>spare</td>
<td>6</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>AI (51 x)</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>0...15</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9: Degradation Message Type MT 7

The time-out interval of the degradation parameters transmitted in MT 7 are only dependent on the flight phase and are given in the following table 10.

<table>
<thead>
<tr>
<th>Message Type</th>
<th>time-out interval (Non-Precision Approach)</th>
<th>time-out interval (Precision Approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 7</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
</tbody>
</table>

Table 10: Time-Out Intervals for MT 7

The time-out interval of the fast corrections provided in the message types MT 2-5 is variable and depends on the values of the degradation factors transmitted in the message type MT 7. The corresponding time-out intervals are also given in table 11.
### Table 11: Degradation Message Type MT 7

In addition to the degradation of the fast corrections and error bounds due to the existence of Selective Availability, the obvious degradation of all transmitted corrections (fast, slow and ionospheric corrections) over time due to the limited bandwidth of the transmission must be taken into account. A parametric model is provided for all error bounds in the MOPS and the parameters for these models are updated through the transmission of the message type MT 10. The content of the message type MT 10 is listed in the table 12. The application of the transmitted information on the degradation of the fast corrections is provided in the relevant chapters (chapter 5 and chapter 7).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&lt;sub&gt;rrc&lt;/sub&gt;</td>
<td>0</td>
<td>10</td>
<td>0.002</td>
<td>0…2.046</td>
<td>m</td>
</tr>
<tr>
<td>C&lt;sub&gt;ltc&lt;/sub&gt; LSB</td>
<td>10</td>
<td>10</td>
<td>0.002</td>
<td>0…2.046</td>
<td>m</td>
</tr>
<tr>
<td>C&lt;sub&gt;ltc&lt;/sub&gt; V1</td>
<td>20</td>
<td>10</td>
<td>0.00005</td>
<td>0…0.05115</td>
<td>ms&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>I&lt;sub&gt;ltc&lt;/sub&gt; V1</td>
<td>29</td>
<td>9</td>
<td>1</td>
<td>0…511</td>
<td>s</td>
</tr>
<tr>
<td>C&lt;sub&gt;ltc&lt;/sub&gt; V0</td>
<td>39</td>
<td>10</td>
<td>0.002</td>
<td>0…2.046</td>
<td>m</td>
</tr>
<tr>
<td>I&lt;sub&gt;ltc&lt;/sub&gt; V0</td>
<td>48</td>
<td>9</td>
<td>1</td>
<td>0…511</td>
<td>s</td>
</tr>
<tr>
<td>C&lt;sub&gt;geo&lt;/sub&gt; LSV</td>
<td>58</td>
<td>10</td>
<td>0.0005</td>
<td>0…0.5115</td>
<td>m</td>
</tr>
<tr>
<td>C&lt;sub&gt;geo&lt;/sub&gt; V</td>
<td>68</td>
<td>10</td>
<td>0.00005</td>
<td>0…0.05115</td>
<td>ms&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>I&lt;sub&gt;geo&lt;/sub&gt;</td>
<td>77</td>
<td>9</td>
<td>1</td>
<td>0…511</td>
<td>s</td>
</tr>
<tr>
<td>C&lt;sub&gt;er&lt;/sub&gt;</td>
<td>83</td>
<td>6</td>
<td>0.5</td>
<td>0…3.15</td>
<td>m</td>
</tr>
</tbody>
</table>
Table 12: Degradation Message Type MT 10

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIONO STEP</td>
<td>93</td>
<td>10</td>
<td>0.001</td>
<td>0…1023</td>
<td>m</td>
</tr>
<tr>
<td>IONO</td>
<td>102</td>
<td>9</td>
<td>1</td>
<td>0…511</td>
<td>s</td>
</tr>
<tr>
<td>CIONO RAMP</td>
<td>112</td>
<td>10</td>
<td>0.000005</td>
<td>0…0.005115</td>
<td>ms⁻¹</td>
</tr>
<tr>
<td>RSSUDRE</td>
<td>113</td>
<td>1</td>
<td>1</td>
<td>0…1</td>
<td>N/A</td>
</tr>
<tr>
<td>RSSIONO</td>
<td>114</td>
<td>1</td>
<td>1</td>
<td>0…1</td>
<td>N/A</td>
</tr>
<tr>
<td>spare</td>
<td>115</td>
<td>88</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The spare bits in the message type MT 10 may be used in an update of the MOPS [4] to define degradation parameters applicable to the GLONASS satellites. The time-out interval of the degradation parameters transmitted in MT 10 are only dependent on the flight phase and are given in the following table 13.

Table 13: Time-Out Intervals for MT 10

<table>
<thead>
<tr>
<th>Message type</th>
<th>time-out interval (Non-Precision Approach)</th>
<th>time-out interval (Precision Approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 10</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
</tbody>
</table>

4.5 Application of Message Types

If possible, the MOPS [4] recommends that the equipment apply differential corrections whenever available. Any message may be optionally used in flight phases for which they are not specifically required, but must then conform to the relevant requirements. If an optional message is used, the loss of those messages must not cause the loss of the navigation functionality.

The message types MT 0 (Don’t Use), MT 1 (PRN Mask), MT 2-5 (Fast Corrections), MT 6 (Integrity), MT 7 (UDRE Degradation), MT 9 (SBAS Ephemeris), MT 17 (SBAS Almanac), MT 24 (Mixed Corrections), MT 25 (Slow Corrections) and MT 27 (service region) must be used for all navigation modes. Additionally, for the Precision Approach flight phase, the message type MT 10 (Correction Degradation), MT 18 (Ionospheric Grid) and MT 26 (Ionospheric Corrections) must be applied.

In the current implementation of the ESTB version 1.2, there is a deviation from the requirements of the MOPS [4]. Thus, in order to be able to use the PEGASUS developments for the implemented version of the ESTB, the following specifications are established.

The current ESTB version 1.2 does transmit message type MT 27 data (service region) as it is defined in the MOPS [3] or [4]. However, the intended application of the message information differs from the requirements of the specification: it is used to define regions outside the applicable service region where a certain type of operation might be supported. No further information was available at the time of the updating of this document. Thus, the presence of this message type will be treated as optional (i.e. read in and processed, if there), but will not be required for any flight phase.
5 Fast Corrections

5.1 Message Types and Format

The fast corrections must be utilised in all navigation modes from En Route to Precision Approach. They are necessary to correct the fast changing errors – usually the satellite clock error due to the (now no longer existing) degradation through selective availability SA. The fast corrections, provided as range correction values, are applied directly to the range measurements.

Together with the fast corrections, the FC message types contain integrity indicators in the form of User Differential Range Error UDRE estimates. These UDRE are an upper bound – in form of a standard deviation - on the residual error of the pseudorange after the application of fast corrections, including the possibility that any messages are missed by the user. The UDRE are used to compute protection levels and warning flags indicating that an individual PRN should not be used in the position solution.

The message types which contain the information for the fast corrections and UDRE are MT 2-5, MT 6 and MT 7. The message types MT 2, MT 3, MT 4 and MT 5 have the same format. Each message contains range correction values and the associated UDRE values for 13 out of a total of 51 GNSS satellites. The fast correction message format is shown in the following table 14. A fast correction message type will only be sent if the number of satellites designated in the PRN mask (message type MT 1) requires it, i.e. message type MT 5 will only be broadcast, if 40 or more satellites are designated. In any case, the last block of message type MT 5 will be empty, since only a total of 51 satellites are supported by the SBAS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>IODF</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>IODP</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>PRC (13 x)</td>
<td>4</td>
<td>12</td>
<td>0.125</td>
<td>-256.000......+255.875</td>
<td>metres</td>
</tr>
<tr>
<td>UDREI (13 x)</td>
<td>160</td>
<td>4</td>
<td>1</td>
<td>0...15</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 14: Fast Correction Message Types MT 2-5

The IODP is used to associate the pseudorange corrections and UDRE which are designated by the PRN Mask Numbers with the real PRN number as shown in chapter 4 and table 7. The Issue of Data Fast Corrections IODF will change with each new set of the Message and is used for synchronising the UDRE from that Message with the ones from a succeeding message type MT 6.

<table>
<thead>
<tr>
<th>UDREI</th>
<th>$\sigma_{i,UDRE}^2$</th>
<th>UDREI</th>
<th>$\sigma_{i,UDRE}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0520 m²</td>
<td>8</td>
<td>2.5465 m²</td>
</tr>
<tr>
<td>1</td>
<td>0.0924 m²</td>
<td>9</td>
<td>3.3260 m²</td>
</tr>
<tr>
<td>2</td>
<td>0.1444 m²</td>
<td>10</td>
<td>5.1968 m²</td>
</tr>
<tr>
<td>3</td>
<td>0.2830 m²</td>
<td>11</td>
<td>20.7870 m²</td>
</tr>
<tr>
<td>4</td>
<td>0.4678 m²</td>
<td>12</td>
<td>230.9661 m²</td>
</tr>
<tr>
<td>5</td>
<td>0.8315 m²</td>
<td>13</td>
<td>2078.695 m²</td>
</tr>
<tr>
<td>6</td>
<td>1.2992 m²</td>
<td>14</td>
<td>satellite not monitored !</td>
</tr>
<tr>
<td>7</td>
<td>1.8709 m²</td>
<td>15</td>
<td>do not use satellite !</td>
</tr>
</tbody>
</table>

Table 15: Conversion from UDREI to $\sigma_{i,UDRE}^2$
As can be seen in the table 14, it is not the UDRE itself that is transmitted, but the User Differential Range Error Indicator UDREI value. The conversion from the indicators UDREI to the actual $\sigma^2_{\text{UDRE}}$ can be derived from table 15.

In addition, a satellite shall not be used for the precision approach flight phase, if the UDREI associated with that satellite is 12 or greater. The exclusion of a satellite using such an approach allows the service provider to designate satellites not to be used for precision approach flight phases under certain failure conditions.

If there are 6 or less pseudorange corrections to be transmitted, these values are placed according to the MOPS [4] in the first half of the message type 24 (Mixed Corrections). The format of that message type is similar to that shown in table 14, but contains only 6 pseudorange corrections and UDREI values and additionally one 2-bit Block ID. This Block ID is used to associate the transmitted correction values with the corresponding message type 2,3,4 or 5 (and thus, with a corresponding PRN number).

Although the ESTB SIS V1.2 does not transmit the message type MT 24 information, the option of incorporating the message type MT 24 into the fast correction calculation is implemented in the PEGASUS development in order to accommodate future versions and configurations of the ESTB Signal-In-Space.

The UDREI values can also be transmitted via the message type MT 6. The message type MT 6 contains an update of the error bound values for all possible 51 satellites at once. According to the MOPS [4], it is transmitted to provide integrity information in the case where the message types MT 2-5 need not be transmitted at an frequent interval (due to absence of S/A, etc.). The format of the message type MT 6 is shown in table 16.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>IODF_1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>IODF_2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>IODF_3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>IODF_4</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>0...3</td>
<td>1</td>
</tr>
<tr>
<td>51 x UDREI</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>0...15</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 16: Integrity Message Type MT 6

This message contains UDREI values for each PRN Mask Number. Thus, the integrity values and alarms for multiple satellites can be transmitted at once if necessary. To guarantee the correct assignments with the preceding MT 2-5, there are 4 IODF parameters included in the message. These Issue of Data Fast Corrections IODF are used to associate the UDREI values with the corresponding pseudorange correction values in the message types MT 2-5.

An Issue of Data PRN IODP is missing in the message type MT 6. When interpreting a message type MT 6, the user equipment shall use the most recently received PRN mask for which corrections have been received.

There is a time gap of several seconds between the update of pseudorange corrections PRC and UDRE for each satellite. To take the degradation of the correction values over time into account, the message type MT 7 transmits degradation factors for the User Differential Range Error UDRE. Apart from the degradation of UDRE with time, the message type MT 7 is used to determine the time-out intervals of the fast correction information in the message types MT 2-5. The format of the message type MT 7 data is given in table 9 in the chapter 4.

Again an IODP is included in the message type MT 7 to assign the degradation factor indicators AI to the correct PRN satellite numbers. The system latency time TL is included for the correct calculation of the
degradation of the UDRE with time, since the time of application of the UDRE is not identical with the time of application of the degradation factors AI. As can be seen from the table 9, it is not the degradation of the UDRE that is transmitted, only indicator values are provided. The actual degradation can be found using the following table 17.

<table>
<thead>
<tr>
<th>degradation factor indicator $a_i$</th>
<th>UDRE degradation factor $a_i$</th>
<th>degradation factor indicator $a_i$</th>
<th>UDRE degradation factor $a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00000 m/s²</td>
<td>8</td>
<td>0.00060 m/s²</td>
</tr>
<tr>
<td>1</td>
<td>0.00005 m/s²</td>
<td>9</td>
<td>0.00090 m/s²</td>
</tr>
<tr>
<td>2</td>
<td>0.00009 m/s²</td>
<td>10</td>
<td>0.00150 m/s²</td>
</tr>
<tr>
<td>3</td>
<td>0.00012 m/s²</td>
<td>11</td>
<td>0.00210 m/s²</td>
</tr>
<tr>
<td>4</td>
<td>0.00015 m/s²</td>
<td>12</td>
<td>0.00270 m/s²</td>
</tr>
<tr>
<td>5</td>
<td>0.00020 m/s²</td>
<td>13</td>
<td>0.00330 m/s²</td>
</tr>
<tr>
<td>6</td>
<td>0.00030 m/s²</td>
<td>14</td>
<td>0.00460 m/s²</td>
</tr>
<tr>
<td>7</td>
<td>0.00045 m/s²</td>
<td>15</td>
<td>0.00580 m/s²</td>
</tr>
</tbody>
</table>

**Table 17**: Degradation Message Type MT 7

The time-out intervals of the indicator values UDREI (message types MT 2-5, MT 6) and AI parameter (message type MT 7) themselves are only dependent on the flight phase and are given in the following table 18.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>time-out interval (Non-Precision Approach)</th>
<th>time-out interval (Precision Approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UDREI</td>
<td>18 seconds</td>
<td>12 seconds</td>
</tr>
<tr>
<td>AI</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
</tbody>
</table>

**Table 18**: Time-Out Intervals for UDREI and AI

The calculation of the fast corrections PRC to the pseudorange measurements is explained in the chapter 5.2, whereas the application of the UDRE and AI degradation factors and the degradation due to message type MT 10 information are described in chapter 5.3.

### 5.2 Pseudorange Corrections

The pseudorange corrections PRC and UDRE are sorted by PRN mask numbers. In the message type MT 2 the values transmitted correspond to the first 13 satellites supported as indicated by the PRN mask (i.e. the first 13 slots in the PRN mask set to “1”). In the message type MT 3, the values transmitted are corresponding to the second set of 13 satellites supported (i.e. the second 13 slots in the PRN mask set to “1”). The same reasoning applies to the values transmitted in the message types MT 4 and MT 5. If the message type MT 24 is used, then this message will contain some indication for what PRN slots the corrections are intended. The assignment from the PRN mask number to the real PRN number via the PRN Mask message MT 1 is already described in the chapter 4.2.

The fast corrections are updated using a certain interval which is in the provision of the service provider. The maximum update interval is given in the MOPS [4] as 60 seconds. However, range-rate corrections are not transmitted by the SBAS satellite. The equipment must compute these rates-of-change by differencing successively received fast correction PRC – regardless of the Issue of Data Fast Correction IODF. This procedure is shown in the following figure 3.
Figure 3: Pseudorange Corrections and Pseudorange-Rate Corrections

\[ RRC(t_{of}) = \frac{PRC_{current} - PRC_{previous}}{t_{of, current} - t_{of, previous}} \] (1)

with

- \( RRC \) Range-Rate Corrections
- \( PRC \) Pseudorange Corrections
- \( t_{of, current} \) time of applicability of the most recent received pseudorange correction
- \( t_{of, previous} \) time of applicability of the previously received pseudorange correction

The time of applicability of the pseudorange corrections is the start of the epoch of the SBAS Network time second that is coincident with the transmission at the SBAS satellite of the first bit of the message block. Most of the current GNSS receivers are receive-time receivers. Thus, the instance of time in which the signal is transmitted by the satellite is not known exactly and can only be estimated using the measured pseudorange. Since the signal transition time is (even for SBAS satellites) very small, the time of applicability of the pseudorange-rate corrections are taken as the start of the epoch of the time of reception of the signal at the receiver for the PEGASUS tools.

However, if the fast correction degradation parameter \( AI \) for a particular satellite is set to zero, then the pseudorange rate correction \( RRC \) shall be equal to zero as well:

\[ AI = 0 \Rightarrow RRC = 0 \] (2)

The most recent received fast correction \( PRC_{current} \) must be used when computing the range rate corrections. In selecting the previously received fast correction \( PRC_{previous} \) to be used in determining the range rate corrections, the user should select the fast correction which minimizes the degradation due to fast corrections and range rate corrections according to chapter 5.3

(a) If the current IODF of the fast corrections is equal to 3 (which signals an “alarm condition” for the fast corrections), it is likely that several corrections (i.e. transmissions of a fast correction message type) will be set over a short period of time. In this case, in order to minimizes the noise effect on the range-rate corrections, the previous fast correction closest to \( t_{fc} / 2 \) seconds prior to the current fast correction shall
be selected, where $I_{fc}$ is the time-out interval for the fast corrections determined through the fast correction degradation factors $Al$ in message type MT 7 and table 11.

Thus, a buffer for at least $I_{fc}/2$ seconds must be available in order to store the received fast corrections. Using the nominal update rates of the transmissions for message types MT 2-5 and the maximum time-out intervals for the fast corrections according to table, a buffer of maximum 20 fast corrections data sets must be stored. This buffer will be searched for the transmitted fast correction which is closest to $I_{fc}/2$ seconds previous to the most recently received fast corrections.

(b) If the current IODF of the fast corrections is not equal to 3, the user should select the most recently previous transmitted fast correction to determine the range rate corrections according to equation (1).

![Figure 4: Time History of Pseudorange Corrections](image_url)

The figure 4 summarises the approach to selecting the previous pseudorange correction in order to compute the range rate correction in equation (1).

At the beginning of data collection or after the reception of a “not monitored” or “do not use” message (see table 15), the calculation of the range-rate correction $RRC$ must be initialised. During re-initialisation, the $RRC$ shall not be used and the variance of the associated satellite $\sigma^2_{i,flt}$ is set to $60^2$ m$^2$ (the variance is used to calculate the protection levels).

Particular attention is drawn to situations where a satellite previously is considered either “not monitored” or “unsuitable” and then its status is changed and a UDREI below 12 is transmitted for that satellite. The above mentioned approach requires that the variance of such a satellite is set to a very high value. This will have an impact on the calculated protection levels, which will experience a significant increase.

It is proposed to not to use a satellite where the range-rate corrections must be initialised for precision approach flight phases. This can be accomplished by setting the UDREI in the first transmission after a “not monitored” or after a “do not use” to a value of either 12 or 13. This will allow the re-initialisation of the range rate corrections, but will prevent the use of the satellite for precision approach flight phases.
In the MOPS [4] there is no guidance on the re-initialisation of the range rate corrections in the situation after a timeout of a pseudorange correction PRC (i.e. in situations, where the most recently received PRC is valid, but the previously received PRC has timed out. It is assumed that the range rate corrections must be re-initialised in these situations.

In addition to the time-out interval for the fast corrections determined by table 11, there are three time-out conditions applicable to the range rate corrections:

\[ t - t_{of} - 1 > I_{fc} \implies \text{RRC time - out} \] (3)

\[ t - t_{of} - 1 > 8 \Delta t \implies \text{RRC time - out} \] (4)

\[ \Delta t > I_{fc} \implies \text{RRC time - out} \] (5)

\[ \Delta t = t_{of} - t_{of, previous} \] (6)

with:
- \( t \) current user epoch
- \( t_{of} \) time of applicability of current fast corrections
- \( t_{of, previous} \) time of applicability of previous fast corrections
- \( I_{fc} \) time-out interface as determined by the fast correction degradation parameter AI

In the MOPS [4] there is no guidance on the re-initialisation of the range rate corrections in situations described in the last paragraph above, i.e. in situations where the range rate corrections have timed out. Furthermore, it is not clear from the MOPS [4] whether the satellite should be used for positioning or not.

