



International Civil Aviation Organization

**The Nineteenth Meeting of the Regional Airspace Safety Monitoring
Advisory Group (RASMAG/19)**

Pattaya, Thailand, 27-30 May 2014

Agenda Item 3: Reports from Asia/Pacific RMAs and EMAs

BOBASMA SAFETY REPORT

(Presented by BOBASMA/India)

SUMMARY

This paper presents the periodic safety assessment for the implementation of 50 NM Reduced Horizontal Separation in the Bay of Bengal Arabian Sea Indian Ocean airspace for the period January to December 2013.

1. INTRODUCTION

1.1 The annual safety review of the implementation of 50 NM reduced horizontal separation (RHS) on sixteen routes in the Bay of Bengal Arabian Sea Indian Ocean airspace was conducted using the TSD of December 2013 and the Large Lateral Deviation (LLD) and Large Longitudinal Error (LLE) reports received by BOBASMA for the period 1st January 2013 to 31st December 2013.

2. DISCUSSION

2.1 The report of the safety assessment conducted for monitoring the horizontal safety risks is attached as **Attachment A**.

Executive Summary

2.2 **Table 1** provides the Bay of Bengal Arabian Sea Indian Ocean airspace horizontal risk estimates. **Figure 1** presents the lateral and longitudinal collision risk estimate trends for BOBASIO airspace during the period January 2013 to December 2013.

Bay of Bengal Arabian Sea Indian Ocean Airspace – estimated annual flying hours = 5,20,210 hours			
<i>(note: estimated hours based on Dec 2013 traffic sample data)</i>			
Risk	Risk Estimation	TLS	Remarks
<i>RASMAG 18 Lateral Risk</i>	0.74428×10^{-9}	5.0×10^{-9}	<i>Below TLS</i>
<i>RASMAG 18 Longitudinal Risk</i>	0.895888×10^{-9}	5.0×10^{-9}	<i>Below TLS</i>
Lateral Risk	0.759155×10^{-9}	5.0×10^{-9}	Below TLS
Longitudinal Risk	4.0239×10^{-9}	5.0×10^{-9}	Below TLS

Table 1: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

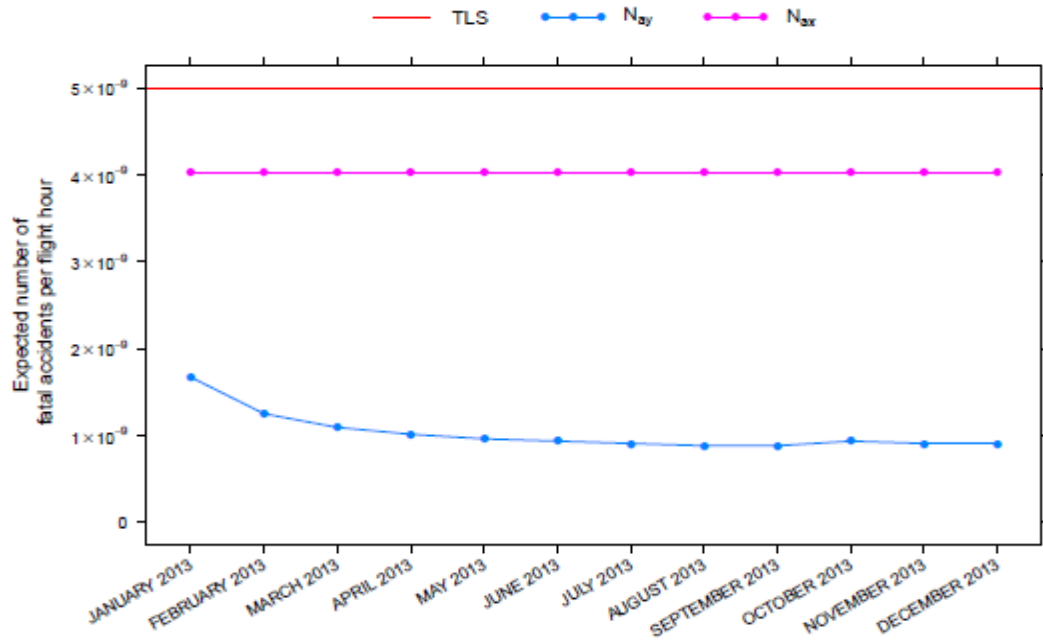


Figure 1: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

- a) note the information contained in this paper; and
- b) discuss any relevant matters as appropriate.

