International Civil Aviation Organization

The Nineteenth Meeting of the Regional Airspace Safety Monitoring Advisory Group (RASMAG/19)

Pattaya, Thailand, 27-30 May 2014

Agenda Item 5: Airspace Safety Monitoring Activities/Requirements in the Asia/Pacific Region

RNP4 SAFETY ASSESSMENT

(Presented by BOBASMA/ INDIA)

SUMMARY

This paper presents the Pre-implementation safety assessment for the introduction of 30 NM Reduced Longitudinal Separation on four routes M300, N571, P570 & P574.

1. INTRODUCTION

1.1 In the third meeting of SAIOACG in February 2013, India presented a working paper on the Proposal to introduce 30 NM Longitudinal Separation in the Bay of Bengal Arabian Sea Indian Ocean (BOBASIO) airspace. India had proposed that states first introduce 30 NM longitudinal separation on the existing RNP routes in a phased manner and then progress to reducing the lateral separation to 30 NM. As a first step India expressed its readiness to implement 30 NM Longitudinal Separation between aircraft with FANS/1A data link capability on an opportunity basis on four routes N571, M300, P570 & P574.

2. DISCUSSION

2.1 The four routes M300, N571, P570 & P574 traverse the entire BOBASIO airspace in an east - west direction over an average distance of 2,050NM and an average flying time of 4 hours 30 minutes. These four routes are used by long haul aircrafts flying between airports in South East Asia and the Middle East & Europe and the distance flown across the Indian FIRs of Chennai & Mumbai accounts for a major portion of their flying time.

2.2 The Pre-implementation safety assessment has been conducted using the TSD of December 2013 of Chennai and Mumbai FIRs. **Table 1** provides the horizontal risk estimates for the four routes.

Estimated annual flying hours = 2,02,823 hours *(note: estimated hours based on Dec 2013 traffic sample data)*			
Risk	**Risk Estimation**	**TLS**	**Remarks**
Lateral Risk	**0.9010673 x 10^{-9}**	5.0 x 10^{-9}	**Below TLS**
Longitudinal Risk	**1.623786 x 10^{-9}**	5.0 x 10^{-9}	**Below TLS**

Table 1: Horizontal Risk Estimates on four routes

2.3 The Safety Assessment for 30 NM longitudinal separation is provided at **Attachment A**.

3. ACTION BY THE MEETING

1. The meeting is invited to:

a) note the information contained in this paper; and

b) discuss any relevant matters as appropriate.

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Pre-Implementation Safety Assessment
For introduction of 30NM RHS on RNP 10 Routes
M300, N571, P570 & P574

1. Lateral and Longitudinal Collision Risk Assessment

1.1. Traffic sample data from Chennai, and Mumbai FIRs for the month of December2013 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

- Records with Exit time less than Entry time were removed.
- Records with missing data on entry/exit points, entry/exit levels, and entry/exit times were removed.
- Records with flight level less than F290 were removed.
- Records whose entry/exit routes were inconsistent were removed.

11031 records that were retained after filtering were considered for the subsequent statistical analysis.

1.2. Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata and Mumbai FIRs, as summarized in Table 1.

YEAR	MONTH	FIR	Flights	LLD	LLE
2013	JANUARY	CHENNAI	4424	0	0
	FEBRUARY		4062	0	0
	MARCH		4278	0	0
	APRIL		4285	0	0
	MAY		4292	0	0
	JUNE		4406	0	0
	JULY		4527	0	0
	AUGUST		4483	0	0
	SEPTEMBER		4439	0	0
	OCTOBER		4556	0	0
	NOVEMBER		4586	0	0
	DECEMBER		4616	0	0
2013	JANUARY	MUMBAI	14025	0	0
	FEBRUARY		12480	0	0
	MARCH		13982	0	0
	APRIL		18638	0	0
	MAY		19461	0	0
	JUNE		18926	0	0
	JULY		18957	0	0
	AUGUST		14981	0	0
	SEPTEMBER		15040	0	0
	OCTOBER		14153	0	0