It is assumed that the range rate corrections must be re-initialised after a time-out of the range-rate corrections. The satellite shall not be used in these circumstances.

At a specified user time \( t \), which is usually greater than the time of applicability \( t_{of} \), the total range corrections are calculated using a linear interpolation scheme (see as well figure 3).

\[ RC(t) = PRC(t_{of}) + RRC(t_{of})(t - t_{of}) \] (7)

with
- \( RC \) total range correction
- \( t \) user time
- \( t_{of} \) time of applicability of the most recent fast corrections

The total range correction according to the equation (7) is then used in the correction scheme for measured pseudorange as shown in chapter 9.

### 5.3 Correction Error Estimation

The fast corrections and the slow corrections are designed to provide the most recent information to the user. However, the limited bandwidth used for the transmission of the information provides for situations where the user does not have the data for the current epoch available. In addition, there is always the possibility that the user will fail to receive one of these messages, either due to momentary shadowing or a random bit error.
In order to guarantee integrity even when some messages are not received, the user must apply models of degradation to the correction information transmitted. The system provider, in turn, will monitor the old data to ensure that it remains valid until it times out. For the fast corrections and the slow corrections, the model of degradation computes an upper bound on the residual error (after application of fast and slow corrections) in the form of a standard deviation.

The fast correction message types (MT 2-5, MT 6 and MT 7) not only provide the correction values that must be applied to the measured pseudorange but – in combination with the message type MT 10 – also provide a means to calculate this upper bound on the pseudorange error after the fast and slow corrections have been applied.

The contribution of the fast corrections and slow corrections to variance of range measurements is determined mainly by the actual residual variance of each range correction and by taking the degradation of the variance with respect to time into account.

\[
\sigma_{\text{var}}^2 = \left\{ \begin{array}{l}
\sigma_{\text{UDRE}}^2 + \delta \sigma_{\text{UDRE}}^2 + \varepsilon_{\text{fc}}^2 + \varepsilon_{\text{rrc}}^2 + \varepsilon_{\text{ltc}}^2 + \varepsilon_{\text{er}}^2 , & \text{RSS}_{\text{UDRE}} = 0 \\
(\sigma_{\text{UDRE}}^2 + \delta \sigma_{\text{UDRE}}^2)^2 + \varepsilon_{\text{fc}}^2 + \varepsilon_{\text{rrc}}^2 + \varepsilon_{\text{ltc}}^2 + \varepsilon_{\text{er}}^2 , & \text{RSS}_{\text{UDRE}} = 1
\end{array} \right.
\]

with

- $\sigma_{\text{var}}^2$: variance of residual error model for fast and slow corrections
- $\sigma_{\text{UDRE}}^2$: variance of User Differential Range Error UDRE (message types MT 2-5, MT 6)
- $\delta \sigma_{\text{UDRE}}^2$: increment for the variance of the UDRE (message types MT 27)
- $\varepsilon_{\text{fc}}$: degradation parameter for fast correction data (message type MT 7)
- $\varepsilon_{\text{rrc}}$: degradation parameter for range rate correction data (message type MT 10)
- $\varepsilon_{\text{ltc}}$: degradation parameter for long term correction data (message type MT 10)
- $\varepsilon_{\text{er}}$: degradation parameter for flight phases en-route through non-precision approach (message type MT 10)
- $\text{RSS}_{\text{UDRE}}$: root-sum-square flag for User Differential Range Errors (message type MT 10)

It should be noted that airborne equipment which does not miss any messages will have zero values in equation (8) for all terms except the standard variation of the residual error $\sigma_{\text{UDRE}}$ and the degradation of the fast corrections $\varepsilon_{\text{fc}}$, which typically will be less than 0.35 meters.

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the message type MT 27. The implementation this message type still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the currently supported message type MT 27. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

The standard deviation of the residual error $\sigma_{\text{UDRE}}$ is derived using table 15 and the User Differential Range Error Indicator values that are contained in the message types MT 2-5, MT 6 and MT 24. If no message type MT 6 is received (as is the case for the ESTB SIS), then the UDREI values will be taken from the corresponding message types MT 2-5 and MT 24 (for each satellite). If, however, the message type MT 6 is transmitted the MOPS [4] specifies the following approach to the selection of the UDREI values. In the message type MT 6, the Issue Of Data Fast Corrections IODFj is provided for each block of 13 UDREI.
If the IODF\(_j\) for one block of 13 provided UDREI values is set to 3, then the UDREI values must be taken from this message type MT 6 block. The time of applicability for the degradation of the UDRE is taken as that from the MT 2-5 and MT 24.

If the IODF\(_j\) for one block of 13 provided UDREI values is set to a value less than 3:

- if the IODF\(_j\) for the block of UDREI values in message type MT 6 is equal to the IODF of the corresponding message type MT 2-5 of MT 24, then UDREI values from the message type MT 6 should be selected. The time of applicability for the degradation of the UDRE is taken as that from MT 6.
- if the IODF\(_j\) for the block of UDREI values in message type MT 6 is not equal to the IODF of the corresponding message type MT 2-5 or MT 24, then none of the transmitted data (MT 2-5, MT 24 and MT 6) should be applied to the corresponding satellite.

The degradation parameter for the fast correction data can be determined using the parameters transmitted in the message type MT 7 (see chapters 4.4 and 5.1, table 17). They are defined as:

\[
\varepsilon_{fc} = a_i \frac{(t-t_u+t_{lat})^2}{2}
\]

with
- \(a_i\) fast correction degradation parameter (from MT 7)
- \(t\) current time
- \(t_u\) time of reception of the last UDRE data (MT 2-5, 6 or 24)
- \(t_{lat}\) system latency time (from MT 7)

The time of reception of the last UDRE data in equation (9) must be the time the last UDRE data was received with an IODF that matches the one associated with the fast correction being used. For UDRE broadcast in MT 2-5 or MT 24, this time equals the time of applicability of the fast corrections, since they are in the same message. For UDRE broadcast in MT 6 when the IODF equals 3, this time equals as well the time of applicability of the fast corrections. For UDRE broadcast in MT 6 when the IODF is not equal to 3, this time is defined to be the time of transmission of the first bit of the MT 6 at the SBAS satellite.

If the fast correction degradation parameter \(A_i\) is equal to zero, then the range rate correction degradation parameter \(\varepsilon_{rrc}\) is as well equal to zero.

\[
a_i = 0 \implies \varepsilon_{rrc} = 0
\]

Otherwise, the range rate degradation is divided into two situations. The first case covers the situation where the IODF of both the current and the previous fast corrections are not equal to 3. This situation is again divided into two subcategories: in one category where the user does not miss consecutive transmissions of the appropriate fast correction message types MT 2-5 or MT 24 and in the other category where the user misses one or several consecutive transmissions.

\[
\varepsilon_{rrc} = \begin{cases} 
0, & (IODF_{current} - IODF_{last}) \ MOD \ 3 = 1 \\
\frac{a_i L_{fc}}{4} + B_{rrc} \frac{(t_t - t_{of})}{\Delta t}, & (IODF_{current} - IODF_{last}) \ MOD \ 3 \neq 1 
\end{cases}
\]
The second case covers the situation where at least one of the IODF is equal to 3. This situation is as well divided into two subcategories: in one category where the appropriate fast corrections for the determination of the range rate corrections are chosen optimally (see chapter 5.2) and in the other category where non-optimal fast corrections are selected to determine the range rate corrections. In the latter case, there are many possible results for the degradation parameter $\varepsilon_{rrc}$, since there are many possible choices of fast corrections as alarm conditions do not necessarily occur at regular spaced intervals.

$$
\varepsilon_{rrc} = \begin{cases}
0, & \left(\Delta t - \frac{I_{fc}}{2}\right) = 0 \\
\left(\frac{a_i \Delta t - I_{fc}}{2} + \frac{B_{rrc}}{\Delta t}\right) (t - t_{of}), & \left(\Delta t - \frac{I_{fc}}{2}\right) \neq 0
\end{cases}
$$

(12)

with $a_i$ fast correction degradation parameter (from MT 7)
$t$ current time
$t_{of}$ time of applicability of range rate corrections
$\Delta t$ time interval between current time and time of applicability of RRC
$I_{fc}$ time-out interval of fast corrections (from MT 7)
$B_{rrc}$ relative estimation noise and round-off error parameter (from MT 10)
$IODF_{current}$ IODF associated with most recent fast corrections
$IODF_{previous}$ IODF associated with previous fast corrections

In the MOPS [4], an ambiguity can be found with regard to range rate correction degradation parameter. In the first line of section A.4.5.1.2 of the MOPS [4], the following requirement is found: “If the RRC = 0, then the range rate correction degradation is also equal to zero”. However, in the second line of the same paragraph, the following comment can be found: “(i.e. $\varepsilon_{RRC} = 0$, if $a_i = 0$, see section A.4.4.5)”.

There can be situations where the equation (1) is applied (i.e. the condition in equation (2) is not met), and the computed range rate is still equal to zero (i.e. if the pseudorange fast corrections used to compute the range rate corrections are of equal value). Following the first line of the section A.4.5.1.2, the range rate correction degradation parameter must be set to zero. Following the second line of the same section, the range rate correction degradation parameter must be computed according to the equations provided above.

Checking other sources of information (e.g. [12]) provided no further information. Without the availability of further guidance on the solution of this problem, the range rate correction degradations are set in the PEGASUS tools only to zero, if the fast correction degradation parameter $a_i$ is set to zero.

The degradation associated with slow corrections is covered for GNSS satellites by two cases depending on whether both offset and velocity information (MT 24 or MT 25 velocity code 1) or only offset information (MT 24 or MT 25 velocity code 0) is included in the message.

$$
\varepsilon_{ltc,VC0} = C_{ltc,lsb} \frac{t - t_{ltc}}{I_{ltc,VC0}}
$$

(13)

$$
\varepsilon_{ltc,VC1} = C_{ltc,lsb} + C_{ltc,vl1} \max\left(0, t_0 - t, t - t_0 - I_{ltc,vl1}\right)
$$

(14)
with \( C_{ltc_{-lab}} \) maximum round-off error due to resolution of slow corrections (from MT 10)
\( C_{ltc_{-v0}} \) bound on position error between two successive transmissions (from MT 10)
\( C_{ltc_{-vl1}} \) bound on velocity error between two successive transmissions (from MT 10)
\( t \) current time
\( t_{of} \) time of applicability of slow corrections
\( t_{ltc} \) time of transmission of slow corrections
\( I_{ltc_{-v0}} \) update interval for velocity code 0 information (from MT 10)
\( I_{ltc_{-vl1}} \) update interval for velocity code 1 information (from MT 10)
\[ \left\lfloor \frac{\max(\cdots)}{L} \right\rfloor \] function to obtain the floor or greatest integer less than input argument
\[ \max(\cdots) \] function to obtain the maximum value of input arguments

Typically, a slow correction message will be broadcast once before \( t_0 \) and two more times in the interval starting at \( t_0 \) and ending at \((t_0+I_{ltc})\). If no slow corrections are missed, it is always possible to have the current time in between the interval starting at \( t_0 \) and ending at \((t_0+I_{ltc})\). In this case, a zero degradation value is determined. If the airborne equipment misses slow correction messages, the equipment may be forced to either use a slow correction transmitted before \( t_0 \) or use the slow corrections transmitted after \((t_0+I_{ltc})\).

The degradation associated with the slow corrections for the SBAS satellites is provided only using the approach for long term corrections with velocity code 1 information and the same principles apply as to the periods of applicability.

\[
\varepsilon_{ltc_{SBAS}} = \begin{cases} 
0 & t_0 < t < t_0 + I_{geo} \\
C_{geo_{-lab}} + C_{geo_{-v1}} \max\left( 0, t_0 - t, t - t_0 - I_{geo} \right) & \text{otherwise}
\end{cases}
\]  \( (15) \)

with \( C_{geo_{-lab}} \) maximum round-off error due to resolution of slow corrections (from MT 10)
\( C_{geo_{-v1}} \) bound on velocity error between two successive transmissions (from MT 10)
\( I_{geo} \) update interval for velocity code 1 information (from MT 10)
\( t \) current time
\( t_0 \) time of applicability of SBAS ephemeris navigation message

This degradation function is similar to that of the slow correction degradation for GNSS satellites in equation (14) and the same principles apply as to the periods of applicability.

When using fast or slow corrections which have timed out for precision approach modes, but have not timed out for other navigation modes, an extra "catch all" degradation factor is applied. This degradation is:

\[
\varepsilon_{cr} = \begin{cases} 
0 & \text{condition (a)} \\
C_{cr} & \text{condition (b)}
\end{cases}
\]  \( (16) \)
The condition (a) describes a situation where neither fast corrections nor slow corrections have timed out for precision approach, whereas the condition (b) describe the situation where at least either the fast corrections or the slow corrections have timed out for the precision approach modes.

Using this degraded estimation of the variance on the residual pseudorange error after the application of the fast and slow corrections, the protection levels can be calculated. This is shown in chapter 10.

### 5.4 Application of Corrections

In order to apply the pseudorange corrections according to equation (7) and the variance on the residual pseudorange error after the application of the fast and slow corrections according to equation (8), the following conditions must be met:

- the time-out periods for the fast corrections must be determinable, i.e. a message type MT 7 must have been received and must be valid (determined through their corresponding time-out interval).
- the corresponding UDREI indicator values must have been received and must be valid (determined through their corresponding time-out interval).
- the corresponding UDREI indicator values are not set to 14 ("not monitored!") or 15 ("do not use!").
- for precision approach modes, the UDREI indicator value must be smaller than 12 for a particular satellite to be used in the position solution.
- for precision approach modes, the message type MT 10 must be received and valid (determined through their corresponding time-out interval).
- the pseudorange correction values must be valid (determined through their corresponding time-out interval).
- the pseudorange rate correction values must be valid as determined through their corresponding time-out intervals. If there are initialisation periods, the satellite can be used for positioning (although with a very high variance $\sigma_{i,flt}^2$).

As a reminder, the time-out interval of the fast corrections PRC provided in the message types MT 2-5 or MT 24 is variable and depends on the values of the degradation factors transmitted in the message type MT 7. The time-out intervals are given in table 11. The time-out interval of the indicator values UDREI (message types MT 2-5, MT 6) and AI (message type MT 7) are only dependent on the flight phase and are given in the table 18. The time-out interval of the general degradation parameters MT 10 are only dependent on the flight phase and are given in table 13.
6 Slow Corrections

6.1 Message Types and Format

The purpose of the slow or long term corrections is to correct for the slowly varying errors such as the satellite position errors caused by errors in the ephemeris transmissions and the degradation of satellite position calculations with the time. Another error source which is corrected by the slow corrections is the update of the satellite clock error, although there is already a correction of the satellite clock error contained in the GPS navigation message.

The slow corrections are provided in the message type MT 25 and in the second half of the mixed correction message type MT 24. In the current ESTB SIS version 1.2, there is no message type MT 24 broadcast, only the message type 25 is discussed. However, the application of the slow corrections in message type MT 24 is identical.

The correction information for one satellite in the message type MT 25 can consist of only position correction and satellite clock bias correction, which is indicated by the velocity code 0. If the velocity code is set to 0, then correction data for two satellites can be transmitted using 106 bits. The format of the message type MT 25 is shown in table 19 for velocity code 0 for these 106 bits (i.e. one half of the total message type MT 25).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity Code = 0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0..1</td>
<td>1</td>
</tr>
<tr>
<td>PRN Mask No</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0..51</td>
<td>1</td>
</tr>
<tr>
<td>issue of data (IOD)</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>0.255</td>
<td>1</td>
</tr>
<tr>
<td>DX (ECEF)</td>
<td>15</td>
<td>9</td>
<td>0.125</td>
<td>±32</td>
<td>metres</td>
</tr>
<tr>
<td>DY (ECEF)</td>
<td>24</td>
<td>9</td>
<td>0.125</td>
<td>±32</td>
<td>metres</td>
</tr>
<tr>
<td>DZ (ECEF)</td>
<td>33</td>
<td>9</td>
<td>0.125</td>
<td>±32</td>
<td>metres</td>
</tr>
<tr>
<td>clock bias DAF0</td>
<td>42</td>
<td>10</td>
<td>$2^{-32}$ ($\approx 2.33 \times 10^{-10}$)</td>
<td>±$2^{-22}$</td>
<td>seconds</td>
</tr>
<tr>
<td>PRN Mask No</td>
<td>52</td>
<td>6</td>
<td>1</td>
<td>0..51</td>
<td>1</td>
</tr>
<tr>
<td>issue of data (IOD)</td>
<td>58</td>
<td>8</td>
<td>1</td>
<td>0.255</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>IODP</td>
<td>104</td>
<td>2</td>
<td>1</td>
<td>0..3</td>
<td>1</td>
</tr>
<tr>
<td>spare</td>
<td>106</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 19: Message Type MT 25 Velocity Code 0

If the message type includes the corresponding drifts (i.e. satellite velocity corrections and satellite clock drift corrections), this will be indicated by the velocity code 1. Then only correction data for one satellite can be transmitted using 106 bits. The format of the message type MT 25 is shown in table 20 for velocity code 1 for these 106 bits (i.e. one half of the total message type MT 25).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity Code = 1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>PRN Mask No</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0.51</td>
<td>1</td>
</tr>
<tr>
<td>issue of data (IOD)</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>0.255</td>
<td>1</td>
</tr>
<tr>
<td>DX (ECEF)</td>
<td>15</td>
<td>11</td>
<td>0.125</td>
<td>±128</td>
<td>metres</td>
</tr>
<tr>
<td>DY (ECEF)</td>
<td>26</td>
<td>11</td>
<td>0.125</td>
<td>±128</td>
<td>metres</td>
</tr>
<tr>
<td>DZ (ECEF)</td>
<td>37</td>
<td>11</td>
<td>0.125</td>
<td>±128</td>
<td>metres</td>
</tr>
<tr>
<td>clock bias DAF0</td>
<td>48</td>
<td>11</td>
<td>$2^{-32}$ ($= 2.33 \times 10^{-10}$)</td>
<td>±$2^{-21}$</td>
<td>seconds</td>
</tr>
<tr>
<td>DX Rate of Change</td>
<td>59</td>
<td>8</td>
<td>$2^{-11}$ ($= 4.88 \times 10^{-4}$)</td>
<td>±0.0625</td>
<td>metres/second</td>
</tr>
<tr>
<td>DY Rate of Change</td>
<td>67</td>
<td>8</td>
<td>$2^{-11}$ ($= 4.88 \times 10^{-4}$)</td>
<td>±0.0625</td>
<td>metres/second</td>
</tr>
<tr>
<td>DZ Rate of Change</td>
<td>75</td>
<td>8</td>
<td>$2^{-11}$ ($= 4.88 \times 10^{-4}$)</td>
<td>±0.0625</td>
<td>metres/second</td>
</tr>
<tr>
<td>clock drift DAF1</td>
<td>83</td>
<td>8</td>
<td>$2^{-39}$ ($= 1.82 \times 10^{-12}$)</td>
<td>±$2^{-32}$</td>
<td>second/second</td>
</tr>
<tr>
<td>time of applicability</td>
<td>91</td>
<td>13</td>
<td>16</td>
<td>0 … 86384</td>
<td>seconds</td>
</tr>
<tr>
<td>IODP</td>
<td>104</td>
<td>2</td>
<td>1</td>
<td>0.3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 20: Message Type MT 25 Velocity Code 1**

Thus, one message type MT 25 can contain correction data for a minimum of 1 and a maximum of 4 satellites, depending on the velocity codes in both halves of the message and how many satellites are being corrected. The error estimates are accompanied by the IODP associated with the PRN mask in message type MT 1. The particular satellites are identified using the PRN mask number – i.e. a value between 1 … 51 (see chapter 4 for more information on the application of the PRN mask). As opposed to the data in message types MT 2-5 and MT 24, the data in the message type MT 25 does not have to appear in sequence. Error corrections for satellites with faster changing long term errors can be repeated at a higher rate than one with slower changing long term errors. Nevertheless, the IODP of the message type MT 25 must agree with the IODP associated with the message type MT 1.