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Lateral and Longitudinal Collision Risk Assessment of Bay of Bengal

1. Introduction

This safety assessment is conducted jointly by Airports Authority of India (AAI) and Indian Statistical Institute, Delhi Centre. The goal of this study is to confirm that the Target Level of Safety (TLS) which is 5×10^{-9} fatal accidents per flight hour, is currently met.

Reduced horizontal separation (RHS) was introduced in two phases on the following routes;

- Phase 1: N571 and P762 from 30th June 2011, and
- Phase 2: L301, L507, L509, L510, L759, M300, M770, N563, N877, N895, P570, P574, P628 and P646 from 15th December 2011.

The current separation standards are as follows

- for lateral separation it is at least 50 NM between all parallel routes;
- for longitudinal it has been reduced to 50 NM between front and behind aircrafts.

In this assessment we carry out the quantitative risk analysis based on two types of datasets supplied by six FIRs in the region.

2. Traffic Sample Data (TSD):

Traffic sample data from Chennai, Mumbai, Kolkata, Yangon and Colombo FIRs for the month of December 2013 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

- Duplicate records were removed.
- Records with Exit time less than Entry time were removed.
- Records with flight level less than F280 were removed.

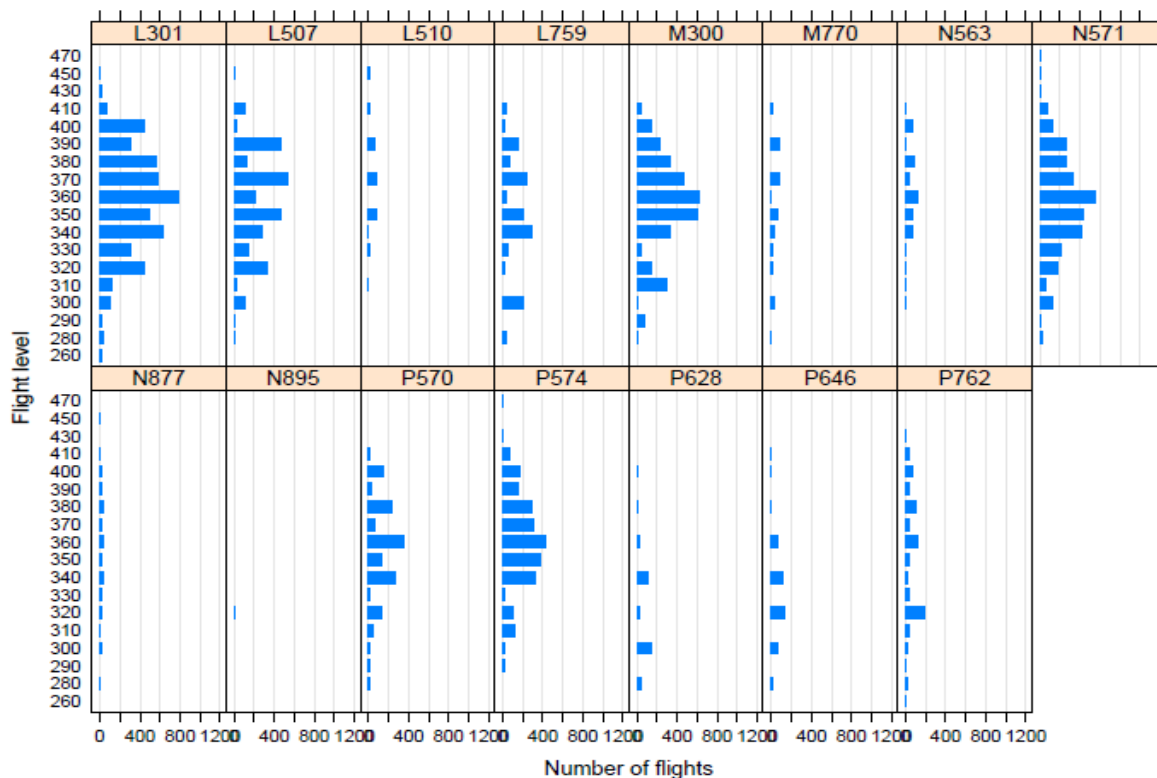


Figure 1: Number of flights by route and flight level based on December 2013 TSD.

39752 records that were retained after filtering were considered for the subsequent statistical analysis. Figure 1 provides a graphical summary of the number of flights by route and flight level for RHS routes.

- Gross Navigational Error (GNE) Data:

Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata, Mumbai, Yangon and Bangkok FIRs, as summarized in Table 1.