	NOVEMBER		13521	0	0
	DECEMBER		16025	0	0

YEAR	MONTH	FIR	Flights	LLD	LLE
2013	JANUARY	KOLKATA	2521	0	0
	FEBRUARY		2161	0	0
	MARCH		2408	0	0
	APRIL		2129	0	0
	MAY		1994	0	0
	JUNE		1913	0	0
	JULY		1878	0	0
	AUGUST		2064	0	0
	SEPTEMBER		2003	0	0
	OCTOBER		1942	0	0
	NOVEMBER		2248	0	0
	DECEMBER		2520	0	0

Table 1: Summary of reports of Gross Navigational Errors.

In Section 3.3 and 3.4 we discuss the risk assessment for the lateral and longitudinal direction.

3.1 Lateral Collision Risk Assessment

2.1. Lateral Collision Risk Model

In order to compute the level of safety for lateral deviations of operations on the BOBASIO region we use the Reich Lateral Collision Risk Model. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}\left(S_y\right)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{2\overline{|V|}}{2\lambda_x} + \frac{\overline{|\dot{y}\left(S_y\right)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] \right\}. \quad (1)$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.
The parameters in the equation (1) are defined as follows:

- Nay := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned 8_y NM lateral separation.
- S_y := Minimum planned lateral separation.
- λ_x := Average length of an aircraft flying in BOBASIO region.

- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λz := Average height of an aircraft flying on BOBASIO region.
- Py (Sy) := The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by Sy.
- Pz (0) := Probability that two aircraft assigned to same flight level are at same geometric height.
- Sx := Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- Ey (same) := Same direction lateral occupancy at same assigned flight level.
- Ey (opp) := Opposite direction lateral occupancy at same assigned flight level.
- $|\Delta V|$:= Average relative speed of two aircraft flying on parallel routes in same direction.
- $|V|$:= Average ground speed on an aircraft.
- $|\dot{y}\ (Sy)|$:= Average relative lateral speed of aircraft pair at loss of planned lateral separation of Sy.
- $|\dot{z}|$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, Pz (0) is the probability of vertical overlap; Py (Sy) is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x}E_y\,(\mathrm{same})$ and $\frac{\lambda_x}{S_x}E_y\,(\mathrm{opp})$ relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\overline{|\Delta V|}}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time $\frac{2\overline{|V|}}{2\lambda_x}$.

The model is based on the following hypothesis:

- All routes are parallel. [1]
- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.
- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2. Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.03055548 NM	Estimated from TSD (see Section 2.3).
λ_y	0.02836613 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008699019 NM	Estimated from TSD (see Section 2.3).
P_y (50)	2.520977× 10-8	Estimated using a mixture model (see Section 2.4).
P_z (0)	0.538	Conservative value used in previous safety assessments (s Section
S_x	30NM	Reduced longitudinal separation.
E_y (same)	0.06227963	Estimated from the TSD (see Section 2.6).
E_y (opp)	0	No opposite directional lateral occupancy at same assign flight level.
$\lvert\Delta V\rvert$	28knots	Value obtained from TSD (see Section 2.7).
$\lvert \dot{y}(50)\rvert$	75 knots	Conservative value taken from EMA Handbook (see Secti 2.8).
$\lvert\dot{z}\rvert$	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk Nay.

$$\mathbf{N_{ay} = 0.9010673 \times 10^{-9}}$$

2.3. Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

Type	Length	Wingspan	Height	Flights
B77W	73.9	64.8	18.5	3860
B738	39.2	34.4	12.57	3597
A320	37.57	34.1	11.76	2924
A332	58.8	60.3	17.4	2513
A333	63.6	60.3	16.85	1660