It should be noted that the ranges of the clock offset and position component error estimates when the velocity code is equal to 0 (see table 19) are less than the ones if the velocity code is equal to 1 (see table 20). The reason for this is data rate efficiency. Usually, the necessity for clock drift and velocity component error estimates is small. Only the clock offset and position component error estimates will be broadcast, unless any of the errors (i.e. position, velocity, clock offset or clock drift) will be large enough to warrant their use on a satellite-by-satellite basis.

For synchronisation purposes with the GPS ephemeris, the message type MT 25 also provides the Issue Of Data IOD. This IOD has to correspond with the IODE of the current GPS ephemeris for that satellite and with the (least 8 significant bits of) the IODC of the GPS ephemeris. If the IODE and IODC of the GPS broadcast ephemeris data do not match the IOD of the SBAS transmission, it is an indication that the GPS ephemeris data sets have changed. The equipment must continue to use the old matched data from the previous broadcasts (i.e. the old GPS ephemeris data sets as well !), until a new message type MT 25 with matching IOD is broadcast for the new GPS ephemeris data set.

Upon the transmission of new clock and ephemeris data from GPS satellites, the SBAS will continue to broadcast corrections to the old long term clock and ephemeris data for a period of 2 minutes. This delay will enable the user equipment to acquire the new GPS ephemeris data. The new matching messages will be broadcast by the service provider within the time constraints for user initialisation.
In contrast to the fast corrections (see chapter 5), where the time of applicability for the corrections is taken to be the reception time of the signal at the receiver, the time of applicability for the slow corrections is—at least for the velocity code 1 transmissions—contained in the message type itself (see table 20). For transmissions with the velocity code 0, however, the time of applicability cannot be found in the content of the transmission (see table 19).

The choice of the time of applicability for the message type MT 25 velocity code 0 data is therefore not obvious. In the PEGASUS implementation, the reception time of the MT 25 velocity code 0 transmission is taken as the time of applicability.

Using the actual user time and the time of applicability for the slow correction, the validity of the slow corrections can be determined. The time-out interval for the slow corrections is shown in the following table 21.

<table>
<thead>
<tr>
<th>MT</th>
<th>time-out interval (Non-Precision Approach)</th>
<th>time-out interval (Precision Approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
</tbody>
</table>

Table 21: Message Type MT 25 Time-Out Interval

There are no such corrections transmitted by an SBAS service provider for SBAS satellites operated by that service provider itself. Instead, the message type 9 will be updated as required to prevent slow varying satellite errors. However, for SBAS satellites not operated by a particular service provider, there are slow corrections transmitted (if the satellite is supported).

In situations where slow corrections are transmitted by a SBAS service provider for geostationary satellites not operated by the service provider itself, there must be a mechanism to link the transmitted slow corrections to a particular set of ephemeris for that satellite. At the current moment it is unclear how the user receiver should be prevented from applying ephemeris and slow correction data which are not matching. A possible solution would be either to use the parameter IODN (contained in the message type 9). This is however not foreseen, since the parameter IODN is not contained in further evolutions of the SBAS SIS requirements (e.g. [12]).

For the current implementation of the PEGASUS tools, this problem is not applicable, since data from only one SBAS satellite can be processed. However, further evolutions of the PEGASUS tools will certainly have to work with data of more than one SBAS satellite.

According to the MOPS [4], the slow corrections have to be applied for position calculations. They are applied (a) to the calculated satellite position and velocity in WGS84 co-ordinates and (b) to the pseudorange measurements (the additional clock error estimate). The uncertainty of the slow corrections is represented in the UDRE included in the message types MT 2-5 and MT 6.

### 6.2 Satellite Position and Clock Corrections

The equipment must compare the Issue Of Data IOD in the message type 25 for each GPS satellite with the IODE of that GPS satellite being utilised by the equipment. There are three possible outcomes:

- The SBAS IOD and the GPS IODE match (i.e. the usual condition), in which case the SBAS corrections must be applied using the current GPS IODE to compute the satellite position.
- The SBAS IOD and the GPS IODE do not match, but the SBAS IOD matches the previous GPS IODE (a condition which might happen for a few minutes each hour), in which case the SBAS corrections shall be applied using the previous GPS IODE to compute the satellite position.
The SBAS IOD and the GPS IODE do not match, nor does the SBAS IOD match the previous GPS IODE (a rare condition). Then the equipment must not apply the fast or slow corrections.

The SBAS will generate each satellite correction based on the old ephemeris for approximately two minutes after the GPS broadcast ephemeris has changed in order to ensure that the user equipment has decoded the new ephemeris. During this period, the equipment needs to retain the old ephemeris for at least 5 minutes or until a match between the SBAS IOD and the GPS IODE is obtained in order to ensure that the pseudorange measurements and the slow corrections for that satellite are based on the same ephemeris data set.

The requirement to retain old ephemeris information for at least 5 minutes requires that at least 3 data sets of broadcast ephemeris may be stored, as GPS satellite data can be updated twice within a 5 minute period. In the PEGASUS implementation, all GPS ephemeris data sets are stored internally until they are timed-out based on their appropriate application interval (4 hours or 6 hours).

The equipment must use the active slow correction with the latest time of applicability which is less (i.e. earlier) than the current time whenever possible. If slow corrections are received with a time of applicability in the future, those corrections should not be used until (1) the current time equals the time of applicability or (2) the previous slow corrections time out.

The application of the corrections is straightforward, once the synchronisation is ensured. At one point of the user position calculation, the satellite positions for a user time \( t \) have been calculated. They will be available in the ECEF co-ordinate system as \( X, Y \) and \( Z \) as a function of time.

To compensate for some of the inherent errors, the satellite position corrections are calculated using a linear interpolation scheme and added to the satellite positions.

\[
DX(t) = DX + DX_{ROC} (t - t_0)
\]
\[
DY(t) = DY + DY_{ROC} (t - t_0)
\]
\[
DZ(t) = DZ + DZ_{ROC} (t - t_0)
\]

with

\[
DX(t), \ DY(t), \ DZ(t) \quad \text{actual corrections for satellite position}
\]
\[
DX, \ DY, \ DZ \quad \text{transmitted position corrections}
\]
\[
DX_{ROC}, \ DY_{ROC}, \ DZ_{ROC} \quad \text{transmitted velocity corrections}
\]
\[
t \quad \text{user time}
\]
\[
t_0 \quad \text{time of applicability for velocity corrections}
\]

If the message type MT 25 does not contain velocity corrections (i.e. a transmission with velocity code 0), then the velocity corrections are zeroed and the correction scheme shown above can still be applied.

It should be noted that the time of applicability \( t_0 \) in the message type MT 25 will be typically approximately 2 minutes in the future of the transmission time of the message, minimizing resolution errors for at least 4 minutes. Additionally, the time of applicability in the message type MT 25 represents “seconds into the day”, whereas the user time is – in most cases – provided as “seconds into the week”. Hence, one of them has to be corrected before the application of the corrections. Additionally, it should be noted that, since the time of applicability of the slow corrections might be in the future with respect to the current user time, the time interval for the linear interpolation scheme might be negative.
In a similar and straightforward way, the satellite clock corrections have to be applied. According to the MOPS [4], the user equipment will compute the clock time error estimate as:

\[
\delta \Delta t_{SV}(t) = \delta a_{f_0} + \delta a_{f_1} \left( t - t_0 \right)
\]

(20)

with \( \delta a_{f_0}, \delta a_{f_1} \) clock correction parameters supplied by message type MT 25, \( \delta \Delta t_{SV} \) resulting clock correction, \( t_0 \) time of applicability of clock correction parameters (corrected for clock rollovers), \( t \) user time.

If the message type MT 25 does not contain velocity corrections (i.e. a transmission with velocity code 0), then the clock drift correction is zeroed and the correction scheme shown above can still be applied.

This calculated clock correction term is then, according to the MOPS [4], added to the satellite clock correction calculated previously using the standard GPS ephemeris data (see section 2.5.5.2 of [14]). The following scheme is used to correct the measurements for the effects of satellite clock errors.

The standard satellite clock error model for the GPS satellite (which is contained in the GPS ephemeris data) and the additional SBAS clock error model for the GPS satellite are computed separately. Thus, the resulting two range corrections are added separately as the final pseudorange correction.

\[
RC_{clock} = \delta \Delta t_{SV}(t) c
\]

(21)

with \( RC_{clock} \) pseudorange correction resulting from SBAS clock correction, \( c \) speed of light.

### 6.3 Correction Error Estimation

As stated already in chapter 5 (application of the fast corrections), the fast corrections and the slow corrections are designed to provide the most recent information to the user. Due to the limitation of the bandwidth and the possibility of missing some transmissions of the SBAS SIS (bit errors, masking, etc.), the user must apply models of degradation to the correction information transmitted. The appropriate algorithms are provided in the chapter 5.3, where the effects of the application of the fast corrections and slow corrections are combined to provide an upper bound on the residual error in the form of a standard deviation.

### 6.4 Application of Correction

The slow corrections are tightly coupled with the fast corrections discussed in chapter 5. For all flight phases, the slow corrections must be applied. If they can not be used for any reason (see below), no fast correction and slow correction can be applied to the satellite and this particular satellite must be de-selected for the position computation. Upon the reception of the slow correction, the following general checks must be made before these slow corrections can be used in the correction scheme.

- The time-out interval for the slow correction is dependent on the flight phase and is provided in table 21.
- The IODP transmitted in the message type MT 25 must match with the IODP transmitted in the message type MT 1.
The equipment must compare the Issue Of Data IOD in the message type 25 for each GPS satellite with the IODE of that GPS satellite being utilised by the equipment. There are three possible outcomes:

- The SBAS IOD and the GPS IODE match (i.e. the usual condition). In which case the SBAS corrections must be applied using the current GPS IODE to compute the satellite position.

- The SBAS IOD and the GPS IODE do not match, but the SBAS IOD matches the previous GPS IODE (a condition which might happen for a few minutes each hour), in which case the SBAS corrections shall be applied using the previous GPS IODE to compute the satellite position.

- The SBAS IOD and the GPS IODE do not match, nor does the SBAS IOD match the previous GPS IODE (a rare condition). Then the equipment must not apply the fast or slow corrections and the satellite can not be used for positioning.
7  Ionospheric Corrections

7.1  General Approach to the Ionospheric Corrections

In contrast to the calculation of the fast and slow corrections (which are very straightforward), the determination of the ionospheric error corrections is more complex. The slant delay and the slant residual ionospheric error caused by the ionosphere are calculated using several steps.

The SBAS ionospheric correction messages are related to an interpolation grid above the earth’s surface at an altitude of the ionosphere. The grid points for the interpolation are defined through the use of the message type MT 18 and the interpolation values at each grid point are provided through the message type MT 26.

For each satellite, the pierce point of the direct line-of-sight from the satellite to the user’s position through the ionosphere is determined. This pierce point is then used to determine the appropriate grid points for the interpolation.

The interpolation uses a weighting scheme for all four neighbouring grid points to determine an estimation of the vertical delay of the pseudorange at the ionospheric pierce point. This vertical delay is finally modified to take the elevation of a particular satellite into account. Thus, a slant delay for the ionospheric error model and an estimation about its residual error are calculated.

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the ionospheric corrections using the message types MT 18 and MT 26. The implementation these corrections still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the currently supported ionospheric correction scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

7.2  The Ionospheric Grid

The SBAS ionospheric correction messages utilise an interpolation grid above the earth’s surface at an altitude of 350 km in the ionosphere. The ionospheric delay corrections are broadcast as vertical delay estimates at specified Ionospheric Grid Points IGP. In order to facilitate flexibility in the location of these IGP, a fixed definition of densely spaced IGP locations is used, resulting in a large number of possible IGP.

The density of these predefined IGP is dictated by the possible large variation in the ionospheric vertical delay during periods of high solar activity, especially at lower latitudes. The IGP used at any time are based upon the current variations of the ionosphere between IGP locations. Since it would be impossible to broadcast IGP delays for all possible locations, a mask is broadcast to define the IGP locations providing the most efficient model of the ionosphere at the time.

There are 1808 predefined possible IGP locations, given in latitude and longitude co-ordinates, as shown in figure 5. These IGP locations must be stored permanently by the user. The IGP locations are denser at lower latitudes due to the fact that the distance represented by a degree of longitude becomes smaller at higher latitudes. The IGP grid at the equator has a spacing of 5° x 5°, increasing to a spacing of 10° x 10° north of latitude N55° and south of latitude S55°. Finally, the IGP are spaced by 90 degrees at N85° and S85°, with the IGP at southern latitudes offset by 40 degrees longitude to accommodate an even distribution of bands as described below.

The grid points for the interpolation are defined through the use of message type MT 18. However, the total IGP grid represents too many IGP for broadcasting in a single message. Thus, the grid is divided into nine bands (band 0 to band 8) – in each band there are 201 possible IGP locations contained. Each band covers 40 degrees of longitude (see figure 5).
A given SBAS satellite will only support IGP in bands that cover the observable IGP visible from the intersection of its footprint and the controlling system’s service volume. At the edge of the SBAS satellite footprint, the ionospheric pierce points IPP could be located beyond IGPs of the band being broadcast. However, because of the overlap of SBAS footprints, those IPPs would be covered by an adjacent SBAS satellite broadcasting an adjacent band. The adjacent SBAS satellite itself would be at a higher elevation angle. It should be noted that the user only has to collect and save the vertical delays for IGPs located within about ± 20 degrees of his location, which would all be located in one or two bands.

### 7.3 Message Types and Format

Message type MT 18 provides a mask for any of these nine bands, while message type MT 26 provides the interpolation values for the supported grid points. The following table 22 contains the format for message type MT 18.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bands</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0 .. 9</td>
<td>-</td>
</tr>
<tr>
<td>Band Number</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0 .. 8</td>
<td>-</td>
</tr>
<tr>
<td>IODI</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>0 .. 3</td>
<td>-</td>
</tr>
<tr>
<td>IGP Mask</td>
<td>10</td>
<td>201</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Spare</td>
<td>211</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 22: Message Type MT 18*
The first parameter of message type 18, the 4-bit Number-Of-Bands, indicates how many band masks will be broadcast by the particular SBAS satellite, so that a user knows whether all available data has been received or whether to wait for another band mask. Additionally, the actual band number of the broadcast is provided. If the number of bands is 0, then the message is used to indicate that no ionospheric delay corrections are being provided — thus indicating, as well, that the precision approach service is not being provided by the broadcasting SBAS satellite.

Within each band, the IGP are numbered from 1 to 201, counting from the southwest corner up each longitude column of the band from south to north and continuing for each column from west to east. The application of the IGP mask is identical to the decoding of the PRN mask (see chapter 4). Thus, if the bit position X in the transmitted IGP mask is set, then the corresponding IGP number X is supported using the current SBAS SIS. Table 23 contains the IGP numbers for the band number 3, 4 and 5 (i.e. the bands covering the ECAC area).

<table>
<thead>
<tr>
<th>Longitude</th>
<th>Latitude</th>
<th>IGP Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>1-27</td>
</tr>
<tr>
<td>55 W</td>
<td>55S, ..., 55N</td>
<td>28-50</td>
</tr>
<tr>
<td>50 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>51-78</td>
</tr>
<tr>
<td>45 W</td>
<td>55S, ..., 55N</td>
<td>79-101</td>
</tr>
<tr>
<td>40 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N, 85N</td>
<td>102-128</td>
</tr>
<tr>
<td>35 W</td>
<td>55S, ..., 55N</td>
<td>129-151</td>
</tr>
<tr>
<td>30 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>152-178</td>
</tr>
<tr>
<td>25 W</td>
<td>55S, ..., 55N</td>
<td>179-201</td>
</tr>
<tr>
<td>Band 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>1-27</td>
</tr>
<tr>
<td>15 W</td>
<td>55S, ..., 55N</td>
<td>28-50</td>
</tr>
<tr>
<td>10 W</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>51-77</td>
</tr>
<tr>
<td>5 W</td>
<td>55S, ..., 55N</td>
<td>78-100</td>
</tr>
<tr>
<td>0</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N, 85N</td>
<td>101-128</td>
</tr>
<tr>
<td>5 E</td>
<td>55S, ..., 55N</td>
<td>129-151</td>
</tr>
<tr>
<td>10 E</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>152-178</td>
</tr>
<tr>
<td>15 E</td>
<td>55S, ..., 55N</td>
<td>179-201</td>
</tr>
<tr>
<td>Band 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 E</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>1-27</td>
</tr>
<tr>
<td>25 E</td>
<td>55S, ..., 55N</td>
<td>28-50</td>
</tr>
<tr>
<td>30 E</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>51-77</td>
</tr>
<tr>
<td>35 E</td>
<td>55S, ..., 55N</td>
<td>78-100</td>
</tr>
<tr>
<td>40 E</td>
<td>85S, 75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>101-128</td>
</tr>
<tr>
<td>45 E</td>
<td>55S, ..., 55N</td>
<td>129-151</td>
</tr>
<tr>
<td>50 E</td>
<td>75S, 6S, 55S, ..., 55N, 65N, 75N</td>
<td>152-178</td>
</tr>
<tr>
<td>55 E</td>
<td>55S, ..., 55N</td>
<td>179-201</td>
</tr>
</tbody>
</table>

Table 23: IGP numbering for Band 4 and Band 5

Each message also contains an Ionospheric mask Issue Of Data IODI to ensure that the ionospheric corrections are properly decoded. The IODI will sequence through the range from 0 to 3, changing each time
the IGP mask changes (which is expected to happen very rarely). The user must assure that the IODI of all received bands agree and that they agree with the IODI in the message type MT 26 before applying the vertical delays to the model.

Message type 26 provides the user with vertical delays and an upper bound on the residual error after application of the corrections (in the form of a standard deviation) at the geographically defined IGP identified by the band number and IGP number. The format of message type MT 26 is shown in the following table 24.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band Number</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0 .. 9</td>
<td>-</td>
</tr>
<tr>
<td>Block ID</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0 .. 13</td>
<td>-</td>
</tr>
<tr>
<td>Vertical Delay (15 x)</td>
<td>8 ... 9</td>
<td>9</td>
<td>0.125</td>
<td>0 .. 63.875</td>
<td>metres</td>
</tr>
<tr>
<td>GIVEI (15 x)</td>
<td>17 ... 19</td>
<td>4</td>
<td>1</td>
<td>0 .. 15</td>
<td>-</td>
</tr>
<tr>
<td>IODI</td>
<td>203</td>
<td>2</td>
<td>1</td>
<td>0 .. 3</td>
<td>-</td>
</tr>
<tr>
<td>Spare</td>
<td>205</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 24: Message Type MT 26

Each message contains a band number and a 4-bit block ID, which cycles through the range of 0 to 13. Using one message type MT 26, only 15 vertical delays and error bounds can be transmitted (i.e. not the whole band of 201 values). Each band is therefore divided into a maximum of 14 blocks. Thus, the block with the ID 0 contains the IGP corrections for the first 15 IGP locations in the band mask, the block with the ID 1 contains the IGP corrections for the second 15 IGP locations (i.e. IGP 16-30) designated in the band mask, etc. Corrections associated with slot numbers that exceed the number of IGPs indicated in the IGP band mask should be ignored.

It should be noted that the block with the ID 0 denotes the first 15 IGP vertical delays that are actually supported by the corresponding IGP mask in MT 18, and so on. Thus, not the IGP numbering, but the sequence of supported IGP is indicated by that Block ID. The assignment of the IGP location from the IGP Mask Number in combination with the MT 26 Block ID and the corresponding vertical delay is shown in the following table 25.

<table>
<thead>
<tr>
<th>IGP in Mask</th>
<th>1</th>
<th>...</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>...</th>
<th>49</th>
<th>50</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mask itself</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
<td>1</td>
<td>...</td>
</tr>
<tr>
<td>MT 26 Block ID</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
<td>1</td>
<td>...</td>
</tr>
<tr>
<td>Place in Block</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>...</td>
<td>15</td>
<td>1</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 25: IGP Locations and MT 26 Block ID

The Issue of Data Ionosphere IODI is used for synchronisation purposes with message type MT 18. The user must assure that the IODI of all received bands of MT 18 agree and that they agree with the IODI in the message type MT 26 before applying the vertical delays to the model.