Year	Month	FIR	Flights	LLD	LLE
2013	JANUARY	CHENNAI	4424	0	0
2013	FEBRUARY	CHENNAI	4062	0	0
2013	MARCH	CHENNAI	4278	0	0
2013	APRIL	CHENNAI	4285	0	0
2013	MAY	CHENNAI	4292	0	0
2013	JUNE	CHENNAI	4406	0	0
2013	JULY	CHENNAI	4527	0	0
2013	AUGUST	CHENNAI	4483	0	0
2013	SEPTEMBER	CHENNAI	4439	0	0
2013	OCTOBER	CHENNAI	4556	0	0
2013	NOVEMBER	CHENNAI	4586	0	0
2013	DECEMBER	CHENNAI	4616	0	0

Year	Month	FIR	Flights	LLD	LLE
2013	JANUARY	MUMBAI	14025	0	0
2013	FEBRUARY	MUMBAI	12480	0	0
2013	MARCH	MUMBAI	13982	0	0
2013	APRIL	MUMBAI	18638	0	0
2013	MAY	MUMBAI	19461	0	0
2013	JUNE	MUMBAI	18926	0	0
2013	JULY	MUMBAI	18957	0	0
2013	AUGUST	MUMBAI	14981	0	0
2013	SEPTEMBER	MUMBAI	15040	0	0
2013	OCTOBER	MUMBAI	14153	0	0
2013	NOVEMBER	MUMBAI	13521	0	0
2013	DECEMBER	MUMBAI	16025	0	0
2013	JANUARY	KOLKATA	2521	0	0
2013	FEBRUARY	KOLKATA	2161	0	0
2013	MARCH	KOLKATA	2408	0	0
2013	APRIL	KOLKATA	2129	0	0
2013	MAY	KOLKATA	1994	0	0
2013	JUNE	KOLKATA	1913	0	0
2013	JULY	KOLKATA	1878	0	0
2013	AUGUST	KOLKATA	2064	0	0
2013	SEPTEMBER	KOLKATA	2003	0	0
2013	OCTOBER	KOLKATA	1942	0	0
2013	NOVEMBER	KOLKATA	2248	0	0
2013	DECEMBER	KOLKATA	2520	0	0
2013	JANUARY	YANGOON	10025	0	0
2013	FEBRUARY	YANGOON	9224	0	0
2013	MARCH	YANGOON	10319	0	0
2013	APRIL	YANGOON	9231	0	0
2013	MAY	YANGOON	9064	0	0
2013	JUNE	YANGOON	8860	0	0
2013	JULY	YANGOON	9072	0	0
2013	AUGUST	YANGOON	9052	0	0
2013	SEPTEMBER	YANGOON	9180	0	0
2013	OCTOBER	YANGOON	10243	0	0
2013	NOVEMBER	YANGOON	11239	0	0
2013	DECEMBER	YANGOON	11520	0	0
2013	JANUARY	BANGKOK	13221	0	0
2013	FEBRUARY	BANGKOK	11527	0	0

Year	Month	FIR	Flights	LLD	LLE
2013	MARCH	BANGKOK	12571	0	0
2013	APRIL	BANGKOK	11074	0	0
2013	MAY	BANGKOK	11012	0	0
2013	JUNE	BANGKOK	10695	0	0
2013	JULY	BANGKOK	10281	0	0
2013	AUGUST	BANGKOK	10458	0	0
2013	SEPTEMBER	BANGKOK	10615	0	0
2013	OCTOBER	BANGKOK	12613	0	0
2013	NOVEMBER	BANGKOK	13869	0	0
2013	DECEMBER	BANGKOK	15094	0	0
		Total	528983	0	0

Table 1: Summary of reports of Gross Navigational Errors.

In Section 2 we discuss the risk assessment for the lateral direction and Section 3 gives the same for the longitudinal direction.

2 Lateral Collision Risk Assessment

2.1 Lateral Collision Risk Model

In order to compute the level of safety for lateral deviations of operations on the BOBASIO region we use the Reich Lateral Collision Risk Model. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y(S_y) P_z(0) \frac{\lambda_x}{S_x} \left\{ E_y(\text{same}) \left[\frac{|\Delta V|}{2\lambda_x} + \frac{|\dot{y}(S_y)|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right] + E_y(\text{opp}) \left[\frac{2|V|}{2\lambda_x} + \frac{|\dot{y}(S_y)|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right] \right\}. \quad (1)$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.

The parameters in the equation (1) are defined as follows:

- N_{ay} := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned 8_y NM lateral separation.
- S_y := Minimum planned lateral separation.
- λ_x := Average length of an aircraft flying in BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λ_z := Average height of an aircraft flying on BOBASIO region.
- $P_y(S_y)$:= The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y .
- $P_z(0)$:= Probability that two aircraft assigned to same flight level are at same geometric

height.

