SEPARATION AND AIRSPACE SAFETY PANEL (SASP)
MEETING OF THE WORKING GROUP OF THE WHOLE
TWENTIETH MEETING
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Agenda Item 2: En-route separation minima and procedures - horizontal

The Longitudinal Reich Collision Risk Model

(Presented by Mr. Harry Daly)

(Prepared by Ms. Jacky Civil)

SUMMARY

The work carried out by UK NATS Operational Analysis (OA) department provides input to the safety assessment for the North Atlantic (NAT) Reduced Longitudinal Separation Minimum (RLongSM) project by providing a collision risk estimation based solely on theoretical collision risk modelling. This paper describes the derivation of the general Reich collision risk model and the modifications required for the specific longitudinal form of the model.

1 Introduction

The Reich Collision Risk Model (CRM) is approved by ICAO for the purpose of collision risk modelling and is well known by practitioners in the field. However, it is helpful to consider the derivation of the model to allow a complete understanding of each of the parameters. This is particularly true in the case of the longitudinal form of the model, for which it is helpful to understand the origin of each component.
2 The Reich CRM

2.1 The Conceptual Framework

The Reich model assumes there are two aircraft of identical size and orientation within an unconstrained space. The aircraft are represented as boxes with dimensions equal to that of aircraft dimensions such that the aircraft are fully contained within the boxes. The dimensions are as follows (demonstrated in Figure 1):

\[
\lambda_x = \text{average aircraft length};
\]
\[
\lambda_y = \text{average aircraft width};
\]
\[
\lambda_z = \text{average aircraft height}.
\]

Figure 1: The Box Representation of Aircraft

The aircraft are assumed to be moving randomly through the space independently of each other, although maintaining their orientation throughout. Occasionally, the motion will cause one of the aircraft to “pass through” the other one. Each occurrence of this where any part of one box passes through the other is considered to be a single collision. The model assumes local smoothness of motion such that the relative velocity of the aircraft will be effectively constant during the collision.

2.2 Collision Frequency

As the aircraft are represented by boxes, there are three distinct ways in which a collision can occur, laterally, vertically and longitudinally (Figure 2). These are distinct events that cannot occur simultaneously; therefore the frequency of each type of collision can be calculated independently.
Consider a longitudinal collision. This requires that the two aircraft pass in the x direction while they are already overlapping in the y and z directions. Thus, the expected number of longitudinal collisions in an hour will be the expected number of occasions they pass in the x direction, multiplied by the probability that each passing occurs during vertical and lateral overlap.

Define:

\[ f_x = \text{number of times the aircraft pass in the x direction in one hour}; \]
\[ p_y = \text{probability the aircraft are in lateral overlap at any given time point}; \]
\[ p_z = \text{probability the aircraft are in vertical overlap at any given time point}. \]

Then,

\[ \text{Number of longitudinal collisions per hour} = f_x p_y p_z \]

And further,

\[ \text{Number of collisions per hour in any dimension} = f_x p_y p_z + f_y p_x p_z + f_z p_x p_y \]

Equation 2-1
2.3 Passing Frequency

We have defined $p_x$ as the probability that the aircraft are in longitudinal overlap at any given time point. As the motion of the aircraft is completely random, this is equivalent to the proportion of time spent in longitudinal overlap, and since we are considering a single hour, this is also equivalent to the total time (hours) spent in longitudinal overlap during the hour.

Define:

$t_x = \text{average time (hours) spent in longitudinal overlap during a single passing}$

Then,

$$p_x = f_x t_x$$

$$\Rightarrow f_x = \frac{p_x}{t_x} \quad \text{Equation 2-2}$$

The relative distance travelled between the two aircraft during a single longitudinal passing will be $2\lambda_x$. Define:

$|\dot{x}| = \text{average relative passing speed in the x direction.}$

Then,

$$t_x = \frac{2\lambda_x}{|\dot{x}|} \quad \text{Equation 2-3}$$

From Equation 2-2 and Equation 2-3,

$$f_x = p_x \frac{|\dot{x}|}{2\lambda_x} \quad \text{Equation 2-4}$$

2.4 Collision Risk

From Equation 2-1 and Equation 2-4,

$$\text{Number of collisions per hour in any dimension}$$

$$= p_x \frac{|\dot{x}|}{2\lambda_x} p_y p_z + p_y \frac{|\dot{y}|}{2\lambda_y} p_x p_z + p_z \frac{|\dot{z}|}{2\lambda_z} p_x p_y$$

$$= p_x p_y p_z \left( \frac{|\dot{x}|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right)$$
Each collision represents two fatal accidents, hence the collision risk in units of number of fatal accidents per flight hour ($N_a$) can be stated as:

$$N_a = 2p_xp_yp_z \left( \frac{|\dot{x}|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right)$$  \hspace{1cm} \text{Equation 2-5}

This is the general form of the Reich Collision Risk Model.