Not the complete resolution of the vertical delay can be used for the determination of the error corrections. A vertical delay of 63.750 meters is used to indicate an IGP not monitored event, whereas a vertical delay of 63.875 meters will indicate an do not use IGP event.
The Grid Ionospheric Vertical Error Indicator GIVEI is used to transmit an upper bound on the residual error after the application of the ionospheric corrections in the form of a standard deviation. Identical to the procedure with the fast corrections, not the actual error bound is broadcast, but an indicator value for this error bound. The table 26 contains the relationship between the Grid Ionospheric Vertical Error Indicator GIVEI and the actual Grid Ionospheric Vertical Error $\sigma^2_{\text{GIVE}}$.

<table>
<thead>
<tr>
<th>GIVEI</th>
<th>$\sigma^2_{\text{GIVE}}$</th>
<th>GIVEI</th>
<th>$\sigma^2_{\text{GIVE}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0084 m$^2$</td>
<td>8</td>
<td>0.6735 m$^2$</td>
</tr>
<tr>
<td>1</td>
<td>0.0333 m$^2$</td>
<td>9</td>
<td>0.8315 m$^2$</td>
</tr>
<tr>
<td>2</td>
<td>0.0749 m$^2$</td>
<td>10</td>
<td>1.1974 m$^2$</td>
</tr>
<tr>
<td>3</td>
<td>0.1331 m$^2$</td>
<td>11</td>
<td>1.8709 m$^2$</td>
</tr>
<tr>
<td>4</td>
<td>0.2079 m$^2$</td>
<td>12</td>
<td>3.3260 m$^2$</td>
</tr>
<tr>
<td>5</td>
<td>0.2994 m$^2$</td>
<td>13</td>
<td>20.7870 m$^2$</td>
</tr>
<tr>
<td>6</td>
<td>0.4075 m$^2$</td>
<td>14</td>
<td>187.0826 m$^2$</td>
</tr>
<tr>
<td>7</td>
<td>0.5322 m$^2$</td>
<td>15</td>
<td>Not monitored</td>
</tr>
</tbody>
</table>

Table 26: Conversion from GIVEI to $\sigma^2_{\text{GIVE}}$

The Grid Ionospheric Vertical Error Indicator GIVEI is used to transmit an upper bound on the residual error after the application of the ionospheric corrections in the form of a standard deviation. Identical to the procedure with the fast corrections, not the actual error bound is broadcast, but an indicator value for this error bound. The table 26 contains the relationship between the Grid Ionospheric Vertical Error Indicator GIVEI and the actual Grid Ionospheric Vertical Error $\sigma^2_{\text{GIVE}}$.

![Figure 6: IGP Grid over ECAC Area (actual supported grid may differ)](image-url)

The ESTB SIS version 1.2 supports only IGP locations of the ECAC area. The figure 6 illustrates the location of the appropriate IGP and their corresponding numbering in each band and block at the time of the generation of this image. It should be mentioned that the form of the supported IGP grid and the numbering of the message type MT 26 Block ID and delay number inside that block is depending on the actual supported
IGP grid, whereas the IGP numbering is fixed (see table 23). At the time of the generation of this image, extended trials in the Mediterranean Area had been scheduled. Thus, the ionospheric grid was extended for that area.

The time-out interval of message type MT 18 and message type MT 26 are not depending on the flight phase and given in the following table 27.

<table>
<thead>
<tr>
<th>message type</th>
<th>time-out</th>
<th>message type</th>
<th>time-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 18</td>
<td>1200</td>
<td>MT 26</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 27: Time-Out Intervals for MT 18 and MT 26

7.4 The Pierce Point Computation

Considering the satellite and user locations, the user must first determine the location of the ionospheric pierce point of the signal path from the satellite. First, the latitude of the pierce point is determined to be:

$$
\Phi_{pp} = \sin^{-1}\left( \sin(\Phi_u) \cos(\Psi_{pp}) + \cos(\Phi_u) \sin(\Psi_{pp}) \cos(A) \right)
$$

(22)

with

- $\Phi_{pp}$: pierce point latitude (in radian)
- $\Phi_u$: user latitude
- $A$: azimuth of satellite
- $\Psi_{pp}$: central earth angle (in radian)

where, as shown as well in figure 7, the angle $\Psi_{pp}$ is the earth’s central angle between the user position and the earth projection of the pierce point computed as:

$$
\Psi_{pp} = \frac{\pi}{2} - E_i - \sin^{-1}\left( \frac{R_e}{R_e + h_i} \cos(E_i) \right)
$$

(23)

with

- $E_i$: elevation of satellite
- $R_e$: earth radius (assumed to be 6378 km)
- $h_i$: height of maximum electron density (assumed to be 350 km)

The longitude of the pierce point is then:

$$
\lambda_{pp} = \lambda_u + \sin^{-1}\left( \frac{\sin(\Psi_{pp}) \sin(A)}{\cos(\Phi_{pp})} \right)
$$

(24)

with

- $\lambda_{pp}$: pierce point longitude (in radian)
- $\lambda_u$: user longitude
7.5 The Selection of Interpolation Grid Points

Although the data base broadcast to the user is in the form of vertical IGP delays, these points do not generally correspond to the computed IPP location (see section before). Thus, it is necessary for the equipment to interpolate from the broadcast IGP delays to that at the computed IPP location. It should be noted that the user only has to collect and save the vertical delays for IGPs located within about ±20° of his location, which would all be located in one or two bands.

After determining the location of the user ionospheric pierce point, the user must select the IGP to be used to interpolate the ionospheric correction value and its corresponding error bound. This selection is done based only on the information provided in the mask, and must be done without regard to whether or not the selected IGP are monitored, not monitored or a “do not use” event is issued.

- if the latitude of an ionospheric pierce point is between N55° and S55°:
  - if four IGP that define a 5° x 5° rectangular cell around the IPP are set in the IGP mask, they are selected, else
  - if three IGP that define a 5° x 5° triangular cell around the IPP are set in the IGP mask, they are selected, else
  - if four IGP that define a 10° x 10° rectangular cell around the IPP are set in the IGP mask, they are selected. There are four potential 10° x 10° cells that must be checked, any one of which may be used. There is no hierarchy for selecting among multiple cells if they are defined. Else
  - if three IGP that define a 10° x 10° triangular cell around the IPP are set in the IGP mask, they are selected, else
  - an ionospheric correction is not available
if the latitude of an ionospheric pierce point is between N55° and N75° or between S55° and S75°

if four IGP that define a 10° x 10° rectangular cell around the IPP are set in the IGP mask, they are selected, else

if three IGP that define a 10° x 10° triangular cell around the IPP are set in the IGP mask, they are selected, else

an ionospheric correction is not available

if the latitude of an ionospheric pierce point is between N75° and N85° or between S75° and S85°

if the two nearest IGP at 75° and the two nearest IGP at 85° are set in the IGP mask, a 10° x 10° rectangular cell is created by linearly interpolating between the IGP at 85° to obtain virtual IGP at longitudes equal to the longitudes of the IGP at 75°, else

an ionospheric correction is not available

if the latitude of an ionospheric pierce point is greater than N85°

if the four IGP at N85° and longitudes of W180°, W90°, 0° and E90° are set in the IGP mask, they are selected, else

if any three of the four points at N85° that describe a semicircle that circumscribes the IPP are set in the IGP mask, they are selected, else

an ionospheric correction is not available

if the latitude of an ionospheric pierce point is greater than S85°

if the four IGP at S85° and longitudes of W140°, W50°, E40° and E130° are set in the IGP mask, they are selected, else

if any three of the four points at S85° that describe a semicircle that circumscribes the IPP are set in the IGP mask, they are selected, else

an ionospheric correction is not available

It should be noted that if four IGPs were selected and one of the four is identified as “not monitored”, then the triangular interpolation should be used if the user’s pierce points is within the triangular region covered by the three corrections that are provided. If one of the four selected IGPs is identified as “don’t use”, the entire square must not be used.
7.6 The Interpolation Scheme

Once the nodes of an interpolation cell of the IGP grid that surround the IPP to a satellite have been determined (see chapter before), the equipment must interpolate from those nodes to the pierce point using the following algorithm. This situation is illustrated in figures 13 for the square cell and figure 15 for the triangular cell.

For the four-point interpolation, the mathematical formulation for the interpolated vertical IPP delay as a function of the IPP latitude and longitude is given by:

$$
\tau_{vipp}(\Phi_{pp}, \lambda_{pp}) = \sum_{i=1}^{4} W_i(x_{pp}, y_{pp}) \tau_{vi}
$$

with

- $\tau_{vipp}$: vertical ionospheric delay at pierce point
- $\tau_{vi}$: vertical ionospheric delay at grid points
- $W_i$: weighting function

A weighting function approach for the modelling of irregular surfaces provides a simple procedure for approximating an irregular surface from regularly spaced data. The general equation for the weighting function is given by:

$$
W_i(x, y) = x \cdot y
$$

$$
W_2(x, y) = (1 - x) \cdot y
$$

$$
W_3(x, y) = (1 - x) \cdot (1 - y)
$$

$$
W_4(x, y) = x \cdot (1 - y)
$$
with \( x_{pp} \) relative longitude in the interpolation grid
\( y_{pp} \) relative latitude in the interpolation grid
\( W_i(x, y) \) weighting function

and displayed in figure 14. The relative longitude \( x_{pp} \) and relative latitude \( y_{pp} \) in the interpolation grid can be determined as follows. For IPP's between N85° and S85°, the relative longitude and latitude in the interpolation grid are computed as:

\[
\begin{align*}
    x &= \frac{\lambda_{pp} - \lambda_1}{\lambda_2 - \lambda_1} \\
y &= \frac{\Phi_{pp} - \Phi_1}{\Phi_2 - \Phi_1}
\end{align*}
\] (30) (31)

For IPP's north of N85° or south of S85°, these values are computed as:

\[
\begin{align*}
    x &= \frac{\lambda_{pp} - \lambda_3}{90°} \cdot (1 - 2y_{pp}) + y_{pp} \\
y &= \frac{\Phi_{pp} - 85°}{10°}
\end{align*}
\] (32) (33)

with \( \lambda_{pp}, \lambda_1, \lambda_2 \) longitude of pierce point and grid points (see figure 13)
\( \Phi_{pp}, \Phi_1, \Phi_2 \) latitude of pierce point and grid points (see figure 13)
The MOPS [3] provides guidance on the calculation of the relative longitude and relative latitude for two regions: (a) between N75 and S75 – see equations (30) and (31) – and (b) north of N85 and south of S85. There is no guidance on how to compute the relative latitude and longitude in the region between N75 and N85 or in the region between S75 and S85.

In order to implement the required algorithms, the MOPS [4] were used as a guideline to compute the relative longitude and latitude for these regions according to equations (30) and (31) as well.

For the triangular interpolation, the pierce points are numbered as shown in figure 15 so that grid point #2 is always the vertex opposite the hypotenuse and the relative longitude and relative latitude are always determined relative to the distance to grid point #2. It should be noted that there are an additional three orientations of the triangle that are not shown in figure 15.

For the three-point interpolation, the mathematical formulation for the interpolated vertical IPP delay as a function of the IPP latitude and longitude is given by:

\[
\tau_{ipp}(\Phi_{pp}, \lambda_{pp}) = \sum_{i=1}^{3} W_i(x_{pp}, y_{pp}) \tau_{vi}
\]

(34)

The general equation for the weighting function is given by :

\[
W_1(x, y) = 1 \cdot y
\]

(35)

\[
W_2(x, y) = 1 \cdot (1 - x - y)
\]

(36)

\[
W_3(x, y) = x \cdot 1
\]

(37)

with the relative longitude \(x_{pp}\) and relative latitude \(y_{pp}\) as defined with the four-point interpolation. The resulting weights using this set of functions are displayed in figure 16.
Figure 15: IGP interpolation cell and IPP

Figure 16: Weighting Functions W1 – W3 for three-point interpolation
7.7 The Ionospheric Delay

Once the equipment establishes the vertical delay at the pierce point, the user must multiply this vertical delay by the obliquity factor (displayed in figure 17) to obtain the slant delay, which then can be subtracted from the pseudorange measurement:

\[
RC_{\text{iono}} = \left( 1 - \left( \frac{R_c \cos(E)}{R_c + h_t} \right)^2 \right)^{\frac{1}{2}} \tau_{\text{vpp}} 
\]

with \( RC_{\text{iono}} \) slant ionospheric delay at pierce point

![Figure 17: Obliquity Factor for Ionospheric Corrections](image)

7.8 Degradation of Correction Information

As stated already in chapter 5 (application of the fast corrections), the ionospheric corrections are designed to provide the most recent information to the user. Due to the limitation of the bandwidth and the possibility of missing some transmissions of the SBAS SIS (bit errors, masking, etc.), the user must apply models of degradation to the correction information transmitted.

The message type 26 provides the user – in addition to the vertical delays – with an upper bound on the residual error after application of the corrections (in the form of a standard deviation) at the geographically defined IGP. In a first step, the degradation over time of this transmitted information is calculated using data from the message type MT 10:
\[
\sigma_{\text{iono, grid}}^2 = \begin{cases} 
(\sigma_{\text{GIVE}} + \epsilon_{\text{iono}})^2, & \text{RSS}_{\text{iono}} = 0 \\
\sigma_{\text{GIVE}}^2 + \epsilon_{\text{iono}}^2, & \text{RSS}_{\text{iono}} = 1
\end{cases}
\]  \tag{39}

with
- \(\sigma_{\text{GIVE}}^2\) grid ionospheric vertical error bound
- \(\sigma_{\text{ionogrid}}^2\) grid ionospheric vertical error bound with degradation over time
- \(\epsilon_{\text{iono}}\) degradation of ionospheric correction information
- \(\text{RSS}_{\text{iono}}\) root-sum-square flag for ionospheric information (from MT 10)

The degradation of the ionospheric correction information is determined by:

\[
\epsilon_{\text{iono}} = C_{\text{iono, step}} \left( \frac{t - t_{\text{iono}}}{I_{\text{iono}}} \right) + C_{\text{iono, ramp}} \left( t - t_{\text{iono}} \right)
\]  \tag{40}

with
- \(C_{\text{iono, step}}\) bound on the update error for ionospheric grid delay values (from MT 10)
- \(C_{\text{iono, ramp}}\) rate of change for the ionospheric grid delay values (from MT 10)
- \(t\) current time
- \(t_{\text{iono}}\) time of transmission of the first bit of the MT 26
- \(I_{\text{iono}}\) update interval for ionospheric corrections MT 26 (from MT 10)
- \([\cdots]\) function to obtain the floor or greatest integer less than input argument

Furthermore, the user must use the interpolation scheme described in chapter 7.6 to determine an upper bound on the vertical error for the location of the ionospheric pierce point IPP:

\[
\sigma_{\text{UIVE}}^2(\Phi_{\text{pp}}, \lambda_{\text{pp}}) = \sum_{i=1}^{4} W_i(x, y) \sigma_{\text{ionogrid}}^2
\]  \tag{41}

\[
\sigma_{\text{UIVE}}^2(\Phi_{\text{pp}}, \lambda_{\text{pp}}) = \sum_{i=1}^{3} W_i(x, y) \sigma_{\text{ionogrid}}^2
\]  \tag{42}

Finally, the user must take the slant direction of the signal into account. The User Ionospheric Range Error estimate is calculated by:

\[
\sigma_{\text{UIRE}}^2 = \left( 1 - \left( \frac{R_s \cos(E_s)}{R_s + h_l} \right)^2 \right) \cdot \sigma_{\text{UIVE}}^2
\]  \tag{43}

with
- \(\sigma_{\text{UIVE}}^2\) interpolated user ionospheric vertical error bound
- \(\sigma_{\text{UIRE}}^2\) interpolated user ionospheric range error bound
7.9 Application of Corrections

In order to apply the ionospheric corrections according to equation (38) and the User Ionospheric Range Error UIRE estimate according to equation (43), the following conditions must be met:

- Message type MT 26 must not be timed-out. The time-out interval for MT 26 is given in table 27. The same condition must be met for MT 18.
- The equipment must be able to find an interpolation cell with four or at least three supported grid points following the logic described in chapter 7.5.
- None of the four grid points surrounding the pierce point may have a “do not use” flag in MT 26: If one or more selected IGPs do have a “do not use” flag, no ionospheric corrections can be computed.
- If a square interpolation mode is selected and if only one of the selected IGPs is flagged as “not monitored” in MT 26, then the interpolation mode can revert to a triangular interpolation, provided that the IPP can be found to be inside the resulting triangle. If the IPP is outside the resulting triangle or if more than one IGP is flagged as “not monitored” in MT 26, then no ionospheric corrections can be computed.

Only if the these conditions are met, the ionospheric corrections can be applied. They are applied by adding them to the pseudoranges instead of applying the KLOBUCHAR model. The estimation on the upper error bound (in combination with the degradation over time) is used to in the computation of the protection levels.
8 Tropospheric Corrections

8.1 General Approach to the Tropospheric Corrections

Because tropospheric refraction is a local phenomenon, all users will compute their own tropospheric delay correction which is depending only on their current position. The meteorological parameters used in the tropospheric correction model are interpolated using tabulated values.

The slant delay and the slant residual tropospheric error caused by the refraction is calculated using several steps. The meteorological parameters are interpolated using the latitude of the current user position and the day-of-year of the user. Then, the zero-altitude zenith delay terms for the wet and the dry part of the error compensations are calculated. These computed delays are then modified to account for the user’s altitude. Finally, the elevation of the particular satellite is considered, transforming the estimation of the vertical delay into an estimation of the slant delay.

8.2 The Linear Interpolation Scheme

The starting point of the determination of the tropospheric corrections is the linear interpolation of five different meteorological parameters from tabulated values.

- the meteorological pressure $P$
- the temperature $T$
- the water vapour pressure $e$
- the temperature lapse rate $\beta$
- the water vapour lapse rate $\lambda$

Each of these five parameters are given in the MOPS [4] through a tabulated mean value and a tabulated seasonal variation. The independent parameter for the tabulated values is the user latitude. The table 28 contains the tabulated mean meteorological values and the table 29 contains the tabulated seasonal variation for each meteorological parameter:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15 ° or less</td>
<td>1013.25</td>
<td>299.65</td>
<td>26.31</td>
<td>$6.30 \times 10^{-3}$</td>
<td>2.77</td>
</tr>
<tr>
<td>30 °</td>
<td>1017.25</td>
<td>294.15</td>
<td>21.79</td>
<td>$6.05 \times 10^{-3}$</td>
<td>3.15</td>
</tr>
<tr>
<td>45 °</td>
<td>1015.75</td>
<td>283.15</td>
<td>11.66</td>
<td>$5.58 \times 10^{-3}$</td>
<td>2.57</td>
</tr>
<tr>
<td>60 °</td>
<td>1011.75</td>
<td>272.15</td>
<td>6.78</td>
<td>$5.39 \times 10^{-3}$</td>
<td>1.81</td>
</tr>
<tr>
<td>75 ° or more</td>
<td>1013.00</td>
<td>263.65</td>
<td>4.11</td>
<td>$4.53 \times 10^{-3}$</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Table 28: Meteorological Mean Parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15 ° or less</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>$0.00 \times 10^{-3}$</td>
<td>0.00</td>
</tr>
<tr>
<td>30 °</td>
<td>-3.75</td>
<td>7.00</td>
<td>8.85</td>
<td>$0.25 \times 10^{-3}$</td>
<td>0.33</td>
</tr>
</tbody>
</table>
### Table 29: Meteorological Seasonal Variation Parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>45 °</td>
<td>-2.25</td>
<td>11.00</td>
<td>7.24</td>
<td>0.32 $10^{-3}$</td>
<td>0.46</td>
</tr>
<tr>
<td>60 °</td>
<td>-1.75</td>
<td>15.00</td>
<td>5.36</td>
<td>0.81 $10^{-3}$</td>
<td>0.74</td>
</tr>
<tr>
<td>75 ° or more</td>
<td>-0.50</td>
<td>14.50</td>
<td>3.39</td>
<td>0.62 $10^{-3}$</td>
<td>0.30</td>
</tr>
</tbody>
</table>

In order to interpolate the meteorological parameters, the user latitude is used. For latitude less than 15 ° or more than 75 °, the values for the mean and seasonal variation are taken directly from the tables 28 and 29. For latitudes in the range between 15 ° and 75 °, the values for the mean and seasonal variation are calculated by linear interpolation between the values for the two closest latitudes.

\[
\xi_0(\Phi) = \xi_0(\Phi_i) + \left( \xi_0(\Phi_{i+1}) - \xi_0(\Phi_i) \right) \frac{\Phi - \Phi_i}{\Phi_{i+1} - \Phi_i} \tag{44}
\]

\[
\Delta \xi(\Phi) = \Delta \xi(\Phi_i) + \left( \Delta \xi(\Phi_{i+1}) - \Delta \xi(\Phi_i) \right) \frac{\Phi - \Phi_i}{\Phi_{i+1} - \Phi_i} \tag{45}
\]

with
- $\xi_0$ mean meteorological parameter
- $\Delta \xi$ seasonal variation meteorological parameter
- $\Phi$ user latitude
- $\Phi_i$ latitude of first interpolation point
- $\Phi_{i+1}$ latitude of second interpolation point

#### 8.3 The Meteorological Parameters

Having established a scheme for the linear interpolation, the five current meteorological parameters that influence the tropospheric correction model are calculated using the contribution of the interpolated mean value and the contribution of the interpolated seasonal variation.

\[
\xi(\Phi, D) = \xi_0(\Phi) - \Delta \xi(\Phi) \cos \left( \frac{2\pi (D - D_{\text{min}})}{365.25} \right) \tag{46}
\]

with
- $\xi(\Phi, D)$ actual meteorological parameter
- $\xi_0(\Phi)$ mean meteorological parameter from equation (44)
- $\Delta \xi(\Phi)$ seasonal variation of the meteorological parameter from equation (45)
- $D$ day-of-year
- $D_{\text{min}}$ constant offset

\[
D_{\text{min}} = \begin{cases} 
28 & \text{for northern latitudes} \\
211 & \text{for southern latitudes} 
\end{cases} \tag{47}
\]
Using this scheme for the determination of the five meteorological parameters, the vertical zero-altitude zenith delay due to the tropospheric refraction can be calculated.