- S_x := Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- E_y (same) := Same direction lateral occupancy at same assigned flight level.
- E_y (opp) := Opposite direction lateral occupancy at same assigned flight level.
- $|\overline{\Delta V}|$:= Average relative speed of two aircraft flying on parallel routes in same direction.
- $|V|$:= Average ground speed on an aircraft.
- $|\overline{y}(S_y)|$:= Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y .
- $|\overline{z}|$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, $P_z(0)$ is the probability of vertical overlap; $P_y(S_y)$ is the probability of lateral overlap and the combinations of $\frac{\lambda_x E_y(\text{same})}{S_x}$ and $\frac{\lambda_x E_y(\text{opp})}{S_x}$ relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example,

$\frac{|\overline{\Delta V}|}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time $\frac{2\overline{V}}{2\lambda_x}$.

The model is based on the following hypothesis:

- All routes are parallel.¹
- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.
- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.03076899 NM	Estimated from TSD (see Section 2.3).
λ_y	0.02849246 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008719131 NM	Estimated from TSD (see Section 2.3).
$P_y (50)$	2.49117×10^{-8}	Estimated using a mixture model (see Section 2.4).
$P_z (0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
S_x	50 NM	Reduced longitudinal separation.
E_y (same)	0.05995443	Estimated from the TSD (see Section 2.6).
E_y (opp)	0	No opposite directional lateral occupancy at same assigned flight level.
$ \Delta V $	27 knots	Value obtained from TSD (see Section 2.7).
$ \dot{y} (50) $	75 knots	Conservative value taken from EMA Handbook (see Section 2.8).
$ \dot{z} $	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk N_{ay} .

$$N_{ay} = 0.759155 \times 10^{-9}$$

2.3 Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

Type	Length	Wingspan	Height	Flights
B77W	73.9	64.8	18.5	4464
B738	39.2	34.4	12.57	3886
A320	37.57	34.1	11.76	3238
A332	58.8	60.3	17.4	2647
A333	63.6	60.3	16.85	2040
B772	63.7	60.9	18.4	1769
B744	70.6	64.8	19.4	1218

Type	Length	Wingspan	Height	Flights
A388	73	79.8	24.1	1214
A321	44.51	34.1	11.76	1000
A343	63.6	60.3	16.85	770
B763	54.9	47.6	15.9	653
B77L	63.7	64.8	18.3	595
B773	73.9	60.9	18.4	450
A319	33.84	34.1	11.76	401
A346	75.3	63.45	17.3	256
A306	54.1	44.84	16.54	157
MD11	61.2	51.7	17.6	124
A310	46.66	43.9	15.8	121
B752	47.3	38.1	13.6	99
A345	67.9	63.45	17.1	56
B737	33.6	34.3	12.6	39
CL60	20.85	19.6	6.3	29
GLF5	29.4	28.5	7.5	26
GLF4	26.9	23.7	7.4	25
GLEX	30.3	26.9	7.6	22
F2TH	20.2	19.3	7.1	21
B74S	56.3	59.6	20	19
GL5T	28.69	28.65	7.7	17
C17	53	51.8	16.8	17
A380	73	79.8	24.1	15
F900	20.2	19.3	7.6	14
GALX	19	17.4	6.4	7
E135	26.3	20.2	6.7	7
B742	70.6	59.6	19.3	7

Table 2: Dimensions of aircraft types, along with number of records in the TSD

2.4 Estimating Probability of Lateral Overlap: $P_y(S_y)$

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y , is denoted by $P_y(S_y)$ and is defined as

$$P_y(S_y) := \mathbf{P}(|S_y + Y_1 - Y_2| \leq \lambda_y), \quad (2)$$

where Y_1 and Y_2 are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y . We assume that Y_1 and Y_2 are identically distributed but statistically independent with a distribution F_y .

We model F_y as mixture distribution having a core distribution G_y and a non-core distribution H_y .

- The core distribution G_y , represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.
- The non-core distribution H_y , represents Gross Navigation Errors (GNE), that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution

may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)G_y(y) + \alpha H_y(y). \quad (3)$$

The mixing parameter α is the probability of a gross navigational error.