Type	Length	Wingspan	Height	Flights
B772	63.7	60.9	18.4	1490
A388	73	79.8	24.1	1150
B744	70.6	64.8	19.4	988
A321	44.51	34.1	11.76	898
B77L	63.7	64.8	18.3	553
A343	63.6	60.3	16.85	467
B773	73.9	60.9	18.4	367
B763	54.9	47.6	15.9	365
A319	33.84	34.1	11.76	356
A346	75.3	63.45	17.3	147
A306	54.1	44.84	16.54	139
MD11	61.2	51.7	17.6	126
A310	46.66	43.9	15.8	113
A345	67.9	63.45	17.1	62
B752	47.3	38.1	13.6	49
B737	33.6	34.3	12.6	36
GLF5	29.4	28.5	7.5	22
GLF4	26.9	23.7	7.4	21
CL60	20.85	19.6	6.3	20
F2TH	20.2	19.3	7.1	18
GLEX	30.3	26.9	7.6	17
B74S	56.3	59.6	20	16
GL5T	28.69	28.65	7.7	15
F900	20.2	19.3	7.6	14
A380	73	79.8	24.1	13
GALX	19	17.4	6.4	7
E135	26.3	20.2	6.7	7
B742	70.6	59.6	19.3	7
C17	53	51.8	16.8	5

Table 2: Dimensions of aircraft types with number of records in the TSD

2.4. Estimating Probability of Lateral Overlap: Py(Sy)

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by Sy, is denoted by Py (Sy) and is defined as

$$P_y(S_y) := \mathbf{P}\left(|S_y + Y_1 - Y_2| \le \lambda_y\right), \tag{2}$$

where Y1 and Y2 are assumed to be the lateral deviations of two aircraft which are nominally separated by Sy. We assume that Y1 and Y2 are identically distributed but statistically independent with a distribution Fy. We model Fy as mixture distribution having a core distribution Gy and a non-core distribution Hy. The core distribution Gy, represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision. The non-core distribution Hy, represents Gross Navigation Errors (GNE), that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)G_y(y) + \alpha H_y(y)\,. \qquad (3)$$

The mixing parameter α is the probability of a gross navigational error.

The core lateral deviation distribution Gy is modeled by a Double Exponential distribution with a parameter βy> 0 as the rate, that is, if Y1 ~ Gy then

$$\mathbf{P}\left(|Y_1| > y\right) = e^{-\beta_y y}\,,$$

Where y ≥ 0. In other words we assume that the core distribution has a density of the f form

$$g_y\left(y\right) = \frac{\beta_y}{2} e^{-\beta_y |y|}\,.$$

Finally the non-core distribution Hy is modeled by a “Separated Double Exponential” distribution with parameters μy> 0, representing the “separation and γy> 0 the rate parameter, that is, if Y2 ~ Hy then

$$\mathbf{P}\left(Y_2 > \mu_y + y\right) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$\mathbf{P}\left(Y_2 < -\mu_y - y\right) = \frac{1}{2} e^{\gamma_y y}\,,$$

where y ≥ 0. This really means that the non-core distribution H_y gives no mass in $[-\mu_y,\mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y.
The density of this distribution is given by

$$h_y\left(y\right) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y (y+\mu_y)} & \text{if } \; y < -\mu_y \\ 0 & \text{if } \; -\mu_y \le y \le \mu_y \\ \frac{\gamma_y}{2} e^{-\gamma_y (y-\mu_y)} & \text{if } \; y > \mu_y \end{cases}.$$

This modeling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors.