### 8.4 The Slant Tropospheric Delay

The zero-altitude zenith delay contribution for the wet and dry delays are calculated as follows:

\[
\begin{align*}
z_{\text{hyd}} &= \frac{10^{-6} k_1 R_d P}{g_m} \\
z_{\text{wet}} &= \frac{10^{-6} k_2 R_d e}{g_m (\lambda + 1) - \beta R_d T}
\end{align*}
\]

(48) \quad (49)

with

- \(z_{\text{hyd}}\) dry contribution to zero-altitude zenith delay
- \(z_{\text{wet}}\) wet contribution to zero-altitude zenith delay
- \(P\) meteorological pressure using equation (46)
- \(e\) water vapour pressure using equation (46)
- \(T\) temperature using equation (46)
- \(\beta\) temperature lapse rate using equation (46)
- \(\lambda\) water vapour lapse rate using equation (46)

The following table 30 contains the physical and meteorological constants that have been used in equation (48) and (49):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_1)</td>
<td>77.604 k/mbar</td>
<td>(g_m)</td>
<td>9.784 m/s²</td>
</tr>
<tr>
<td>(k_2)</td>
<td>38200 k²/mbar</td>
<td>(g)</td>
<td>9.80665 m/s²</td>
</tr>
<tr>
<td>(R_d)</td>
<td>287.054 J/kg/K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 30: Meteorological Constants Used by the Tropospheric Model

The zero-altitude zenith delay must then be modified to take the user’s altitude into account. This is done by the following equations.

\[
\begin{align*}
d_{\text{hyd}} &= \left(1 - \frac{\beta H}{T}\right)^{\frac{g}{R_d\beta}} z_{\text{hyd}} \\
d_{\text{wet}} &= \left(1 - \frac{\beta H}{T}\right)^{\frac{(\lambda+1)g}{R_d\beta} - 1} z_{\text{wet}}
\end{align*}
\]

(50) \quad (51)

with

- \(d_{\text{hyd}}\) dry contribution to zenith delay
- \(d_{\text{wet}}\) wet contribution to zenith delay
- \(H\) user altitude expressed in meters above mean sea level
The tropospheric delay estimate takes then the following form:

$$RC_{\text{tropo}} = - (d_{\text{hyd}} + d_{\text{wet}}) \frac{1.001}{\sqrt{0.002001 + \sin^2(E_i)}}$$  \hspace{1cm} (52)

with 

- $RC_{\text{tropo}}$: tropospheric delay estimate
- $E_i$: elevation angle of a particular satellite

This mapping function for the tropospheric delay estimate is valid for satellite elevation angles of not less than 5 degrees. The model for the residual error for the tropospheric delay estimate for a particular satellite is given as:

$$\sigma_{i,\text{tropo}}^2 = 0.12^2 \left( \frac{1.001}{\sqrt{0.002001 + \sin^2(E_i)}} \right)^2 m^2$$  \hspace{1cm} (53)

This function need not take account a degradation over time, since the calculation of the tropospheric corrections does not depend on the reception of any correction information through the transmission of the SBAS Signal-In-Space.
9 Determination of the Corrected Pseudorange

9.1 Satellite Designations

The selection of individual satellites for the position solution algorithm is performed using a complex logic. The MOPS [3] requires that the equipment must designate each GPS and SBAS satellite as either “SBAS unhealthy”, “SBAS unmonitored” or “SBAS healthy”. These three definitions are listed below:

9.1.1 Designation SBAS Unhealthy

The equipment must designate any GPS or SBAS satellite as “SBAS unhealthy” for a particular navigation mode upon the occurrence of any of the following conditions:

- the equipment has successfully decoded a UDREI of 15, indicating that the SBAS has assessed the satellite’s signals as unusable
- the step detector function has declared a step error
- for SBAS satellites, the failure of parity on four successive messages

The current implementation of PEGASUS contains no possibility to perform a parity check on the ESTB messages, since the receiver native output of the ESTB data does not contain a ESTB parity information. A parity check is performed on the receiver native output message. It is assumed that erroneous ESTB messages can be detected by that mechanism.

The “SBAS unhealthy” status for that satellite shall be changed only after the condition has cleared and none of the above condition exists. When an SBAS satellite is designated as “SBAS unhealthy” due to any of the above conditions, the integrity and correction data can continue to be applied until the normal time-out interval expires.

Although not considered “SBAS unhealthy”, a satellite must not be used for the precision approach mode, if the UDREI associated with that satellite (transmitted in the message types 2-5 and 24) is equal to 12 or even larger. The exclusion of satellites for precision approach under these conditions allows the service provider to designate satellites not to be used for these operations under certain failure conditions.

Due to the formulation of the MOPS [4], it is unclear whether the message types that are transmitted by the ESTB satellite can still be decoded when a UDREI of 15 is transmitted for that particular satellite. The MOPS [4] only states that the integrity and correction data still can be used until the normal time-out periods expires. For the PEGASUS implementation, it is assumed that in such a case the SBAS message types can still be decoded and applied.

9.1.2 Designation SBAS Unmonitored

If not designated already as “SBAS unhealthy”, the equipment must designate any GPS or SBAS satellite as “SBAS unmonitored” for a particular navigation mode upon the occurrence of any of the following conditions:

- the equipment has successfully decoded a UDREI of 14 (i.e. not “not monitored”),
- SBAS data is not provided (i.e. not supported by the current PRN mask in message type MT 1),
- SBAS signals are not received (i.e. a reception loss due to line-of-sight problems, etc.)
- SBAS data has timed out
SBAS IOD and SBAS IODE can not be reconciled

The “SBAS unmonitored” status for that satellite shall be changed only after the condition has cleared and none of the above conditions exists.

### 9.1.3 Designation SBAS Healthy

If not designated already as “SBAS unmonitored”, the equipment must designate any GPS or SBAS satellite as “SBAS healthy” for a particular navigation mode upon the occurrence of any of the following conditions:

- the step detection function has not declared a step error
- the equipment has not received a UDREI of 14 or 15.

The SBAS may declare a satellite as usable under some conditions even when the GPS navigation message indicates that the satellite is unhealthy. In such a case, the user can utilise the satellite in any mode of navigation.

The current implementation of PEGASUS assumes that once the parity check of the receiver native output message has been passed and the range measurements are output, the GPS measurements can be identified as “GPS healthy” – unless it is explicitly designated as “GPS unhealthy” in the appropriate ephemeris data set.

### 9.2 Selection of Satellites

The equipment shall automatically select the satellites for use in the navigational computation. It must select at least one ESTB satellite (if received), which is providing valid integrity information. Remaining satellites shall be selected to ensure continued integrity in the event of the loss of the SBAS satellite. The SBAS satellite is necessary in order to obtain the integrity information. However, it is not required that the SBAS satellite be used in the position determination.

The current implementation of PEGASUS provides two pre-set navigation modes while using the SBAS functionality: the “Non-Precision Approach mode” and the “Precision Approach mode”. The selection of these modes pre-selects as well the corrections to be applied (see chapter 9.6 and 9.7).

The equipment shall not utilise any satellite designated as “SBAS unhealthy”. Otherwise, the selection of the satellites shall be accomplished in the following way:

- If there are enough “SBAS healthy” satellites available to compute a position solution and to provide an protection level below the alarm limit, then use those satellites. Integrity is provided by the SBAS.

If there are not enough “SBAS healthy” satellites available to meet the requirements stated above, then the precision approach mode is not possible in the current situation. The user is notified and the following scheme is applied to obtain a position solution:

- Are there enough “SBAS healthy” and “SBAS unmonitored” satellites available to compute a position solution and to provide integrity by the FDE functionality, then those satellites are used (however, without applying the SBAS corrections and thus reverting to a GPS standalone positioning mode). Integrity is provided by the FDE functionality. Else,

- Are there enough “SBAS healthy” and “SBAS unmonitored” satellites available to compute a position solution and to provide integrity by the FD functionality, then use those satellites (however, without
applying the SBAS corrections and thus reverting to a GPS standalone positioning mode). Integrity is provided by the FD functionality. Else,

- Are there enough "SBAS healthy" and "SBAS unmonitored" satellites available to compute a position solution, then use those satellites (however, without applying the SBAS corrections and thus reverting to a GPS standalone positioning mode). Integrity is not provided. Else,
- no position solution can be computed.

The equipment must always operate in the highest available mode that can satisfy the performance requirements of the particular navigation mode.

In contrast to the method specified above, the current implementation of PEGASUS implements a different kind of selection strategy. The deviation from the selection strategy required in the MOPS [4] can be explained by the use of the PEGASUS tools: They are used more in an investigative approach than in an operative way.

- In the Precision Approach mode, if there are enough "SBAS healthy" satellites available to compute a position solution, then those satellites are used. The protection levels are computed at all epochs a valid position solution can be generated. Integrity is provided by the SBAS, if the protection levels are below the alarm limit (if not, then integrity is not provided and the particular flight phase is designated as "not available").
- In the Non-Precision Approach modes, if there are enough "SBAS healthy" satellites available to compute a position solution, then those satellites are used. The protection levels are computed at all epochs a valid position solution can be generated. Integrity is provided by the SBAS, if the protection levels are below the alarm limit (if not, then integrity is not provided and the particular flight phase is designated as "not available").

For both modes, if there are not enough "SBAS healthy" satellites available to compute a position solution, then a navigation solution for that epoch is not available.

If possible, the MOPS [4] recommends that the equipment apply differential corrections whenever available. Any message may be optionally used in flight phases for which they are not specifically required, but must then conform to the relevant requirements. If an optional message is used, the loss of those messages must not cause the loss of the navigation functionality.

The message types MT 0 (Don't Use), MT 1 (PRN Mask), MT 2-5 (Fast Corrections), MT 6 (Integrity), MT 7 (UDRE Degradation), MT 9 (SBAS Ephemeris), MT 17 (SBAS Almanac) and MT 24 (Mixed Corrections), MT 25 (Slow Corrections) and MT 27 (Service Region) must be used for all navigation modes. Additionally, for the Precision Approach flight phase, the message type MT 10 (Degradation), MT 18 (Ionospheric Grid) and MT 26 (Ionospheric Corrections) must be applied.

In the current implementation of the ESTB version 1.2, there is a deviations from the requirements of the MOPS [4]. Thus, in order to be able to use the PEGASUS developments for the implemented version of the ESTB, the following specifications are established.

The current ESTB version 1.2 does transmit message type MT 27 data (service region) as it is defined in the MOPS [3] or [4]. However, the intended application of the message information differs from the requirements of the specification: it is used to define regions outside the applicable service region where a certain type of operation might be supported. No further information was available at the time of the updating of this document. Thus, the presence of this message type will be treated as optional (i.e. read in and processed, if there), but will not be required for any flight phase.
9.3 Consistency Check of Range Measurements

9.3.1 Quality Control for Range Measurements

In order to use the measured pseudoranges, the measurement values have to pass a consistency check before any other evaluation and calculation process is done. This consistency check is concerned mainly about the behaviour of the measured code pseudorange and the measured carrier phase pseudorange.

Based on the difference in transit times, an estimation on the range rate of the measured code pseudorange is calculated. Additionally, based on the increments in carrier phase measurements (i.e. the integrated doppler or delta range measurements), an estimation on the range rate of the carrier phase pseudorange is determined. It is important to notice that – due to the format description of the measurement data (which can be found in the ICD [8]) – the carrier phase measurements are provided as increments or “delta ranges”. The total value for the carrier phase measurements must be reconstructed by adding consecutive “delta ranges”.

![Figure 19: Approach to Consistency Checks](image)

If the difference between those two estimation of the range rate exceeds a certain threshold, the measurements of that particular satellite are considered not to be applicable to the positioning in that epoch. If a carrier-phase smoothing filter is used, then additionally the ranging filter is re-set. The approach to the consistency check implemented is shown as well in figure 19.

Thus, the implemented consistency check is realised as a step detector as required in the MOPS [4]. It will be described in detail in the next section.

9.3.2 Test for Exclusion

The step detector implemented in PEGASUS is the algorithm proposed by the MOPS [4] It should be able to detect and exclude a range measurement that has a pseudorange step error of 700 meters or more from one measurement to the next. It should be noted that the equipment should not act on smaller step errors to avoid interference with the Fault Detection and Exclusion FDE algorithms.
The pseudorange step detector implemented calculates the pseudorange measurement innovation of an individual satellite by subtracting the predicted pseudorange from the measured pseudorange. The predicted pseudorange is based on the last pseudorange measurement and the last increment of the pseudorange measurement.

\[
\Delta R = R_2 - (R_1 + \delta R)
\]  

(54)

with \(\Delta R\) difference between the measured and predicted pseudorange

\(R_2\) measured pseudorange at epoch “2”

\(R_1\) measured pseudorange at epoch “1”

\(\delta R\) predicted change in the pseudorange

and the predicted change in the pseudorange measurement from epoch 1 to the epoch 2 is based on the extrapolated change in the pseudorange measurement using the previous epoch. Following the proposed implementation of the MOPS [4], the following test is performed:

\[
\begin{align*}
\Delta R & \leq \Delta R_{\text{thres}} \quad \text{test is passed} \\
\Delta R & > \Delta R_{\text{thres}} \quad \text{test is failed}
\end{align*}
\]

(55) (56)

with \(\Delta R_{\text{thres}}\) pseudorange step detector threshold

The setting for the pseudorange step detector is set to the specified value of 73.8 meters in order to obtain a probability of less than 0.001 of accepting a satellite with a pseudorange bias of greater than 700 metres, as required by the MOPS [4].

9.3.3 Test for Re-Inclusion

Once a satellite has been excluded from the position solution due to a failed “step detector” test, it must be monitored for possible future inclusion into the position solution. The MOPS [4] differentiates between the following three situations:

- exclusion with no loss-of-lock
- exclusion with loss-of-lock but re-acquisition within a short period of time
- exclusion with loss-of-lock and re-acquisition after a longer period of time

After a satellite has been excluded without loss-of-lock, it remains excluded until positive integrity is available through autonomous fault detection. Thus, if a valid position solution has been calculated (without using the excluded satellite), the difference between the measured pseudorange and the “predicted” pseudorange for that satellite is computed. The “predicted” pseudorange is determined through the calculated position solution (i.e. based on the satellites which are already in the position solution. The same test threshold settings as in the step detector test is used, i.e. the test threshold is set to 73.8 meters.

The above mentioned approach of a test requires that a valid position solution must be calculated using the other valid (i.e. not excluded) satellites. In the case where a satellite has been excluded due to a failed step detector test and no valid position solution can be calculated, the satellite is “blocked” for a 10 seconds and no measurement data is accepted from that satellite.
After a satellite has been excluded with loss-of-lock and is re-acquired within a short period of time (the MOPS [4] specifies 10 seconds), the above mentioned test for positive re-inclusion into the position solution is performed, but with an augmented test threshold which accounts for a possible dynamic user profile of a constant altitude assumption allowing up to 2 g of acceleration in the horizontal plane. The final test threshold is then set to 433.8 meters.

A discrepancy can be found in the MOPS [4] for the magnitude of the augmented test threshold. An additional value of 360 meters is provided. The maximum allowable time period for re-acquisition was specified to 10 seconds, but the explanation for the derivation of these 360 meters uses a time period of 6 seconds. The interested reader is referred to the Appendix I of the MOPS [4] page I-3 and is invited to provide feedback and any comments.

After a satellite has been excluded with loss-of-lock and is re-acquired only after a longer period of time (the MOPS [4] specifies 10 seconds), the satellite is considered as “newly acquired” and no special test for positive re-inclusion into the position solution is performed. It is assumed that the SBAS is transmitting the necessary integrity information with regard to that satellite within the specified time period.

9.3.4 Surviving Data Gaps

However, every instance the ranging filter had to be restarted, the filtered pseudorange measurement could not be used for a certain amount of time (the time the filter uses for the convergence process of the ranging filter) – this amount of time is directly proportional to the time constant used for the smoothing filter (see chapter 9.4). However, a considerable time constant causes a long transition time in the filter in order to reach the steady state of the smoothed pseudorange, which is, of course, disadvantageous especially for high dynamic applications with frequent loss-of-lock and re-acquiring of GPS signals.

In the current version of the PEGASUS prototypes, the user can specify a maximum allowable time gap, during which the carrier phase range smoothing filter will not be restarted, if there are interruptions in the data stream for one satellite. It is important to notice that:

- There will be no interpolations and extrapolations for missing data sets (i.e. missing input data epochs will result in missing output data epochs)
- After the data interruption, the carrier phase measurements of that satellite will have to pass the above described consistency check (as it is always the case). Only if this consistency check is passed using the current and last (i.e. previously) received phase measurements, then the carrier smoothing filter for that satellite will not be reset. In all other cases, the filter will be reset and thus the range measurements for that satellite will not be available for positioning.
- If the specified period in has been exceeded, the carrier phase smoothing filter for the satellite in question will be reset regardless of the quality of the (then to be) received carrier phase measurements.

9.4 Range Smoothing

The noise of the pseudorange code signal is limited to approximately 50 cm one standard deviation for current receivers (for more information, please refer to [20]). In order to reduce the noise level of the code pseudorange measurements, a smoothing scheme is employed in which the carrier phase measurement are used. The noise level of the carrier phase measurements is limited to approximately 2 mm. In order to the compliant to the SARPS [12], this carrier phase smoothing is applied before the standard and SBAS pseudorange corrections are added on to the pseudorange measurements.

The implemented carrier-smoothing filter follows the specification of the SARPS [12]. It is implemented using the following equation:
\[ PR_k^S = \alpha \ PR_{k-1}^C + (1 - \alpha) \left[ P_k^S + \frac{\lambda}{2\pi} \left( PR_k^p - PR_{k-1}^p \right) \right] \quad (57) \]

\[ \alpha = \frac{dt}{T} \quad (58) \]

with
- \( PR_k^S \) smoothed pseudorange at epoch \( k \)
- \( PR_k^C \) code pseudorange at epoch \( k \)
- \( PR_k^p \) phase pseudorange at epoch \( k \)
- \( \lambda \) wavelength of the GPS carrier frequency at L1
- \( T \) smoothing constant
- \( dt \) sampling interval

It can be shown through a re-organisation of the terms in equation (57) that the implemented smoothing filter can be illustrated as well using the figure 20.