The core lateral deviation distribution G_y is modeled by a Double Exponential distribution with a parameter $\beta_y > 0$ as the rate, that is, if $Y_1 \sim G_y$ then

$$\mathbf{P}(|Y_1| > y) = e^{-\beta_y y},$$

Where $y \geq 0$. In other words we assume that the core distribution has a density of the form

$$g_y(y) = \frac{\beta_y}{2} e^{-\beta_y |y|}.$$

Finally the non-core distribution H_y is modeled by a ‘‘Separated Double Exponential’’ distribution with parameters $\mu_y > 0$, representing the ‘‘separation and $\gamma_y > 0$ the rate parameter, that is, if $Y_2 \sim H_y$ then

$$\mathbf{P}(Y_2 > \mu_y + y) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$\mathbf{P}(Y_2 < -\mu_y - y) = \frac{1}{2} e^{-\gamma_y y},$$

where $y \geq 0$. This really means that the non-core distribution H_y gives no mass in $[-\mu_y, \mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y .

The density of this distribution is given by

$$h_y(y) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y(y+\mu_y)} & \text{if } y < -\mu_y \\ 0 & \text{if } -\mu_y \leq y \leq \mu_y \\ \frac{\gamma_y}{2} e^{-\gamma_y(y-\mu_y)} & \text{if } y > \mu_y \end{cases}.$$

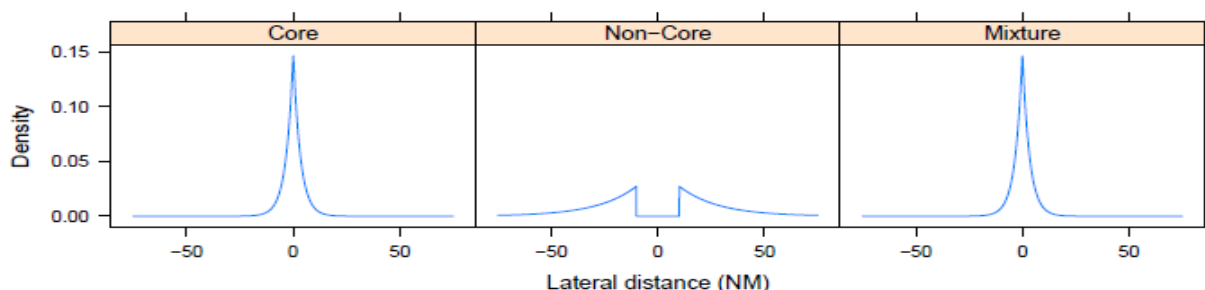


Figure 2: Modeling of lateral deviation

This modeling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors. This in turn gives a better understanding of the mixing parameter α which we estimate by taking the 95% upper confidence limit from the available GNE data. The formula comes out to be

$$\hat{\alpha} = 1 \times 10^{-5},$$

Here we would like to note that even though the non-core distribution H_y has a discontinuous density h_y , it does not create difficulty in this risk assessment.

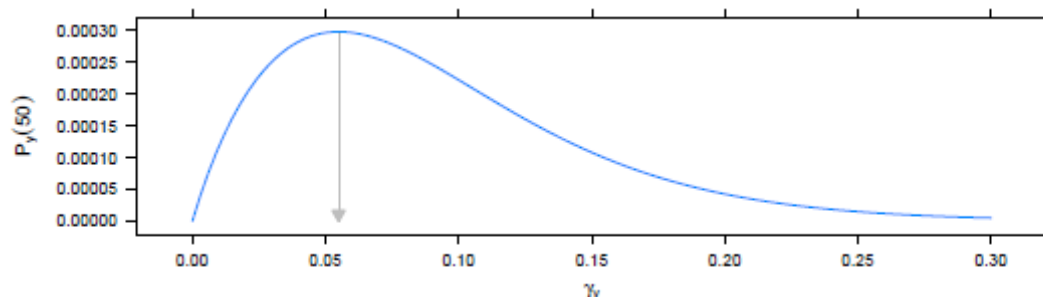


Figure 3: Wingspan overlap probability as a function of γ_y with $S_y = 50$ NM initial separation.

The parameter β_y is estimated under the RNP10 assumption of ± 10 NM deviation with 95% confidence, this leads to the estimate

$$\beta_y = -\log 0.05/10 = 0.2995732$$

The parameter μ_y is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with $S_y = 50$ NM initial separation (see Figure 3). This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γ_y is 0.05489708 leading to the estimated value of $P_y(50)$ as 2.49117×10^{-8} .

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of $P_y(50)$ and N_{ay} had we observed more GNEs. The results, given below, are still well below the TLS.