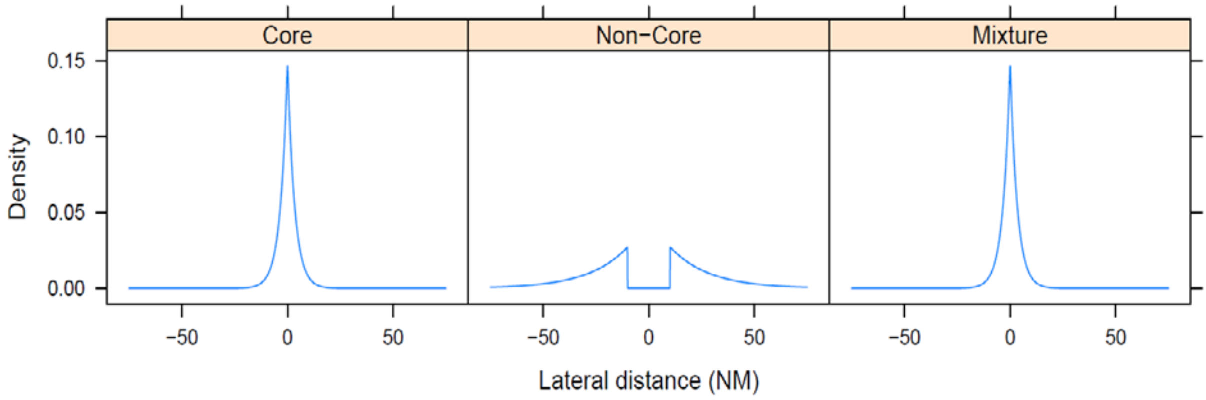

Figure 1: Modeling of lateral deviation

This in turn gives a better understanding of the mixing parameter a which we estimate by taking the 95% upper confidence limit from the available GNE data. The formula comes out to be $\boldsymbol{\alpha = 1-(0.05)^{1/N}} = \mathbf{1.113964 \times 10^{-5}}$

Where **N = 268924** is the number of flights observed and no gross navigational errors were detected. More GNE data with no detected gross navigational error will increase the value of N and hence decrease the value of which will lead to decrease in the risk. Here we would like to note that even though the non-core distribution H_y has a discontinuous density h_y, it does not create difficulty in this risk assessment.

The parameter β_y is estimated under the RNP10 assumption of ±10 NM deviation with 95% confidence, this leads to the estimate

$$\boldsymbol{\beta y = -\log 0.05/10 = 0.299573227}$$

The parameter μ_y is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with S_y = 50 NM initial separation This is a conservative method similar to what has been used by FAA and alsoin EUR/SAM. The estimated value of γ_y is 0.05489708 leading to the estimated value of $\mathbf{P_y}$ **(50)** as $\mathbf{2.520977 \times 10^{-8}}$

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of $P_y(50)$ and N_{ay} had we observed 1,2,3,4 or 5 GNEs. The results, given below, are still well below the TLS.

No. of GNEs	$P_y(50)$	N_{ay}
0	2.520977×10^{-8}	0.9010673×10^{-9}
1	2.7669691×10^{-8}	1.349769×10^{-9}
2	2.9822017×10^{-8}	1.454699×10^{-9}
3	3.125415×10^{-8}	1.524652×10^{-9}
4	3.340513×10^{-8}	1.62958×10^{-9}
5	3.555610×10^{-8}	1.734511×10^{-9}

The estimate of α does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods.

2.5. Estimating Probability of Vertical Overlap: $\mathbf{P}_z(0)$

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by $P_z(0)$. It is defined by

$$P_z(0) = \mathbf{P}(|Z1 - Z2| \leq \lambda_z) ,$$

where Z1 and Z2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z1 and Z2 are statistically independent with distribution F_z. Unlike in the computation of $P_y(S_y)$ where we assume the lateral deviations follow a mixture distribution here we may assume that F_z is a Double Exponential distribution with parameter $\beta_z>0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z |z|} .$$

One can then estimate $\beta_z>0$ by

$$\widehat{\beta_z} = -\frac{\log 0.05}{0.032915} = 91.014196371 .$$

This is under assumption that a typical aircraft stays within ±200 ft = ±0.032915 NM of its assigned flight level 95% of the time. This leads to an estimated value 0.3552838 for $P_z(0)$. Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of **0.538**, as used by MAAR for vertical safety assessment in BOB region.

2.6. Estimating the Lateral Occupancy Parameters: Ey (same) and E_y (opp)

In equation (1) there are two occupancy terms, one for same direction occupancy E_y (same) and another one for opposite direction occupancy E_y(opp).