![Figure 20: Carrier Smoothing Filter Scheme](image)

The quality of the noise reduction is strongly dependent on the time constant \( T \). However, a considerable time constant causes a long transition time in the filter in order to reach the steady state of the smoothed pseudorange, which is, of course, disadvantageous especially for high dynamic applications with frequent loss-of-lock and re-acquiring of GPS signals.

Therefore, a compromise has to be found on the quantity of the filter time constant according to the type of application and on the decision on when to use a “steady-state” smoothed pseudorange. In the PEGASUS implementation, the following scheme is proposed:
\[
\Delta t < \max\left( T, T_{\text{align}} \right) \quad \text{smoothed pseudorange is not used for positioning} \quad (59)
\]
\[
\Delta t \geq \max\left( T, T_{\text{align}} \right) \quad \text{smoothed pseudorange is used for positioning} \quad (60)
\]

with \( \Delta t \) = time since initialisation of the filter for a particular pseudorange
\( T_{\text{align}} \) = filter align time, pre-set to 60 seconds

It should be noted that the specifications MOPS [3] and MOPS [4] do not require a range smoothing. However, in the current SARPs [12] a range smoothing filter with a certain performance is required. Thus, the need for a range smoothing has been recognised and a range smoothing filter has been implemented.

9.5 SBAS Non-Precision Approach Mode

In the Non-Precision Approach mode, only the SBAS fast and slow corrections are applied. If no fast and slow corrections are available (see chapter 5.4 and 6.4), then the particular satellite will not be used in the position solution. The message type MT 10 information will not be used in order to calculate the degradation of the correction information (see chapters 5.3 and 7.8).

With regard to ionospheric corrections models, the user can choose between “no ionospheric correction”, the “standard ionospheric correction” or the “SBAS ionospheric correction”. If SBAS ionospheric corrections are selected and available, then those corrections will be applied. If they are not available for a particular satellite, then the standard GPS ionospheric correction model will be used.

With regard to tropospheric corrections models, the user can choose between “no tropospheric correction”, the “standard tropospheric correction” and the “SBAS tropospheric corrections”. Tropospheric corrections will be used according to the selected model.

The MOPS [4] requires that if the equipment smoothes code or Doppler measurements in developing a pseudorange measurement, the differential corrections must be applied after the filtering and immediately before the determination of the user position. The corrected pseudorange will then be:

\[
\rho = \rho_{\text{meas}} + RC_{\text{fast}} - RC_{\text{iono}} + RC_{\text{tropo}} + RC_{\text{clock}} \quad (61)
\]

with \( \rho \) = pseudorange for position determination application (see chapter 10)
\( \rho_{\text{meas}} \) = measured pseudorange
\( RC_{\text{fast}} \) = fast corrections (see chapter 5)
\( RC_{\text{iono}} \) = ionospheric corrections for selected model
\( RC_{\text{tropo}} \) = tropospheric corrections for selected model
\( RC_{\text{clock}} \) = clock corrections for the particular satellite (either GPS or SBAS)

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the ionospheric corrections using the message types MT 18 and MT 26. The implementation these corrections still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the currently supported ionospheric correction scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.
In contrast to the MOPS [4], the ionospheric corrections are subtracted from the measured pseudoranges. This is due to the fact that the ionospheric corrections itself do not possess a negative sign.

The satellite position will be corrected using the slow corrections (see chapter 6.4) and equation:

\[
\begin{bmatrix}
    x_{c,sv} \\
    y_{c,sv} \\
    z_{c,sv}
\end{bmatrix} = \begin{bmatrix}
    x_{sv} \\
    y_{sv} \\
    z_{sv}
\end{bmatrix} + \begin{bmatrix}
    \Delta X_{sv} \\
    \Delta Y_{sv} \\
    \Delta Z_{sv}
\end{bmatrix}
\]  

(62)

with \( x_{c,sv}, y_{c,sv}, z_{c,sv} \) corrected satellite position
\( x_{sv}, y_{sv}, z_{sv} \) satellite position using GPS ephemeris
\( \Delta X_{sv}, \Delta Y_{sv}, \Delta Z_{sv} \) slow corrections (see chapter 6)

Furthermore, the satellite clock error must as well be corrected for. For a GPS satellite, this can be done by adding the additional SBAS clock correction (from message type MT 25) to the already computed GPS clock correction (see chapter 6.3). It should be noted that the clock error model for the SBAS satellite uses a slightly different error model than for the GPS satellite (see chapter 13).

### 9.6 SBAS Precision Approach Mode

In the Precision Approach mode, the SBAS fast, slow, ionospheric and tropospheric corrections are applied. If no fast and slow corrections are available (see chapter 5.4 and 6.4), then the particular satellite will not be used in the position solution. Additionally, if the SBAS ionospheric corrections are not available for a particular satellite, this satellite will be excluded from the position solution. The message type MT 10 information will be used in order to calculate the degradation of the correction information (see chapters 5.3 and 7.8).

With regard to ionospheric corrections models, the user can choose between "no ionospheric correction", the "standard ionospheric correction" or the "SBAS ionospheric correction". If a non-default ionospheric correction is selected, the PEGASUS tools will issue a warning. If SBAS ionospheric corrections are selected and unavailable, then the particular satellite will not be used for the position computation.

With regard to tropospheric corrections models, the user can choose between "no tropospheric correction", the "standard tropospheric correction" and the "SBAS tropospheric corrections". Tropospheric corrections will be used according to the selected model. If a non-default tropospheric correction is selected, the PEGASUS tools will issue a warning.

The MOPS [4] requires that if the equipment filters code or Doppler measurements in developing a pseudorange measurement, the differential corrections must be applied after the filtering and immediately before the determination of the user position. The corrected pseudorange will then be:

\[
\rho = \rho_{\text{meas}} + RC_{\text{fast}} - RC_{\text{iono}} + RC_{\text{tropo}} + RC_{\text{clock}}
\]  

(63)

with \( \rho \) pseudorange for position determination application (see chapter 10)
\( \rho_{\text{meas}} \) measured pseudorange
\( RC_{\text{fast}} \) fast corrections (see chapter 5)
The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the ionospheric corrections using the message types MT 18 and MT 26. The implementation these corrections still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the currently supported ionospheric correction scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

In contrast to the MOPS [4], the ionospheric corrections are subtracted from the measured pseudoranges. This is due to the fact that the ionospheric corrections itself do not possess a negative sign.

The satellite position will be corrected using the slow corrections (see chapter 6.4) and equation:

\[
\begin{bmatrix}
    x_{c,sv} \\
    y_{c,sv} \\
    z_{c,sv}
\end{bmatrix} = \begin{bmatrix}
    x_{sv} \\
    y_{sv} \\
    z_{sv}
\end{bmatrix} + \begin{bmatrix}
    \Delta X_{sv} \\
    \Delta Y_{sv} \\
    \Delta Z_{sv}
\end{bmatrix}
\]

(64)

with 
\(x_{c,sv}, y_{c,sv}, z_{c,sv}\) \quad corrected satellite position

\(x_{sv}, y_{sv}, z_{sv}\) \quad satellite position using GPS ephemeris

\(\Delta X_{sv}, \Delta Y_{sv}, \Delta Z_{sv}\) \quad slow corrections (see chapter 6)

The satellite clock error must as well be corrected for. For a GPS satellite, this can be done by adding the additional SBAS clock correction (from message type MT 25) to the already computed GPS clock correction (see chapter 6.3). It should be noted that the clock error model for the SBAS satellite uses a slightly different error model than for the GPS satellite (see chapter 13).
10 Navigation Solution and Protection Levels

10.1 General Approach to Protection Levels

The accuracy of a navigation system is defined in terms of Total System Error (TSE) which is referenced to a required flight path defined for each phase of flight. To follow the required path, the aircraft navigation system estimates the aircraft’s position and generates commands (either to a cockpit display or to the autopilot). Errors in the estimation of the aircraft’s position is referred to as Navigation System Error (NSE) which is the difference between the aircraft’s true position and its displayed position (see figure 21).

The difference between the required flight path and the displayed position of the aircraft is called Flight Technical Error (FTE) and contains aircraft dynamics, turbulence effects, man-machine-interface problems, etc. The vector sum of the NSE and the FTE is the Total System Error. Since the actual Navigation System Error cannot be observed without a high-precision reference system (the NSE is the difference between the actual position of an aircraft and its computed position), an approach has to be found with which an upper bound can be found for this error.

- The Horizontal Protection Level (HPL) is the radius of a circle in the horizontal plane (the plane tangent to the WGS84 ellipsoid), with the centre being at the true aircraft position, which describes the region which is assured to contain the indicated horizontal position. It is the horizontal region for which the missed alert requirements can be met.

- The Vertical Protection Level (VPL) is the half length of a segment on the vertical axis (perpendicular to the horizontal plane of the WGS84 ellipsoid), with the centre being at the true aircraft position, which describes the region which is assured to contain the indicated vertical position. It is the vertical region for which the missed alert requirements can be met.

The SBAS protection levels are a function of the satellite constellation and the estimated SBAS performance. Thus, using the SBAS correction data, the protection levels can be determined without using actual pseudorange measurements.

The computed protection levels must be compared to the required Alert Limits (AL) for the particular phase of flight. If the protection level is smaller than the required alert limit, then the phase of flight can be performed. However, if the protection level is greater than or equal to the required alert limit, then the integrity of the position solution can not be guaranteed in the context of the requirements for that particular flight phase.
$XPL < XAL$ integrity can be assured \hfill (65)

$XPL \geq XAL$ integrity can not be assured \hfill (66)

with $XPL$ (horizontal or vertical) protection level

$XAL$ (horizontal or vertical) alert limit

The relevant alert limits, in combination with the required alert limit requirement, are listed in table 31. In particular, the Integrity Requirements will be used later to derive the protection levels. The corresponding situation in the horizontal plane is depicted in the figure 22.

<table>
<thead>
<tr>
<th>Flight Phase</th>
<th>Integrity Requirements ([12])</th>
<th>Horizontal Alert Limit ([12])</th>
<th>Vertical Alert Limit ([12])</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENR</td>
<td>$1 \times 10^{-7}$ per hour</td>
<td>7400 m</td>
<td>N/A</td>
<td>different alert limits for domestic and oceanic flight phases</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3700 m</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850 m</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>TMA</td>
<td>$1 \times 10^{-7}$ per hour</td>
<td>1850 m</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>NPA</td>
<td>$1 \times 10^{-7}$ per hour</td>
<td>600 m</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>APV-I</td>
<td>$1 \times 2 \times 10^{-7}$ per approach</td>
<td>600 m</td>
<td>50 m</td>
<td>new flight phase defined in the current SARPs [12]</td>
</tr>
<tr>
<td>APV-II</td>
<td>$1 \times 2 \times 10^{-7}$ per approach</td>
<td>40 m</td>
<td>20 m</td>
<td>new flight phase defined in the current SARPs [12]</td>
</tr>
<tr>
<td>CAT-I</td>
<td>$1 \times 2 \times 10^{-7}$ per approach</td>
<td>40 m</td>
<td>$10 \text{ m} - 15 \text{ m}$</td>
<td></td>
</tr>
</tbody>
</table>

**Table 31**: Protection Levels for Flight Phases ([4] and [12])

**Figure 22**: Horizontal Protection Levels and Horizontal Alert Limit
It should be noted that the main significance using this approach is not the computation of the protection levels and their comparison with the corresponding alert limit. The major interest should be considered to be on the assurance that the computed protection levels represent an upper bound on the NSE with a certain confidence. “Misleading Information” results only, if the NSE is greater than the alert limit and the protection level does not indicate this fact (for a more complete and detailed description of the “overbounding concept” and problems resulting of it, refer to [21]).

### 10.2 Navigation Equation

The navigation equation of the satellite navigation systems can be found by analysing the geometric distance between a satellite and the (unknown) user position and by taking into account of the additional (unknown) user clock offset:

\[
\rho = \sqrt{(X - X_{sv})^2 + (Y - Y_{sv})^2 + (Z - Z_{sv})^2 + c \Delta T}
\]  

(67)

with
- \(\rho\): measured and corrected pseudorange for one satellite
- \(X, Y, Z\): unknown user position
- \(X_{sv}, Y_{sv}, Z_{sv}\): satellite position
- \(c\): velocity of light
- \(\Delta T\): unknown receiver clock error

The navigation equation can be solved by linearising around a known starting point, preferably the position solution of the last epoch. Thus:

\[
\delta R = H \delta x + \varepsilon
\]

(68)

with
- \(\delta R\): linearised pseudorange using a starting point
- \(H\): observation matrix
- \(\delta x\): unknown innovations for the user position and receiver clock offset using a starting point
- \(\varepsilon\): error in the linearised pseudoranges

The linearised pseudorange is established as the difference between the pseudorange measurements (including carrier smoothing effects and all necessary corrections) and the expected ranging values based on the location of the satellites and the location of the user. A column of the design matrix for a particular satellite constellation is determined to:

\[
H_i = \begin{bmatrix}
-\cos(E)\cos(A) & -\cos(E)\sin(A) & -\sin(E) & 1
\end{bmatrix}
\]

(69)

with
- \(H_i\): a column of the design matrix for a particular satellite
- \(E\): elevation of the satellite
- \(A\): azimuth of the satellite

when positive azimuth is defined clockwise from North. Thus, the co-ordinate system used is a local tangent one, the \(x\)-axis pointing to the east, the \(y\)-axis pointing to the north and the vertical axis complementing the other to in the local vertical (i.e. pointing upwards).
10.3 Unweighted Least Squares Algorithms

The Unweighted Least Squares algorithm is one possible approach to determine the navigation solution (i.e. the user position and the receiver clock offset) from the GNSS pseudorange measurements. The navigation solution using the standard Least Squares can be found to:

\[
\delta x = (H^T H)^{-1} H^T \delta R \tag{70}
\]

The innovations to the unknown user position and receiver clock offset, computed by this equation, are added to the starting point of the linearisation. The process of linearising and solving the linearised equation is performed iteratively until the innovations to the user position can be neglected.

Having achieved a new position solution, the characteristics of the linearised pseudoranges can be assumed to have the following characteristics:

\[
E[\delta R] = 0 \tag{71}
\]

\[
D[\delta R] = \sigma^2 \begin{bmatrix} 1 & \cdots & 1 \\ 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \tag{72}
\]

with 
\(E[\cdot]\) expectation operator
\(D[\cdot]\) covariance operator
\(\sigma\) assumed standard deviation of the pseudoranges

Thus, the linearised pseudoranges are assumed to have a zero mean. Additionally, they are supposed to be uncorrelated and have the same standard deviation. The value of the standard deviation for the SPS of GPS had been assumed by [2] to be 33.3 m, but after the removal of S/A this value obviously has be reviewed. Thus, the covariance of the position solution can be determined to be:

\[
E[\delta x] = 0 \tag{73}
\]

\[
D[\delta x] = E[ (\delta x - E[\delta x]) (\delta x - E[\delta x])^T ] \tag{74}
\]

Thus, the standard deviation of the horizontal and vertical position solution can be determined to the following well-known result:

\[
\sigma_{\text{hor}} = \text{HDOP} \ \sigma \tag{75}
\]

\[
\sigma_{\text{ver}} = \text{VDOP} \ \sigma \tag{76}
\]

with
\(\text{HDOP}\) horizontal dilution of precision
\(\text{VDOP}\) vertical dilution of precision
\(\sigma_{\text{hor}}\) standard deviation of horizontal position
\(\sigma_{\text{ver}}\) standard deviation of vertical position
10.4 Weighted Least Squares Algorithms

In contrast to the standard Least Squares approach to the position solution of satellite navigation systems, the weighted Least Squares does not assume that all pseudorange measurements will have the same standard deviation. The general idea of the weighted Least Squares is that a measurement with a high level of certainty should be weighted higher in a position solution than a measurement with a low level of uncertainty. Thus, a weighting matrix is introduced.

\[
W^{-1} = \begin{bmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_n^2
\end{bmatrix}
\]

with \( W \) weighting matrix
\( \sigma_i^2 \) weighting factor of an individual satellite (i.e. uncertainty of the pseudorange)

The navigation solution using the weighted Least Squares approach can be found to:

\[
\delta x = (H^T W H)^{-1} H^T W \delta R
\]

The innovations to the unknown user position and receiver clock offset, computed by this equation, are added to the starting point of the linearisation. The process of linearising and solving the linearised equation is performed iteratively until the innovations to the user position can be neglected.

If the weighting factors in the weighting matrix of equation (77) represent the actual variance of pseudorange measurements, then the achieved position solution is also a minimum variance position solution. This algorithm assumes that the error sources for each satellite are uncorrelated with the error sources for any other satellite. While this assumption may not be strictly true, it should be a reasonably good approximation.

Because the satellites are weighted unequally, it is no longer possible to separate the expected positioning errors into a geometrical factor (the DOP values) and a user ranging accuracy. The covariance of the position solution can be determined to be:

\[
E[x] = 0
\]

\[
D[x] = E[ (x - E[x])^T (x - E[x]) ]
\]

\[
D[x] = (H^T WH)^{-1}
\]

Using that result, the standard deviation of the position solution in the horizontal and the vertical direction can be computed.

10.5 Weights using SBAS Corrections

10.5.1 Total Modelled Variance

Since the actual variance of the pseudorange measurement can not be observed in real-time, the variance for an individual satellite is modelled based on the correction information supplied by the SBAS. The weighting factors are:
with 

- \( \sigma_{i,\text{flit}}^2 \): total modelled variance of the pseudorange measurement
- \( \sigma_{i,\text{UIRE}}^2 \): variance of the residual error after applications of fast and slow corrections
- \( \sigma_{i,\text{air}}^2 \): variance of the residual error after applications of ionospheric corrections
- \( \sigma_{i,\text{trop}}^2 \): variance of the contributions of the receiver to the residual error
- \( \sigma_{i,\text{trop}}^2 \): variance of the residual error after applications of tropospheric corrections

The remaining residual error after the application of fast and slow corrections, ionospheric corrections and tropospheric corrections and the contribution of the receiver noise and multipath effects are discussed in the following sections.

### 10.5.2 Bound on Fast and Slow Residual Errors

When applying fast and slow corrections and the contents of message type MT 10 are processed, the parameters to be used are determined from the broadcast information. The variance of the residual pseudorange error can be found by equation (8).

For airborne equipment which does not process the message type MT 10 information (and is thus not capable of precision approach flight phases), a conservative bound is used and the following simplified version for the variance of the residual pseudorange error after application of fast and slow corrections can be made:

\[
\sigma_i^2 = (\sigma_{\text{UDR}} + 8 \text{ m})^2
\]

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the service region indication using the message type MT 27. The implementation of these corrections follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the service region determination scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

At the current time, all equipment is required to apply slow and fast corrections when they are available. However, future systems may support an integrity-only mode, where message type MT 6 information is broadcast to provide integrity monitoring without corrections. If no corrections are applied, a conservative estimate of the value of the corrections has been chosen based upon the maximum correction values. The maximum magnitude of the fast corrections is 255 m, clock correction is at maximum 25 m (calculated over the time-out period of 360 seconds), root-sum-square long-term ephemeris corrections are 222 m (rotated to line-of-sight) and the maximum slow correction is 143 m (for velocity code 1).

Since these quantities are independent, a reasonable maximum can be calculated by the root-sum-square of these terms, which is 368 m. Dividing this total budget by the typical K values for the flight phases en-route though non-precision approach yields the following upper bound of approximately 60 m. Thus, the variance of the residual pseudorange error after application of fast and slow corrections shall be:

\[
\sigma_i^2 = 3600 \text{ m}^2
\]
In the current PEGASUS development, when selecting the precision approach modes, the variance of the residual error after the application of slow and fast corrections is computed according to equation (8). If message type MT 10 information need not be applied, i.e. in all other flight phases except precision approach, the variance of the residual error after the application of slow and fast corrections is computed according to equation (82).

### 10.5.3 Bound on Ionospheric Residual Errors

When using the SBAS ionospheric corrections, the residual pseudorange error after application of the corrections can be found to follow equation (43).