No. of GNEs	$P_y(50)$	N_{ay}
1	2.49117×10^{-8}	0.948944×10^{-9}
2	2.56318×10^{-8}	0.976378×10^{-9}
3	2.6352×10^{-8}	1.00381×10^{-9}
4	2.77924×10^{-8}	1.05868×10^{-9}
5	2.85126×10^{-8}	1.08611×10^{-9}

The estimate of α does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods.

2.5 Estimating Probability of Vertical Overlap: $P_z(0)$

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by $P_z(0)$. It is defined by

$$P_z(0) = P(|Z_1 - Z_2| \leq \lambda_z),$$

where Z_1 and Z_2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z_1 and Z_2 are statistically independent with distribution F_z . Unlike in the computation of $P_y(S_y)$ where we assume the lateral deviations follow a mixture distribution here we may assume that F_z is a Double Exponential distribution with parameter

$\beta_z > 0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z|z|}.$$

One can then estimate $\beta_z > 0$ by

$$\hat{\beta}_z = -\frac{\log 0.05}{0.032915} = 91.014196371.$$

This is under assumption that a typical aircraft stays within ± 200 ft = ± 0.032915 NM of its assigned flight level 95% of the time. This leads to an estimated value 0.3519883 for $P_z(0)$. Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

2.6 Estimating the Lateral Occupancy Parameters: E_y (same) and E_y (opp)

In equation (1) there are two occupancy terms, one for same direction occupancy $E_y(\text{same})$ and another one for opposite direction occupancy $E_y(\text{opp})$.

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing “flying in the same direction” by “flying in the opposite direction”. Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

WP1	WP2	Proximate	Total
BIDEX	ORARA	0	584
IGOGU	IGREX	126	2450
NOPEK	IGOGU	138	2630
GIRNA	IDASO	100	2500
VATLA	ORARA	14	614
IGOGU	EMRAN	2	1880
LIBDI	MEPEL	0	92
RINDA	SAGOD	0	3060
MEPEL	IBITA	0	92
IBITA	TEBOV	0	2112
SAGOD	IBITA	0	6

WP1	WP2	Proximate	Total
POMAN	IGAMA	706	5388
KITAL	LOTAV	238	3980
OPIRA	IGAMA	0	3128
LOTAV	REXOD	104	3748
TOTOX	REXOD	66	2316
TOTOX	PARAR	464	4606
ADPOP	SUGID	0	3134
RASKI	PARAR	714	6030
NOBAT	SUGID	1160	8900
POMAN	ODOLI	0	2260
KITAL	ASPUX	0	486
NISOK	NIXUL	0	140
NIXUL	TOPIN	4	398
SULTO	DUGOS	0	452
ATETA	DEMON	0	224
UDULO	KAKIB	0	646
RIBRO	ELBAB	6	462
VATLA	MABUR	0	428
SAGOD	MEPEL	0	98
MIPAK	LALAT	0	1238

Table 3: Number of laterally proximate flights per route pair, based on TSD.

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator n_y is the number of proximate pairs and the denominator, n , is the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

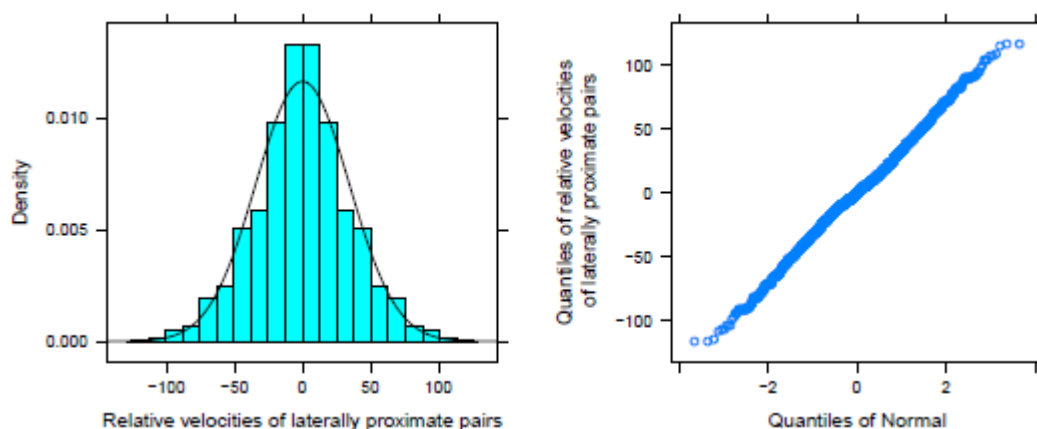


Figure 4: Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

2.7 Estimate of Average Relative Longitudinal Speed: $|\overline{\Delta V}|$

$|\overline{\Delta V}|$ is the average relative longitudinal speed between aircraft flying in the

Same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction (see

Figure 4. $|\overline{\Delta V}|$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 26.27114. We use the conservative value 27. Here we note that the lateral proximate pairs are already determined while estimating the parameter E, (same).