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing "flying in the same direction" by "flying in the opposite direction". Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator $n_{y'}$ is the number of proximate pairs and the denominator, n, is the the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

WP1	WP2	Proximate	Total
MEPEL	IBITA	30	770
TOTOX	REXOD	62	2302
LOTAV	REXOD	84	3434
GIRNA	IDASO	98	2442
NOPEK	IGOGU	116	2402
IGOGU	IGREX	120	2298
KITAL	LOTAV	196	3618
TOTOX	PARAR	480	4694
POMAN	IGAMA	568	4932
RASKI	PARAR	790	6416
NOBAT	SUGID	1130	8764

Table3:Number of laterally proximate flights per route pair, based on TSD.

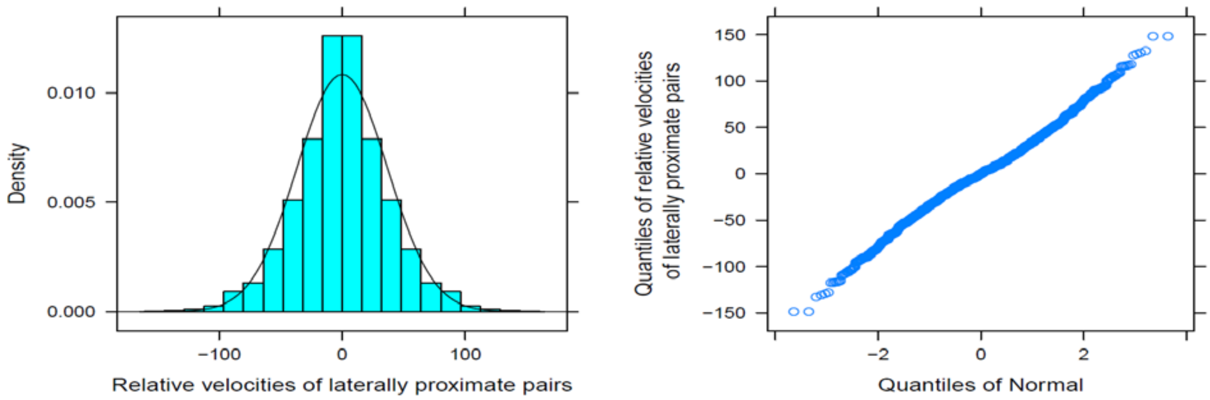

Figure2 :Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

2.7. Estimate of Average Relative Longitudinal Speed: $\overline{|\Delta V|}$

$\overline{|\Delta V|}$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs ofaircraft that constitute a lateral proximate pair in the same direction (see Figure 2) . $\overline{|\Delta V|}$ is estimated as the mean absolute value of all the calculated differences, which turns out to be **26.673**. We use the conservative value **27**. Here we note that the lateral proximate pairs are already determined while estimating the parameter E, (same).

2.8. Estimate of Average Relative Lateral Speed: $\overline{|\dot{y}\,(S_y)|}$

$\overline{|\dot{y}\,(S_y)|}$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value **75** knots as per the EMA Handbook.

2.9. Estimate of Average Relative Vertical Speed: $\overline{|\dot{z}|}$

$\overline{|\dot{z}|}$ denotes the average of the relative vertical speed between a pair of a flight, it can be considered that there is no vertical speed of $\overline{|\dot{z}|}$ with the vertical separation between the aircraft. As noted by various agencies data on $\overline{|\dot{z}|}$ are relatively scarce but typically taken as **1.5** knots which is considered to be conservative (see EMA Handbook).

3. Longitudinal Collision Risk Assessment

In order to compute the level of safety for longitudinal deviations of operations on the BOBASIO region we use the Longitudinal Collision Risk Model. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)\,P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{\overline{|\dot{x}|}}{2\lambda_x}+\frac{\overline{|\dot{y}(0)|}}{2\lambda_y}+\frac{\overline{|\dot{z}|}}{2\lambda_z}\right)\times\left[\sum_{k=m}^{M}2Q(k)\,\mathbf{P}(K>k)\right]. \qquad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- Nax := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum in NM longitudinal separation.