If flight phases other than precision approach are selected by the user, the standard GPS ionospheric error model might be applied. A conservative bound for the vertical delay is used depending on the pierce point latitude. To account for the possibility that erroneous values are broadcast by the GPS in the navigation message, a scaled version of the GPS based ionospheric correction is also used. The residual pseudorange error after application of the standard GPS ionospheric error model is specified to be:

\[
\sigma^2_{UIRE} = \max\left(\frac{T_{\text{iono}}}{5}\right)^2, \left(F_{pp}\tau_{\text{vert}}\right)^2\right) \tag{84}
\]

with

\[
\tau_{\text{vert}} = \begin{cases} 
9\ \text{m}, & 0 \leq |\Phi_m| \leq 20 \\
4.5\ \text{m}, & 20 < |\Phi_m| \leq 55 \\
6\ \text{m}, & 55 < |\Phi_m| 
\end{cases} \tag{85}
\]

and with

- \(T_{\text{iono}}\): ionospheric corrections of standard error model (see [14])
- \(F_{pp}\): obliquity factor (see chapter 7.7 and equation (38))
- \(\tau_{\text{vert}}\): conservative bound on the vertical delay (see [14])
- \(\Phi_m\): geomagnetic latitude of ionospheric pierce point (see [14])

In the current PEGASUS development, the operator has the option to select the standard GPS ionospheric model or no ionospheric correction model at all when using the Non-Precision Approach mode or Precision Approach mode (obviously, the operator is notified about these atypical parameter settings). Using those parameter settings, the error bound given in equation (84) is applied in order to be able to compute protection levels.

### 10.5.4 Bound on Receiver Noise and Multipath Residual Errors

The contribution of the receiver noise to the residual range error shall represent the accuracy performance of the airborne receiver, including receiver noise and multipath. For airborne equipment which meets the accuracy requirements of the MOPS [4] section 2.1.4.1 for precision approach flight phases, the following values shall be used:
\[
\begin{align*}
\sigma^2_{i,\text{air},\text{GPS}} &= \sigma^2_{\text{mp}} + \sigma^2_{\text{noiseGPS}} \quad (86) \\
\sigma^2_{i,\text{air},\text{SBAS}} &= \sigma^2_{\text{mp}} + \sigma^2_{\text{noiseSBAS}} \quad (87)
\end{align*}
\]

with
- \(\sigma_{i,\text{air},\text{GPS}}\): standard deviations receiver noise for a GNSS satellite
- \(\sigma_{i,\text{air},\text{GEO}}\): standard deviation receiver noise for a SBAS satellite
- \(\sigma_{\text{mp}}\): standard deviation modelling the residual multipath error
- \(\sigma_{\text{noiseGPS}}\): standard deviation modelling the tracking accuracy for GPS satellites
- \(\sigma_{\text{noiseSBAS}}\): standard deviation modelling the tracking accuracy for SBAS satellites

For equipment that uses the carrier-smoothing filter specified in the GNSS SARPs [12], the variance of a normal distribution that models the residual multipath error of an airborne subsystem can be obtained to:

\[
\sigma^2_{\text{mp}} = 0.2 \exp\left(-\frac{E}{75.0}\right) m^2 \quad (88)
\]

with
- \(E\): elevation angle of the satellite (in degrees)

The tracking accuracy of the receiver is evaluated as part of the accuracy requirements in the MOPS [4] section 2.1.4.1. The standard deviation modeling the tracking accuracy for either a GPS or a SBAS satellite is depending, among other parameters, on the current signal-to-noise ratio and the time since initialization of the smoothing. It includes any error induced by filtering and code-carrier divergence. This parameter must be variable to reflect the current signal reception conditions. Any degradation of the system accuracy due to abnormal aircraft dynamics must be accounted for within the time to alert. This value should also be inflated for satellites which are incorporated prior to convergence of any smoothing filter.

In order to achieve a predictable and reproducible parameter that reflects the tracking accuracy of a receiver which meets the accuracy requirements of the MOPS [4] section 2.1.4.1, the minimum requirements representing the worst case signal reception conditions are used:

\[
\begin{align*}
\sigma_{i,\text{noise,GPS}} &= 0.4 \ m \quad (89) \\
\sigma_{i,\text{noise,SBAS}} &= 1.8 \ m \quad (90)
\end{align*}
\]

For airborne equipment which does not meet the accuracy requirements of the MOPS [4] section 2.1.4.1 for precision approach flight phases, the following value shall be used:

\[
\sigma^2_{\text{air,3}} = 25 \ m^2 \quad (91)
\]

The current implementation of PEGASUS assumes a receiver which meets the accuracy requirements of the MOPS [4] section 2.1.4.1 for the precision approach flight phases and thus a receiver where the contribution of the receiver noise and multipath effects to the residual pseudorange error can be represented by equations (88), (89) and (90). Furthermore, a steady-state navigation and normal aircraft dynamics are assumed. No satellite will be included in the position solution process until the range smoothing filter for that satellite has converged (and thus, no provision for additional contributions for that effect have been considered).
10.5.5 Bound on Tropospheric Residual Errors

For equipment which applies the SBAS tropospheric model as described in section 8, the bound on the remaining tropospheric error can be found using equation (53). For equipment which does not apply the SBAS tropospheric model, the equipment shall include a model of the residual error that overbound the rare tropospheric delay.

In the current PEGASUS development, the operator has the option to select the standard GPS tropospheric model or no tropospheric correction model at all when using the Non-Precision Approach mode or Precision Approach mode (obviously, the operator is notified about these atypical parameter settings).

In order to be able to compute protection levels using these parameter selections, the SBAS tropospheric error model in equation (53) is applied to compute $\sigma_{i,tropo}^2$ even when (a) applying the standard GPS tropospheric error model or when (b) applying no tropospheric error model at all.✓

10.6 Protection Levels using SBAS Corrections

Since the SBAS correction information is applied to each individual pseudorange in different ways (according to the criteria shown in the chapters before), the assumption on one single value for the standard deviation for all pseudoranges can not be applied. For the calculation of the protection levels using SBAS corrections, the general approach of the weighted least squares is used (see chapter 10.3).

The design matrix is determined using equation (69). The co-ordinate system used is a local tangent one, the x-axis pointing to the east, the y-axis pointing to the north and the vertical axis complementing the other two in the local vertical.

The weighting matrix is given by equation (77). For modes other then precision approach, the weights are undefined. For an unweighted least squares solution, the weighting matrix is a unity diagonal matrix (i.e. the elements on the main diagonal are set to 1). Using a weighted least squares solution, the weights can be determined by the equation (81).

The projection matrix of the position solution is then calculated as:

$$ S = \left(H^T W H\right)^{-1} H^T W $$

(92)

The variances of the model distribution that overbounds the true error distribution in each direction of the local tangent co-ordinate system are calculated as:

$$ d_x^2 = \sum_{i=1}^{N} s_{x,i}^2 \sigma_{i}^2 $$

(93)

$$ d_y^2 = \sum_{i=1}^{N} s_{y,i}^2 \sigma_{i}^2 $$

(94)

$$ d_{xy}^2 = \sum_{i=1}^{N} s_{x,i} s_{y,i} \sigma_{i}^2 $$

(95)

$$ d_z^2 = \sum_{i=1}^{N} s_{z,i}^2 \sigma_{i}^2 $$

(96)
In the horizontal plane, the error distribution will result in an error ellipse. The principal axes of this error ellipse might not be coincident with the north- and east-directions (due to the covariance in north-east direction, see figure 23).

The intent is to determine the semi-major axis of that error ellipse and use that error variance in order to determine a protection level. The error covariance in the horizontal plane is:

\[
P = \begin{bmatrix} d_x^2 & d_{xy} \\ d_{xy} & d_y^2 \end{bmatrix}
\]

Mathematically, an eigenvalue problem of that two-dimensional matrix must be solved. A necessary condition to determine the eigenvalues is:

\[
\det(P - \lambda I) = 0
\]

or:

\[
\det\begin{bmatrix} d_x^2 - \lambda & d_{xy} \\ d_{xy} & d_y^2 - \lambda \end{bmatrix} = 0
\]

Thus:

\[
(d_x^2 - \lambda)(d_y^2 - \lambda) - d_{xy}^2 = 0
\]

If this equation is solved for the eigenvalues \(\lambda_{1,2}\), they are obtained to:

\[
\lambda_{1,2} = \frac{d_x^2 + d_y^2}{2} \pm \sqrt{(d_x^2 - d_y^2)^2 + 4d_{xy}^2}
\]

Taking the root of the larger eigenvalue, the standard deviation of the horizontal position solution in the semi-major axis of the error ellipse is determined.
The horizontal and the vertical protection level are calculated as:

\[
\sigma_{maj} = \sqrt{\lambda_1} = \sqrt{\frac{d_x^2 + d_y^2}{2}} + \sqrt{\left(\frac{d_x^2 - d_y^2}{2}\right)^2 + d_{cv}^2}
\]  

(97)

The horizontal and the vertical protection level are calculated as:

\[
VPL = K_v d_v
\]  

(98)

\[
HPL = K_h \sigma_{maj}
\]  

(99)

with \( K_v = 5.33 \) multiplication factor
\( K_h = 6.18 \) multiplication factor (en-route through non-precision approach)
\( K_h = 6.0 \) multiplication factor (precision approach)

For the precision approach mode, the values of \( K_h \) and \( K_v \) were selected to bound the user’s position in one dimension with a probability of \( 2 \times 10^{-9} \) and \( 10^{-7} \), respectively, assuming that the error is characterised by a Gaussian distribution (see figure 24). A Gaussian distribution is used because the aircraft needs to be protected in the vertical and the lateral axes. Only one dimension is used for the HPL, since the along-track tolerance is so much larger than the cross-track tolerance. The worst-case dimension is used. It has been assumed that there is only one independent sample per approach, that half of the total integrity requirement (i.e. \( 2 \times 10^{-7} \) per approach) has been allocated to the VPL bounding probability and that the HPL bounding probability has been made negligible.

For en-route through non-precision approach modes, the value of \( K_v \) was chosen to bound the user’s position in two dimensions with a probability of \( 5 \times 10^{-9} \) per independent sample, assuming that the error is characterised by a Rayleigh distribution. A Rayleigh distribution is used because the radial error needs to be bounded (both cross-track and along-track errors), using the worst case assumption that the semi-major and semi-minor axes are equal. It has been assumed that there are 10 independent samples per hour, and that half of the total integrity requirement (i.e. \( 10^{-7} \) per hour) has been allocated to the HPL bounding probability.
11 The Service Region Indication

11.1 General Approach to the Service Region Indication

The currently transmitted ESTB SIS V1.2 adheres to the standard DO 229B [4] except for some deviations, in particular the application of the service region indication using the message type MT 27. The implementation these corrections still follows the standard DO 229A [3]. It has been decided that the PEGASUS Project should be able to process the ESTB SIS according to its current specifications, in particular in the implementation of the service region determination scheme. The interested reader is additionally referred to the document [5], where the relevant changes for the ESTB CPF are identified.

In contrast to the original intent of the MOPS [3], the ESTB version 1.1 uses this message type only when it is in an "expansion mode", i.e. supports in addition to the required service region another area with ESTB applications. Further information on the "expansion mode" could not be obtained at the moment of updating this document, but it is expected that this information will be available in the future. The message type MT 27 contains SBAS service indications and UDRE increments for a specified region. There are two applications for these parameters:

- The SBAS service indications shall specify the class of service (i.e. en-route, terminal area, non-precision approach or precision approach) provided in designated regions where the full SBAS service (i.e. the precision approach capability) is not provided.
- Increments for the UDRE will be used to increase the model variance defined by the UDREI parameters (see chapter 5) provided in the message types MT 2-5, MT 6 and MT 24 for designated regions where the full SBAS service is provided. These increments will account for local phenomena, such as anomalous ionospheric or tropospheric conditions or interference, that can not be modelled in the corrections messages.

11.2 Message Types and Format

The service region indication and UDRE increments transmitted by the message type MT 27 contains 21-bit region indications (11 bits for longitude and 10 bits for latitude with a resolution of $\frac{1}{4}$° degree), SBAS service indications and / or UDRE increment indicators and region radius indicators. The format of the message type MT 27 is provided in the table 32.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>for each of 7 regions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>region latitude</td>
<td>0</td>
<td>10</td>
<td>0.25</td>
<td>± 90</td>
<td>degrees</td>
</tr>
<tr>
<td>region longitude</td>
<td>10</td>
<td>11</td>
<td>0.25</td>
<td>± 180</td>
<td>degrees</td>
</tr>
<tr>
<td>region radius indicator</td>
<td>21</td>
<td>3</td>
<td>1</td>
<td>0 .. 7</td>
<td>N/A</td>
</tr>
<tr>
<td>SBAS service indication</td>
<td>24</td>
<td>4</td>
<td>1</td>
<td>0 .. 15</td>
<td></td>
</tr>
<tr>
<td>UDRE increment indication</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of service messages</td>
<td>196</td>
<td>4</td>
<td>1</td>
<td>1 .. 15</td>
<td>N/A</td>
</tr>
<tr>
<td>Issue of Data IODS</td>
<td>200</td>
<td>3</td>
<td>1</td>
<td>0 .. 7</td>
<td>N/A</td>
</tr>
<tr>
<td>spare</td>
<td>203</td>
<td>9</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 32: Fast Correction Message Types MT 2-5
The four bit SBAS service indication / UDRE increment indication will be used by the user to either determine the service provided or to increase the $\sigma_{UDRE}^2$ for that region. The Service Issue Of Data IODS and the number of service messages for that IODS is provided in each message to aid the user in defining his data base.

The SBAS service indication and the UDRE increment indication can be translated to the SBAS service and the UDRE increments with the use of table 33.

<table>
<thead>
<tr>
<th>SBAS Service Indication</th>
<th>UDRE Increments Indication</th>
<th>SBAS Service</th>
<th>UDRE Increments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0 m (clears any previously provided indicator)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.1 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.3 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.4 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.5 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.6 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.8 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.0 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1.5 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.5 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>5.0 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>10.0 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>SBAS supports en-route, terminal area and non-precision approach only</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>SBAS supports en-route, terminal area only</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>SBAS performance not guaranteed for any class of services</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 33: SBAS Service Indication and UDRE Increments**

The region radius indicator will be translated to the region radius in nautical miles with the use of table 34, which defines the boundary about the region centre in which the SBAS service or the UDRE increments are applicable.

<table>
<thead>
<tr>
<th>Region Radius Indicator</th>
<th>Region Radius</th>
<th>Region Radius Indicator</th>
<th>Region Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50 nm</td>
<td>4</td>
<td>400 nm</td>
</tr>
<tr>
<td>1</td>
<td>100 nm</td>
<td>5</td>
<td>800 nm</td>
</tr>
<tr>
<td>2</td>
<td>200 nm</td>
<td>6</td>
<td>1600 nm</td>
</tr>
<tr>
<td>3</td>
<td>300 nm</td>
<td>7</td>
<td>5400 nm</td>
</tr>
</tbody>
</table>

**Table 34: Region Radius Indicator**
There is no time-out interval for the message type MT 27, as is shown in table. Thus, a message type MT 27 shall apply until a subsequent message type MT 27 is received or until broadcast data from the associated service provider is not used.

<table>
<thead>
<tr>
<th>message type</th>
<th>all flight phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 27</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 35: Time-Out Intervals for MT 27

11.3 Service Region Processing

The user must examine the information contained in message type MT 27 to determine whether he is within the region of applicability. The equipment shall not use the SBAS corrections and integrity information in a particular mode, if a message type MT 27 indicates that the navigation mode is not supported by the SBAS service. This does not preclude the use of this data in other modes.

The region designation will indicate the centre of the region. The region radius is the length of an arc on a great circle passing through the centre of the region. The region is that which is enclosed by the union of that circle and the 5° mask footprint of the broadcasting SBAS satellite, which has a radius of 8497 nm. Multiple circles may be required to make up an overall region. Since regions could overlap, the user will use the best service indicated or the smallest UDRE increment of overlapping regions. Thus, the indirect geodetic problem has to be solved for the user location. The solution to that problem formulation is specified in the MOPS [3] and given in the following section.

In areas where no region is defined, the full SBAS service is provided. If the user is not in a region identified by a message type MT 27 (including if not message type MT 27 has been received), the full SBAS service is provided.

11.4 Indirect Geodetic Problem

This section provides an example algorithm for solving the inverse problem of geodesy or the indirect geodetic problem.

"Given the geodetic latitude and longitude of two points on the WGS84 ellipsoid, find the range (i.e. the distance) and the bearing of the shortest path between them. This path is called a geodesic."

The corresponding problem on the surface of a sphere has a well-known and elementary solution. On a non-spherical ellipsoid, the problem is considerably different. It is therefore as well inappropriate to refer to geodesics on a non-spherical ellipsoid as "great circles". The geodesic can, in general, be computed to an arbitrary level of accuracy only iteratively. The algorithm presented here (see [22] for more information) will provide an accuracy within fractions of a millimetre, if the termination criterion is selected appropriately.

The inputs to the solution of the indirect geodetic problem are:

\[
\begin{align*}
\Phi_1 & \quad \text{departure geodetic latitude (in radians)} \\
L_1 & \quad \text{departure geodetic longitude (in radians)} \\
\Phi_2 & \quad \text{arrival geodetic latitude (in radians)} \\
L_2 & \quad \text{arrival geodetic longitude (in radians)}
\end{align*}
\]
The latitude and longitude values for these two points have to be used in radians. Thus, if not already in the correct dimension, these will have to be converted. Next, the difference in longitude and the reduced latitudes are calculated:

\[ \Delta L = (L_2 - L_1) \]  
\[ \beta_1 = \tan^{-1} \left[ (1 - f) \tan(\Theta_1) \right] \]  
\[ \beta_2 = \tan^{-1} \left[ (1 - f) \tan(\Theta_2) \right] \]

with:  
\( \Delta L \) difference in longitude  
\( \beta_1 \) reduced departure latitude  
\( \beta_2 \) reduced arrival latitude  
\( f \) WGS84 flattening (= 3.35281066474 \( \times \) \( 10^{-3} \))

The iteration is initialised with the following value:

\[ \lambda_0 = \Delta L \]

The next couple of equations form the iterative procedure of this algorithm.

\[ \sin \sigma = \sqrt{\left( \cos \beta_2 \sin \lambda_k \right)^2 + \left( \cos \beta_1 \sin \beta_2 - \sin \beta_1 \cos \beta_2 \cos \lambda_1 \right)^2} \]
\[ \cos \sigma = \sin \beta_1 \sin \beta_2 + \cos \beta_1 \cos \beta_2 \cos \lambda_k \]
\[ \sigma = \arctan2(\sin \sigma, \cos \sigma) \]
\[ \sin \alpha_e = \frac{\cos \beta_1 \cos \beta_2 \sin \lambda_k}{\sin \sigma} \]
\[ \cos^2 \alpha_e = 1 - \sin^2 \alpha_e \]
\[ \cos(2\sigma_m) = \begin{cases} \cos \sigma - \frac{2 \sin \beta_1 \sin \beta_2}{\cos^2 \alpha_e} & \cos^2 \alpha_e \neq 0 \\ 0 & \cos^2 \alpha_e = 0 \end{cases} \]
\[ C = \frac{f}{16} \cos^2 \alpha_e \left[ 4 + f \left( 4 - 3 \cos^2 \alpha_e \right) \right] \]
\[ \lambda_{k+1} = \Delta L + (1 - C) f \sin \alpha_e \left\{ \sigma + C \sin \sigma \left[ \cos(2\sigma_m) \cdots \right] \right\} + C \cos \sigma \left[ -1 + 2 \cos^2(2\sigma_m) \right] \]

where the function “atan2” has the following properties:
The iteration is terminated, if the following criterion is met:

\[ |\lambda_{k+1} - \lambda_k| < \varepsilon = 10^{-12} \]  

(112)

After the iterative part of the algorithm is terminated, the following values have to be determined:

\[ u^2 = (e')^2 \cos^2(\alpha) \]  

(113)

\[ A = 1 + \frac{u^2}{16384} \{4096 + u^2 \left[ -768 + u^2 \left( 320 - 175 u^2 \right) \right] \} \]  

(114)

\[ B = \frac{u^2}{1024} \{256 + u^2 \left[ -128 + u^2 \left( 74 - 47 u^2 \right) \right] \} \]  

(115)

\[ \Delta \sigma = B \sin \sigma \left\{ \cos(2\sigma_m) + \frac{1}{4} B \left[ \left. \left. \begin{array}{c} -1 + 2 \cos^2(2\sigma_m) \cos \sigma \cos \Delta \sigma \\ -3 + 4 \sin^2 \sigma \cos(2\sigma_m) \cos(2\sigma_m) \end{array} \right] \right. \right. \right. \} \]  

(116)

with: \( (e')^2 \) square of WGS84 second eccentricity (\( = 6.7394967427 \times 10^{-3} \))

The range \( s \) and the bearings \( \alpha_1 \) and \( \alpha_2 \) at the departure point and arrival point, respectively, may now be computed as follows:

\[ s = b A \left( \sigma - \Delta \sigma \right) \]  

(117)

\[ \alpha_1 = \text{atan2} \left( \cos \beta_2 \sin \lambda_{k+1} \right) \]  

(118)

\[ \alpha_2 = \text{atan2} \left( \cos \beta_1 \sin \lambda_{k+1} \right) \]  

(119)

with \( b \) WGS84 semi-minor axis (\( = 6356752.3142 \) m)

\( s \) distance between two locations on WGS84 (in meter)

\( \alpha_1 \) bearing between departure point and arrival point at the departure point (in radians)

\( \alpha_2 \) bearing between departure point and arrival point at the arrival point (in radians)

\[ \text{It should be noted that the above proposed algorithm enters in an endless loop if the start point and the end point are exactly identical. This special case should be considered before the iteration loop above is entered.} \]
SBAS Ranging
12 The SBAS Raw Data

The use of the SBAS ranging data in the position calculation process described in chapter 10 requires that range measurements from the SBAS satellite are available and that the SBAS satellite position can be calculated. The range measurements of the SBAS is described in this chapter, while the calculation of the SBAS satellite position is provided in the next chapter 13 using the ephemeris and in chapter 14 using the almanac.