3 The Longitudinal Form

3.1 The Conceptual Framework

In the general Reich CRM, two aircraft were assumed to be moving randomly in the same space with no constraints other than local smoothness of motion. For the specific longitudinal form of the model, we assume two aircraft travelling in the same direction, nominally on the same track and flight level, with a non-negligible longitudinal separation. For a pair of aircraft the risk is assessed over a period of travel during which the aircraft are considered to be continuously at risk. This may be across the entire ocean (ignoring controller intervention), between waypoints, or during a fixed reporting period.

The model makes certain assumptions based on realistic aircraft behaviours:

- There is no risk that the following aircraft will pass the leader and collide with another aircraft ahead.
- As the relative speed must be high in order to permit a catch-up given a non-negligible longitudinal separation, assume that the relative longitudinal velocity is never negative (i.e. a longitudinal passing can only occur once for a given pair of aircraft.)

3.2 The Reich CRM

Define:

- $P_y(0) = \text{Probability of lateral overlap for aircraft nominally on the same track.}$
- $P_z(0) = \text{Probability of vertical overlap for aircraft nominally on the same track.}$
- $\Pi_x = \text{Probability of longitudinal overlap for aircraft nominally on the same track.}$
- $|\dot{y}_0| = \text{Average relative lateral speed for aircraft nominally on the same track.}$
- $|\dot{z}_0| = \text{Average relative vertical speed for aircraft nominally on the same track.}$
- $|\dot{x}(m)| = \text{Longitudinal closing speed for aircraft nominally on the same track with minimum separation } m. \text{ (See Section 3.3.2 for discussion)}$

Then, by Equation 2-5, the number of fatal accidents per flight hour for aircraft nominally on the same track with minimum longitudinal separation $m$ is:

$$N_{ax} = 2\Pi_xP_y(0)P_z(0) \left( \frac{|\dot{x}(m)|}{2\lambda_x} + \frac{|\dot{y}_0|}{2\lambda_y} + \frac{|\dot{z}_0|}{2\lambda_z} \right)$$  \hspace{1cm} \text{Equation 3-1}
The value $\Pi_x$ is intended to be analogous to the lateral and vertical probabilities $P_y(0)$ and $P_z(0)$ and is frequently referred to as the “probability of longitudinal overlap”. However, the analogy does not strictly hold and the terminology can be unhelpful and confusing. Instead, it is beneficial to use the alternative interpretation (Section 2.3) of the probability of overlap, i.e. it is the proportion of time spent in overlap.

3.3 **The Proportion of Time Spent in Longitudinal Overlap**

The proportion of time spent in longitudinal overlap during a single at-risk period of travel is the proportion of time spent in longitudinal overlap *given that* a longitudinal overlap event occurs, multiplied by the probability that a longitudinal overlap event occurs.

3.3.1 **The Probability that a Longitudinal Overlap Event Occurs**

Suppose two aircraft have a planned separation at the end of the at-risk period of $s$ minutes. Then, for a longitudinal overlap event to occur, the aircraft must lose $s$ or greater minutes of separation.

Define:

\[
E(s) = \text{the probability distribution of planned separations at the end of the at-risk period for all possible planned separations } s.
\]

\[
Q(s) = \text{the probability distribution of a loss of planned separation of } s \text{ or greater minutes by the end of the at-risk period.}
\]

Then, the probability that a longitudinal overlap event occurs between any pair of aircraft is:

\[
\sum \frac{s}{E(s)Q(s)}
\]  

*Equation 3-2*

This equation can use either summation or integral form, depending on whether $E(s)$ is specified as a discrete or continuous distribution. Please note that in some prior work (Reference 1) the notation for the two distributions is used the other way round, with $Q(s)$ denoting the distribution of planned separations, whereas this work has adopted the notation used in the NAT MNPS Risk Quick Reference Guide (Reference 2).

It should be emphasised that the definition of separation used in Equation 3-2 is that of the *planned or intended* separation at the *end* of the at-risk period, and not the initial separation of the aircraft pair at the start of the at-risk period. This differentiation is not important when using a theoretical model for the two distributions, since it is usual to assume that the two aircraft have identical speed and thus the two separation definitions would be identical. However, when using real aircraft transit data to derive the distributions, the possibility of planned catch-up or pull-away due to differing leader and follower speed must be accounted for. In this circumstance, if the initial separation were being used, the planned catch-up or pull-away would form part of the separation loss distribution, $Q(s)$, and the two distributions could not be considered independent (since, for example, a planned catch-up would not be permitted if the initial separation was small). Defining the separation as
planned separation allows the assumption that the two distributions are independent in all cases, and the joint probability distribution does not need to be considered.

### 3.3.2 The Proportion of Time in Longitudinal Overlap Given that it Occurs

To be conservative, the estimate for the proportion of time spent in longitudinal overlap given that a longitudinal overlap event occurs should be as large as is logically feasible. In Section 3.1, assumptions were made about the relative motion of the aircraft such that only a single longitudinal passing could occur during the at-risk period. Therefore, the greatest amount of time spent in overlap would occur when the relative longitudinal speed is the smallest possible that will still permit a catch-up to occur.