- m := Minimum longitudinal separation in NM.

- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λ_x := Average length of an aircraft flying on BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λ_z := Average height of an aircraft flying on BOBASIO region.
- $P_y(0)$:= Probability that two aircraft assigned at the same route will be at same across-track position.
- $P_z(0)$:= Probability that two aircraft assigned to same flight level are at same geometric height.
- $|\dot{x}|$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by in NM at a reporting point to overtake lead aircraft at the next reporting point.
- $|\dot{y}(0)|$:= Relative across-track speed of same route aircraft pair.
- $|\dot{z}|$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.
- Q (k):= Proportion of aircrafts for which the following aircraft has initial longitudinal separation k.
- P (K > k) := Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions. The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events..

3.1. Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	30 NM	Current minimum longitudinal separation (due to RHS).

M	160 NM	Conservative value corresponding to 20 minutes separation.
λ_x	0.03055548 NM	Estimated from TSD (see Section 2.3).
λ_y	0.02836613 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008699019 NM	Estimated from TSD (see Section 2.3).
$P_y(0)$	0.2	Conservative estimate (see Section 3.2).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$\|x\|$	13 knots	Conservative estimate using speed and distance between way point (see Section 3.3).
$\|\dot{y}(0)\|$	1 knot	RASMAG/9 safety assessment (see Section 3.4).
$\|Z\|$	1.5	Conservative value as per EMA Handbook (see Section 2.9).
Q (k)	See Table 4	Obtained from TSD (see Section 3.5).
P (K > k)	See Table 4	Computed using normal model on speed (see Section 3.6).

Finally this leads to the following estimate for the longitudinal collision risk Nax.

$$\mathbf{Nax = 1.623786 \times 10^{-9}}$$

3.2. Estimating Probability of Lateral Overlap: **Py (0)**

P_y (0) is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting Sy = 0 in the equation (2). This leads to an estimate of P_y (0) as **0.2**.

However as noted earlier in the EUR/SAM report, this factor $P_y(0)$ has a significant effect on the risk estimate. Therefore, it should not be underestimated. Py(0) will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of P_y (0) for the global system performance. We further note that in the EMA Handbook the value has been taken conservatively as 0.2. We take this rather conservative value for our analysis as well.

3.3. Estimation of the Parameter $\overline{|\dot{x}|}$

$\overline{|\dot{x}|}$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and $v_0$is the speed of the front aircraft then $\overline{|\dot{x}|}$ can be computed by the equation.

$$\frac{d-m}{v_0} = \frac{d}{v_0 + \overline{|\dot{x}|}},$$

leading to

$$\overline{|\dot{x}|} = \frac{m v_0}{d-m}.$$

We conservatively estimate it by taking $v_0$as the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is d = 842 NM. With m = 30 NM the final estimate turns out to be $\overline{|\dot{x}|}$ = **13.30049** knots. We use a conservative value of **13** knots.

3.4. Estimation of the Parameter: $\overline{|\dot{y}(0)|}$

$\overline{|\dot{y}(0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of **1** knot as given in the EMA Handbook.

3.5. Estimation of the Parameter Q (k)

Q (k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 3), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q (k) as

$$Q(k) = \frac{\text{number of flights pairs with inter-arrival distance 8k}}{\text{Total number of flight pairs with at least 50 NM separation}}$$