The user equipment must determine the pseudorange to each SBAS satellite that is currently being used in the position computation. These pseudoranges must be referenced (by the receiver) to the same time base as that of the GPS satellites. The GNSS receiver with pseudorange measurement output provides the raw ranging data of the SBAS satellites in the same way as the GPS satellites. The raw ranging data contains the measured code pseudorange, the accumulated carrier phase cycles and the Doppler frequency, which can be handled just like the GPS measurements.

The SBAS satellite uses an own time scale, the SBAS Network Time SNT. Thus, the SBAS pseudorange is transmitted by the satellite which uses the SNT time scale and received by a GNSS receiver which must reference the pseudoranges to the GPS time scale. The system provider must assure that the SBAS Network Time scale will be maintained within 50 nanoseconds of the GPS system time. Estimates of the time difference between the SNT and the UTC will be provided in an appropriate message type (message type 12), the format of which is shown in the following table 36.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_WNT$</td>
<td>0</td>
<td>24</td>
<td>$2^{30}$</td>
<td>±7.45 $10^9$</td>
<td>s/s-</td>
</tr>
<tr>
<td>$A_WNT$</td>
<td>24</td>
<td>32</td>
<td>$2^{30}$</td>
<td>± 1</td>
<td>s</td>
</tr>
<tr>
<td>$T_0$</td>
<td>56</td>
<td>8</td>
<td>$2^{12}$</td>
<td>602112</td>
<td>s</td>
</tr>
<tr>
<td>$WN_t$</td>
<td>64</td>
<td>8</td>
<td>1</td>
<td>256</td>
<td>N/A</td>
</tr>
<tr>
<td>$\Delta_t$</td>
<td>72</td>
<td>8</td>
<td>1</td>
<td>± 127</td>
<td>s</td>
</tr>
<tr>
<td>$WN_{LSF}$</td>
<td>80</td>
<td>8</td>
<td>1</td>
<td>256</td>
<td>N/A</td>
</tr>
<tr>
<td>$DN$</td>
<td>88</td>
<td>8</td>
<td>1</td>
<td>7</td>
<td>N/A</td>
</tr>
<tr>
<td>$\Delta T_{LSF}$</td>
<td>96</td>
<td>8</td>
<td>1</td>
<td>± 127</td>
<td>s</td>
</tr>
<tr>
<td>UTC ID</td>
<td>134</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>N/A</td>
</tr>
<tr>
<td>GPS TOW</td>
<td>104</td>
<td>20</td>
<td>1</td>
<td>86399</td>
<td>s</td>
</tr>
<tr>
<td>GPS WN</td>
<td>124</td>
<td>10</td>
<td>1</td>
<td>1024</td>
<td>N/A</td>
</tr>
<tr>
<td>GLONASS indicator</td>
<td>134</td>
<td>1</td>
<td>1</td>
<td>0 .. 1</td>
<td>N/A</td>
</tr>
<tr>
<td>GLONASS time offset (reserved)</td>
<td>135</td>
<td>74</td>
<td>tbd.</td>
<td>tbd.</td>
<td>tbd.</td>
</tr>
</tbody>
</table>

Table 36: Message Type MT 12 (SBAS time transformation)

The definition of these parameters and the applicable algorithms are in section 2.4.5.5 and 2.5.6 of [14], with the exception that the UTC parameters will correlate the UTC time with the SBAS Network Time SNT. The UTC standard used is indicated by the UTC ID and is given in table 37. Therefore, it must be noted that, if the corrections transmitted by the SBAS satellite are not used, the position solution will be with respect to a composite GPS/SNT time, and the resulting accuracy will be affected by the difference between those two time scales. Only if the corrections transmitted by the SBAS satellite are used, the user’s solution for time will be with respect for the SNT time scale.
According to [14], the raw pseudorange measurements have to be corrected before they can be used in the position calculation algorithms. There are several corrections (see chapter 9) - among others there is the correction for satellite clock errors. The correction for this satellite clock error for the SBAS range measurements is slightly different (as compared with the GPS range measurements). The SBAS correction can be found in chapter 12.

<table>
<thead>
<tr>
<th>UTC ID</th>
<th>UTC standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>UTC as operated by the Communications Research Laboratory, Tokyo, Japan</td>
</tr>
<tr>
<td>1</td>
<td>UTC as operated by the National Institute of Standards and Technology</td>
</tr>
<tr>
<td>2</td>
<td>UTC as operated by the US Naval Observatory</td>
</tr>
<tr>
<td>3</td>
<td>UTC as operated by the International Bureau of Weights and Measures</td>
</tr>
<tr>
<td>4-6</td>
<td>reserved for future uses</td>
</tr>
<tr>
<td>7</td>
<td>UTC not provided</td>
</tr>
</tbody>
</table>

Table 37: UTC Standard for MT 12

The relevant time-out intervals for the MT 12 data is given in the table 38.

<table>
<thead>
<tr>
<th>message type</th>
<th>Flight Phases En-Route, Terminal and Non-Precision Approach</th>
<th>Flight Phase Precision Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 12</td>
<td>86400</td>
<td>86400</td>
</tr>
</tbody>
</table>

Table 38: Time-Out Intervals for MT 12

The current SIS V1.2 of the ESTB transmits the message type MT 0 on a regular basis, due to the fact that the SIS can not and shall not be used for any official positioning service. Receiver software which is designed to the MOPS [4] thus shall not be able to use the ESTB pseudorange measurement for ranging purposes. In order to use the ESTB SIS for the PEGASUS development, the result of the message type MT 0 is disregarded. Nevertheless, the message type MT 0 will be read in and processed. Once the EGNOS transmits the message type MT 0 for its original purpose, the message type MT 0 can be easily re-activated in the PEGASUS development.
13 The SBAS Ephemeris

13.1 Message Type and Formats

The ephemeris of the SBAS satellites are – compared to the ephemeris data set of the GPS satellites - quite different in content. Instead of KEPLER elements, the ephemeris data of the SBAS satellites consist of the ECEF position co-ordinates and their derivatives up to the second order (i.e. satellite velocity and satellite acceleration). The change of position in time is calculated with the use of an second-order polynomial. This relatively simple model of the SBAS satellite’s position is due to the fact that the SBAS satellites are geostationary satellites, i.e. the change in position with respect to their nominal position should be very small.

The content of the Navigation Message MT 9 (i.e. the SBAS ephemeris) is shown in the table 39. It consists of the position, velocity and acceleration of the geostationary satellite and its apparent clock offset and drift with respect to the SBAS network time. Also included is the time of applicability, an Issue Of Data IOD and an accuracy exponent URA representing the estimated accuracy of the message.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Issue of Data</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>0...250</td>
<td>-</td>
</tr>
<tr>
<td>time of applicability I₀</td>
<td>8</td>
<td>13</td>
<td>16</td>
<td>0...86384</td>
<td>s</td>
</tr>
<tr>
<td>accuracy URA</td>
<td>21</td>
<td>4</td>
<td>1</td>
<td>0...15</td>
<td>-</td>
</tr>
<tr>
<td>X₆ (ECEF)</td>
<td>25</td>
<td>30</td>
<td>0.08</td>
<td>42949673</td>
<td>m</td>
</tr>
<tr>
<td>Y₆</td>
<td>55</td>
<td>30</td>
<td>0.08</td>
<td>42949673</td>
<td>m</td>
</tr>
<tr>
<td>Z₆</td>
<td>85</td>
<td>25</td>
<td>0.4</td>
<td>6710886.4</td>
<td>m</td>
</tr>
<tr>
<td>X₆ velocity</td>
<td>110</td>
<td>17</td>
<td>0.000625</td>
<td>40.96</td>
<td>m/s</td>
</tr>
<tr>
<td>Y₆ velocity</td>
<td>127</td>
<td>17</td>
<td>0.000625</td>
<td>40.96</td>
<td>m/s</td>
</tr>
<tr>
<td>Z₆ velocity</td>
<td>144</td>
<td>18</td>
<td>0.004</td>
<td>524.288</td>
<td>m/s</td>
</tr>
<tr>
<td>X₆ acceleration</td>
<td>162</td>
<td>10</td>
<td>0.0000125</td>
<td>0.0064</td>
<td>m/s²</td>
</tr>
<tr>
<td>Y₆ acceleration</td>
<td>172</td>
<td>10</td>
<td>0.0000125</td>
<td>0.0064</td>
<td>m/s²</td>
</tr>
<tr>
<td>Z₆ acceleration</td>
<td>182</td>
<td>10</td>
<td>0.0000625</td>
<td>0.032</td>
<td>m/s²</td>
</tr>
<tr>
<td>clock bias aₒf₀</td>
<td>192</td>
<td>12</td>
<td>2³² (≈2.3×10⁻¹⁰)</td>
<td>0.9537×10⁻⁶</td>
<td>s</td>
</tr>
<tr>
<td>clock bias drift aₒf₁</td>
<td>204</td>
<td>8</td>
<td>2⁴⁰ (≈9.1×10⁻¹³)</td>
<td>1.1642×10⁻¹⁰</td>
<td>s/s</td>
</tr>
</tbody>
</table>

Table 39: Message Type MT 9 (SBAS ephemeris)

The User Range Accuracy URA parameter can also be found in the GPS ephemeris data. For the interpretation of that parameter, see section 2.5.3 of [14].

The SBAS ephemeris data also contain the time of applicability (i.e. the starting point for the polynomial interpolation) and an issue of data IOD. For the purpose of timeout calculation, the data set that contains the SBAS ephemeris must include, as well, the time of reception of the message type MT 9. For the PEGASUS implementation, a new equivalent RINEX ephemeris format for the SBAS ephemeris data - not unlike the GLONASS RINEX2 ephemeris – has been proposed (for more information, please refer to [8]).


The time-out interval for the message type MT 9 is depending on the flight phase and is shown in table 40. It should be noted that the time-out interval starts from the time of message reception, and not, as it might be assumed, from the time of applicability.

<table>
<thead>
<tr>
<th>message type</th>
<th>Flight Phases En-Route, Terminal and Non-Precision Approach</th>
<th>Flight Phase Precision Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 9</td>
<td>360 seconds</td>
<td>240 seconds</td>
</tr>
</tbody>
</table>

Table 40: Time-Out Intervals for MT 9

13.2 Satellite Position Calculation

Due to the geostationary orbit, the SBAS satellites move much slower than the GPS satellites. Furthermore, the update rate of the SBAS ephemeris data is set to 2 minutes. Thus, a very simple and straightforward approach to the calculation of the satellite position in ECEF co-ordinates can be used:

\[
\begin{bmatrix}
X_{Gk} \\
Y_{Gk} \\
Z_{Gk}
\end{bmatrix}
= \begin{bmatrix}
X_G \\
Y_G \\
Z_G
\end{bmatrix}
+ \begin{bmatrix}
\dot{X}_G \\
\dot{Y}_G \\
\dot{Z}_G
\end{bmatrix}
(t_k - t_0)
+ \frac{1}{2} \begin{bmatrix}
\ddot{X}_G \\
\ddot{Y}_G \\
\ddot{Z}_G
\end{bmatrix}
(t_k - t_0)^2
\]

(120)

with

- \(X_{Gk}, Y_{Gk}, Z_{Gk}\) satellite position at time \(t_k\)
- \(X_G, Y_G, Z_G\) satellite position at time \(t_0\)
- \(\dot{X}_G, \dot{Y}_G, \dot{Z}_G\) satellite velocity at time \(t_0\)
- \(\ddot{X}_G, \ddot{Y}_G, \ddot{Z}_G\) satellite acceleration at time \(t_0\)
- \(t_k\) time of day, corrected for day cross-over
- \(t_0\) time of applicability

It should be noted that the time scale for the calculation of the satellite position is based on the “time of the day” scale – and not on the “time of the week” scale of the GNSS satellites. Thus, an appropriate correction term must be added or subtracted.

The position co-ordinates can then be used in the design matrix calculations in the same way as the GPS satellite co-ordinates, as is shown in chapter 10. Since the message type MT 9 will be updated very frequently (each 2 minutes), there are no slow corrections transmitted for them, as already mentioned in chapter 6.

13.3 Satellite Clock Error Calculations

As with the GPS satellites, the clock running in an SBAS satellite is affected by several clock errors. In combination with the SBAS satellite position data, the message type MT 9 contains two parameters, which allow the modelling of the clock error. A relatively simple and straightforward approach of a first-order polynomial is used.

\[
\Delta t_c(t_k) = a_{G/0} + a_{G/1} (t_k - t_0)
\]

(121)
with $\Delta t_G(t_k)$ correction term to satellite clock
$\alpha_{G/0}, \alpha_{G/1}$ clock correction parameters
$t_k$ time of day, corrected for day cross-over
$t_0$ time of applicability

In contrast to the correction approach of the GPS satellite clock error, there are no relativistic effect and no correction for the group delay for the SBAS satellite. Any relativistic effects will be removed by the system provider.

The corrected satellite time $t_G$ is the time associated with the reception time of the raw range measurements. Practical use can be found for the correction term $\Delta t_G$. While the GPS ranges are corrected for the satellite clock error $\Delta t_{SV}$, (see the relevant chapter in [14]) the SBAS pseudoranges are corrected for the $\Delta t_G$ according to:

$$PR_{SBAS,v} = PR_{SBAS} + c \Delta t_G(t_k)$$  \hspace{1cm} (122)

with $PR_{SBAS,v}$ SBAS pseudorange corrected for satellite clock error
$PR_{SBAS}$ SBAS measured pseudorange
$c$ speed of light

This modified pseudorange can then be used in the position calculation process just as any other GPS ranging measurement (see chapter 9 and 10).
14 The SBAS Almanac

14.1 Message Type and Formats

Almanacs for all geostationary satellites will be broadcast periodically to alert the user of their existence, location, the general service provided and their health status. The almanacs will be broadcast in the message type MT 17. These messages will be repeated to include all geostationary satellites. Unused almanacs will have a PRN of zero and should be ignored.

The almanac of the SBAS satellites are – compared to the almanac data set of the GPS satellites - quite different in content. Instead of KEPLER elements, the almanac data of the SBAS satellites consist of the ECEF position co-ordinates and their derivatives up to the first order (i.e. satellite velocity). The change of position in time is calculated with the use of an first-order polynomial. This relatively simple model of the SBAS satellite’s position is due to the fact that the SBAS satellites are geostationary satellites, i.e. the change in position with respect to their nominal position should be very small.

The content of the Navigation Message MT 17 (i.e. the SBAS almanac) is shown in the table 41. It consists of the position and the velocity of up to three geostationary satellite. The relevant time-out intervals for the MT 17 data is given in the table 42.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bit Position</th>
<th>No. of Bits</th>
<th>Scale Factor</th>
<th>Range</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>for 3 satellites</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data ID</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td>N/A</td>
</tr>
<tr>
<td>PRN number</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>210</td>
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</tr>
<tr>
<td>Health and Status</td>
<td>10</td>
<td>8</td>
<td></td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>X₀</td>
<td>18</td>
<td>15</td>
<td>2600</td>
<td>± 42595800</td>
<td>m</td>
</tr>
<tr>
<td>Y₀</td>
<td>33</td>
<td>15</td>
<td>2600</td>
<td>± 42595800</td>
<td>m</td>
</tr>
<tr>
<td>Z₀</td>
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<td>9</td>
<td>26000</td>
<td>± 6630000</td>
<td>m</td>
</tr>
<tr>
<td>X₀ velocity</td>
<td>57</td>
<td>3</td>
<td>10</td>
<td>± 40</td>
<td>m/s</td>
</tr>
<tr>
<td>Y₀ velocity</td>
<td>60</td>
<td>3</td>
<td>10</td>
<td>± 40</td>
<td>m/s</td>
</tr>
<tr>
<td>Z₀ velocity</td>
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<td>4</td>
<td>60</td>
<td>± 480</td>
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<tr>
<td>T₀</td>
<td>201</td>
<td>11</td>
<td>64</td>
<td>86336</td>
<td>s</td>
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</table>

Table 41: Message Type MT 17 (SBAS almanac)

<table>
<thead>
<tr>
<th>message type</th>
<th>Flight Phases En-Route, Terminal and Non-Precision Approach</th>
<th>Flight Phase Precision Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 17</td>
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<td>N/A</td>
</tr>
</tbody>
</table>

Table 42: Time-Out Intervals for MT 17

The Data ID for the current Signal Specification format is ‘00’. Other states of the Data ID are reserved for the possibility of future Signal Specification formats. The Health and Status parameter are defined as follows (table 43):
14.2 Satellite Position Calculation

Due to the geostationary orbit, the SBAS satellites move much slower than the GPS satellites. Thus, a very simple and straightforward approach to the calculation of the satellite position in ECEF co-ordinates can be used:

\[
\begin{bmatrix}
X_{Gk} \\
Y_{Gk} \\
Z_{Gk}
\end{bmatrix} = \begin{bmatrix}
X_G \\
Y_G \\
Z_G
\end{bmatrix} + \begin{bmatrix}
\dot{X}_G \\
\dot{Y}_G \\
\dot{Z}_G
\end{bmatrix} (t_k - t_0)
\]

(123)

with
- \(X_{Gk}, Y_{Gk}, Z_{Gk}\): satellite position at time \(t_k\)
- \(X_G, Y_G, Z_G\): satellite position at time \(t_0\)
- \(\dot{X}_G, \dot{Y}_G, \dot{Z}_G\): satellite velocity at time \(t_0\)
- \(t_k\): time of day, corrected for day cross-over
- \(t_0\): time of applicability

It should be noted that the time scale for the calculation of the satellite position is based on the “time of the day” scale – and not on the “time of the week” scale of the GNSS satellites. Thus, an appropriate correction term must be added or subtracted.
<table>
<thead>
<tr>
<th>Project:</th>
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<tbody>
<tr>
<td>Doc. No.:</td>
<td>PEG-TN-SBAS</td>
</tr>
<tr>
<td>Issue:</td>
<td>I</td>
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<tr>
<td>Date:</td>
<td>17/06/2003</td>
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<tr>
<td>Sheet</td>
<td>96 of 96</td>
</tr>
</tbody>
</table>

Technical Notes on SBAS

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