2.8 Estimate of Average Relative Lateral Speed: $|\overline{\dot{y}(S_y)}|$

$|\overline{\dot{y}(S_y)}|$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.9 Estimate of Average Relative Vertical Speed: $|\overline{\dot{z}}|$

$|\overline{\dot{z}}|$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $|\overline{\dot{z}}|$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $|\overline{\dot{z}}|$ with the vertical separation between the aircraft. As noted by various agencies data on $|\overline{\dot{z}}|$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

3 Longitudinal Collision Risk Assessment

In order to compute the level of safety for longitudinal deviations of operations on the BOBASIO region we use the Longitudinal Collision Risk Model. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0) P_z(0) \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\dot{x}|}{2\lambda_x} + \frac{|\dot{y}(0)|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right) \times \left[\sum_{k=m}^M 2Q(k) P(K > k) \right]. \quad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- N_{ax} := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum in NM longitudinal separation.
- m := Minimum longitudinal separation in NM.
- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λ_x := Average length of an aircraft flying on BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λ_z := Average height of an aircraft flying on BOBASIO region.
- $P_y(0)$:= Probability that two aircraft assigned at the same route will be at same across-track position.
- $P_z(0)$:= Probability that two aircraft assigned to same flight level are at same geometric height.
- $|\dot{x}|$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by in NM at a reporting point to overtake lead aircraft at the next reporting point.
- $|\dot{y}(0)|$:= Relative across-track speed of same route aircraft pair.
- $|\dot{z}|$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.
- $Q(k)$:= Proportion of aircrafts for which the following aircraft has initial longitudinal separation k .
- $P(K > k)$:= Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions. The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events..

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	50 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes separation.
λ_x	0.03076899 NM	Estimated from TSD (See Section 2.3).
λ_y	0.02849246 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008719131 NM	Estimated from TSD (see Section 2.3).
$P_y(0)$	0.2	Conservative estimate (see Section 3.2).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$ \dot{x} $	19 knots	Conservative estimate using speed and distance between way points (see Section 3.3).
$ \dot{y}(0) $	1 knot	RASMAG/9 safety assessment (see Section 3.4).
$ \dot{z} $	1.5	Conservative value as per EMA Handbook (see Section 2.9).
Q(k)	See Table 4	Obtained from TSD (see Section 3.5).
$P(K > k)$	See Table 4	Computed using normal model on speed (see Section 3.6).

Finally this leads to the following estimate for the longitudinal collision risk N_{ax} .

$$N_{ax} = 4.0239 \times 10^{-9}$$

3.2 Estimating Probability of Lateral Overlap: $P_y(0)$

$P_y(0)$ is defined as the probability of lateral overlap of aircraft nominally flying at Adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting $S_y = 0$ in the equation (2). This leads to an estimate of $P_y(0)$ as 0.2.

k (mins)	k (NM)	Q(k)	$P(K > k)$
7	56	0.000227449	1.02103×10^{-4}
8	64	0.000318428	1.16202×10^{-5}

9	72	0.000318428	1.018×10^{-6}
10	80	0.01050812	6.85229×10^{-8}
11	88	0.009689305	3.54073×10^{-9}
12	96	0.010417141	1.40698×10^{-10}
13	104	0.010326161	4.36144×10^{-12}
14	112	0.010417141	1.15139×10^{-13}
15	120	0.010690079	3.89439×10^{-15}
16	128	0.01046263	2.68029×10^{-16}
17	136	0.010690079	3.31081×10^{-17}
18	144	0.010280671	4.08966×10^{-18}
19	152	0.01050812	5.05173×10^{-19}
20	160	0.010599099	6.24013×10^{-20}

Table 4: Estimated values of $Q(k)$ and $P(K > k)$

However as noted earlier in the EUR/SAM report, this factor $P_y(0)$ has a significant effect on the risk estimate. Therefore, it should not be underestimated. $P_y(0)$ will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of $P_y(0)$ for the global system performance. We further note that in the EMA Handbook the value has been taken conservatively as 0.2. We take this rather conservative value for our analysis as well.