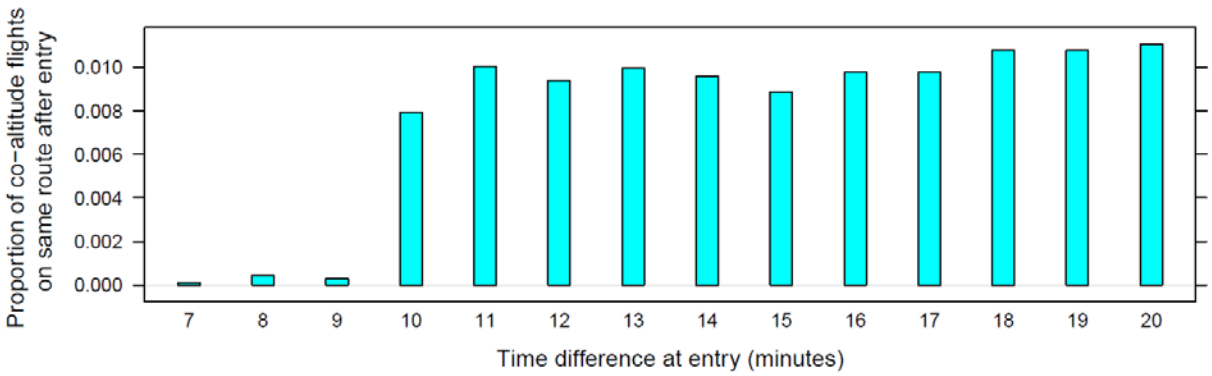

Figure 3: Values of Q(k) estimated from TSD. For co-altitude flights on the same route (after entry / before exit),

the proportion of flights that entered k minutes after the preceding flight is plotted for k = 7,8,9……20 minutes. The final estimated values of Q (k) for k ranging between 7 and 20 minutes are given in the Table 4.

k	N.M	Q (k)	P (K > k)

k	N.M	Q (k)	P (K > k)
7	56	$0.913408x10^{-4}$	9.289737x10-5
8	64	$4.567044x10^{-4}$	1.160193x10-5
9	72	$2.740227x10^{-4}$	1.255691x10-6
10	80	$7.946657x10^{-3}$	1.421109x10-7
11	88	$1.004750x10^{-2}$	2.144779x10-8
12	96	$9.408111x10^{-3}$	4.291844x10-9
13	104	$9.956156x10^{-3}$	9.604582x10-10
14	112	$9.590793x10^{-3}$	2.203050x10-10
15	120	$8.860066x10^{-3}$	5.072133x10-11
16	128	$9.773475x10^{-3}$	1.168254x10-11
17	136	$9.773475x10^{-3}$	2.690923x10-12
18	144	$1.077822x10^{-2}$	6.198197x10-13
19	152	$1.077822x10^{-2}$	1.427675x10-13
20	160	$1.105225x10^{-2}$	3.288466x10-14

Table 4: Estimated values of Q (k) and P (K > k)

3.6. Estimation of the Parameter P(K>k)

To estimate **P** (K > k) we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and V e their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T_0 be the maximum duration of time before ATC intervenes. Then

$$\mathbf{P}(K > k) = \mathbf{P}\left(0 < \frac{k}{V' - V} < T_0\right) = \mathbf{P}\left(V' - V > \frac{k}{T_0}\right).$$

We note here that the value of T_0 is conservatively taken to be **0.5** hours.