If we assume that the relative longitudinal speed is close to constant during the at-risk period (i.e. the following aircraft does not decelerate rapidly just prior to overtaking in order to pass the leader at a slow speed) then the speed necessary for a catch-up to occur can be defined as:

\[ |\dot{x}(m)| = \text{Longitudinal closing speed for aircraft nominally on the same track with minimum separation } m. \]

Note that this term is also used for the average longitudinal passing speed within the collision risk model (Equation 3-1).

If \(|\dot{x}(m)|\) is the longitudinal passing speed, then the total time for a single passing is:

\[
\frac{2\lambda_x}{|\dot{x}(m)|} \tag{Equation 3-2}
\]

If we define \(T\) as the duration of the at-risk period, then the proportion of time spent in longitudinal overlap given that a longitudinal overlap event occurs is:

\[
\frac{2\lambda_x}{|\dot{x}(m)|} \frac{1}{T} \tag{Equation 3-3}
\]

### 3.3.3 The Proportion of Time Spent in Longitudinal Overlap

From Equation 3-2 and Equation 3-3, the proportion of time spent in longitudinal overlap is:

\[
\Pi_x = \frac{2\lambda_x}{|\dot{x}(m)|} \frac{1}{T} \sum_s E(s)Q(s) \tag{Equation 3-4}
\]

Please note that the use of the notation \(m\) for the *minimum separation*, and \(s\) for the dummy variable representing all possible planned separations at the end of the at-risk period, differs from both the prior work (Reference 1) and the NAT MNPS Risk Quick Reference Guide (Reference 2), where both concepts are given the same notation (either \(t\) or \(m\)). Given analytical tractability, it would in fact be possible to bring the entire risk equation within the summation or integral and allow the longitudinal passing speed to vary based on the separation. However, in practice the longitudinal passing speed has always been set at the single, conservative value based on the
minimum longitudinal separation. Thus the common practice of using the same notation for both concepts within the equation merely promotes confusion and has not been replicated within this work.

3.4 The Longitudinal CRM

From Equation 3-1 and Equation 3-4, the longitudinal collision risk is:

$$N_{ax} = 2P_y(0)P_z(0)\left(\frac{|\ddot{x}(m)|}{2\lambda_x} + \frac{|\ddot{y}_0|}{2\lambda_y} + \frac{|\ddot{z}_0|}{2\lambda_z}\right) \frac{2\lambda_x}{|\ddot{x}(m)|} \frac{1}{T} \sum_s E(s)Q(s)$$

4 Parameter Values

Values of the CRM parameters that are currently adopted for use within the NAT are given in Table 4-1. These values are taken from the NAT MNPS Risk Quick Reference Guide (Reference 2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Origin/Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_x$</td>
<td>Average aircraft length</td>
<td>0.03108Nm</td>
<td>Adopted April 2006, NAT SPG/42</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>Average aircraft wing-span</td>
<td>0.02846Nm</td>
<td>Adopted April 2006, NAT SPG/42</td>
</tr>
<tr>
<td>$\lambda_z$</td>
<td>Average aircraft height (includes, incorrectly, the undercarriage)</td>
<td>0.00892Nm</td>
<td>Adopted April 2006, NAT SPG/42</td>
</tr>
<tr>
<td>$</td>
<td>\ddot{x}(m)</td>
<td>$</td>
<td>Longitudinal closing speed between two aircraft nominally on the same track with minimum separation $m$</td>
</tr>
<tr>
<td>$</td>
<td>\ddot{y}_0</td>
<td>$</td>
<td>Average absolute relative cross track speed for aircraft nominally on the same track</td>
</tr>
<tr>
<td>$</td>
<td>\ddot{z}_0</td>
<td>$</td>
<td>Average absolute relative vertical speed for an aircraft pair that have lost all vertical separation</td>
</tr>
<tr>
<td>$P_y(0)$</td>
<td>Probability that two aircraft which are on the same track are in lateral overlap</td>
<td>0.1172</td>
<td>Adopted April 2006, NAT SPG/42</td>
</tr>
<tr>
<td>$P_z(0)$</td>
<td>Probability that two aircraft which are nominally at the same level are in vertical overlap</td>
<td>0.48</td>
<td>Adopted June 2001, NAT SPG/37</td>
</tr>
<tr>
<td>$T$</td>
<td>Average at-risk period</td>
<td>-</td>
<td>For periodic reporting this value will represent the length of the reporting period in hours.</td>
</tr>
<tr>
<td>$E(s)$</td>
<td>Distribution of planned separations at end of at-risk period</td>
<td>-</td>
<td>To be derived. See SASP-WG/WHL/20 WP25</td>
</tr>
<tr>
<td>$Q(s)$</td>
<td>Distribution of loss of planned separation of $s$ or greater by the end of the at-risk period</td>
<td>-</td>
<td>To be derived. See SASP-WG/WHL/20 WP24</td>
</tr>
</tbody>
</table>
5 **Recommendations**

The meeting is invited to note the contents of the paper.

6 **References**