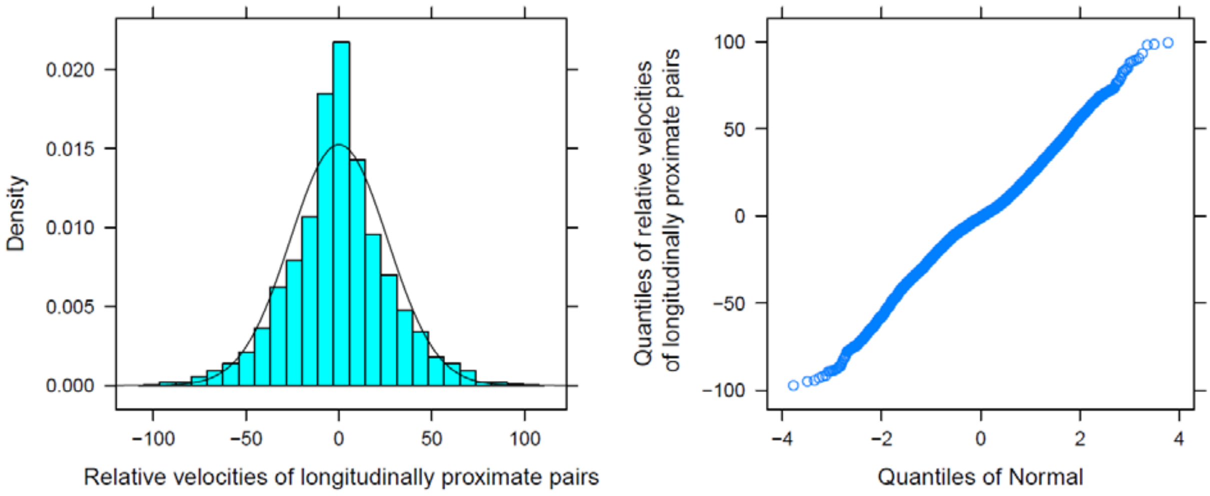

Figure4: Distribution of relative velocities of longitudinally proximate Pairs.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times. We conservatively consider velocity differences of all flight pairs which are separated by 2 hours time at entry. It is to be noted that we observed from the TSD data that two hours is more than the maximum time taken by any aircraft to travel between its entry and exit points.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 4) it is not clear that these differences necessarily normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + (1-p)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion p. We then estimate the parameters of this mixture model by their maximum likelihood estimates (MLEs). Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates.

$\hat{p}$ = **0.3721716** $\hat{\beta}_{\mathbf{v}}$=**0.0917632** $\hat{\sigma}_{\mathbf{v}}$=**30.7923073**

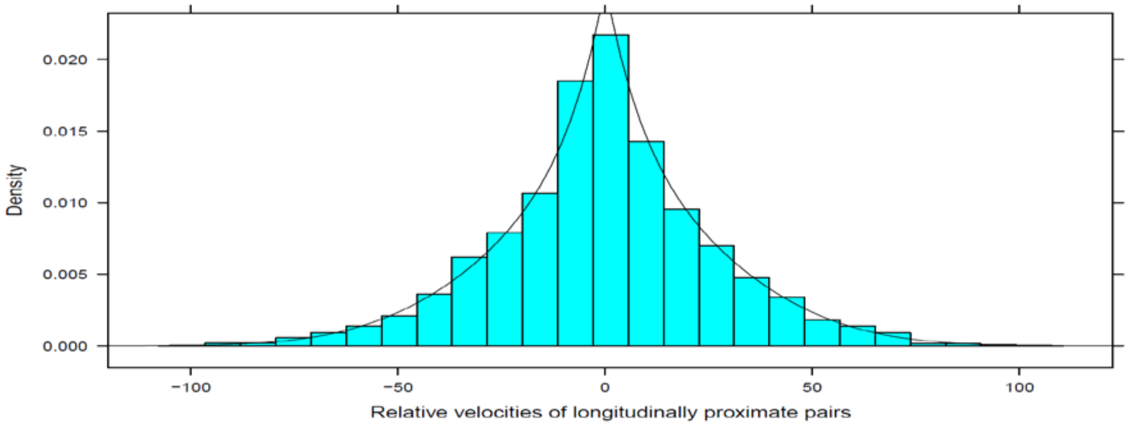

Figure 5: Relative velocities of longitudinally proximate Pairs

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with this value of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 5.

With these we estimate the values of P (K > k) for k ranging between 7 and 20. These arepresented in the Table 4.

4. Summary of the Safety Assessment.

The following table gives a summary of the safety assessment of the BOBASIO region for the month of December 2013.

Type of Risk	**Estimated Risk**	**TLS**	**Remarks**

Lateral Risk	0.9010673×10^{-9}	5 x 10^{-9}	Below TLS
Longitudinal Risk	1.623786×10^{-9}	5 x 10^{-9}	Below TLS