3.3 Estimation of the Parameter $|\overline{\dot{x}}|$

$|\overline{\dot{x}}|$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $|\overline{\dot{x}}|$ can be computed by the equation.

$$\frac{d - m}{v_0} = \frac{d}{v_0 + |\overline{\dot{x}}|},$$

leading to

$$|\overline{\dot{x}}| = \frac{mv_0}{d - m}.$$

We conservatively estimate it by taking v_0 as the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is $d = 842$ NM. With $m = 50$ NM the final estimate turns out to be $|\overline{\dot{x}}| = 19.5439$ knots. We use a conservative value of 19 knots.

3.4 Estimation of the Parameter: $\overline{|\dot{y}(0)|}$

$\overline{|\dot{y}(0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of 1 knot as given in the EMA Handbook.

3.5 Estimation of the Parameter Q (k)

Q (k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 3), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q(k) as

$$Q(k) = \frac{\text{number of flights pairs with inter-arrival distance } 8k}{\text{Total number of flight pairs with at least 50 NM separation}}$$

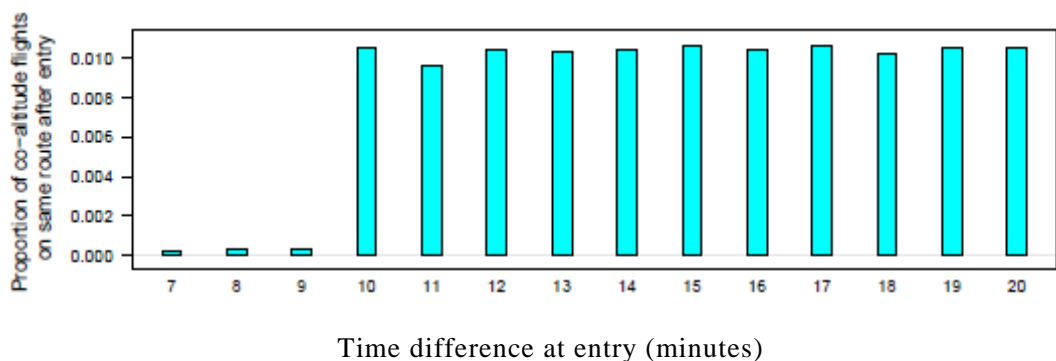


Figure 5: Values of Q(k) estimated from TSD. For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 7,8,9...20 minutes. The final estimated values of Q (k) for k ranging between 7 and 20 minutes are given in the Table 4.

3.6 Estimation of the Parameter P (K > k)

To estimate P (K > k) we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and V e their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T0 be the maximum duration of time before ATC intervenes. Then

$$P(K > k) = P\left(0 < \frac{k}{V' - V} < T_0\right) = P\left(V' - V > \frac{k}{T_0}\right).$$

We note here that the value of T0 is conservatively taken to be 0.5 hours.

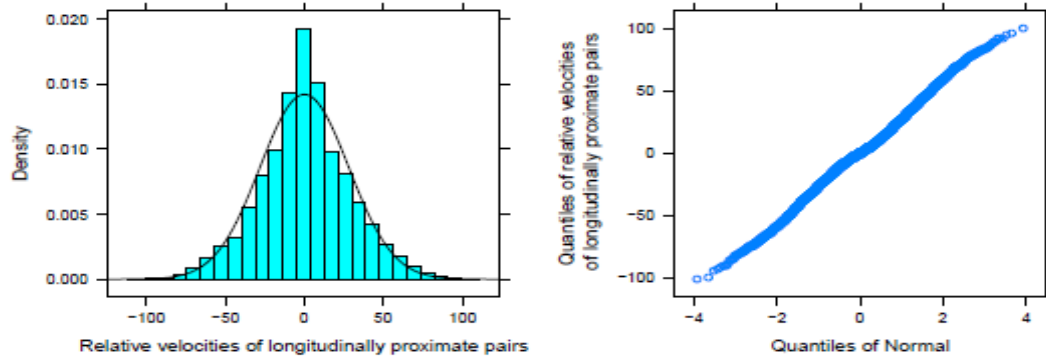


Figure6: Distribution of relative velocities of longitudinally proximate Pairs.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times. We conservatively consider velocity differences of all flight pairs which are separated by 2 hours time at entry. It is to be noted that we observed from the TSD data that two hours is more than the maximum time taken by any aircraft to travel between its entry and exit points.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile-quantile plot (see Figure 4) it is not clear that these differences necessarily normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p \frac{\beta_v}{2} e^{-\beta_v |v|} + (1-p) \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion p . We then estimate the parameters of this mixture model by their maximum likelihood estimates (MLEs). Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates.

$$\hat{p} = 0.1824579 \quad \hat{\beta}_v = 0.1307072 \quad \hat{\sigma}_v = 30.5795$$

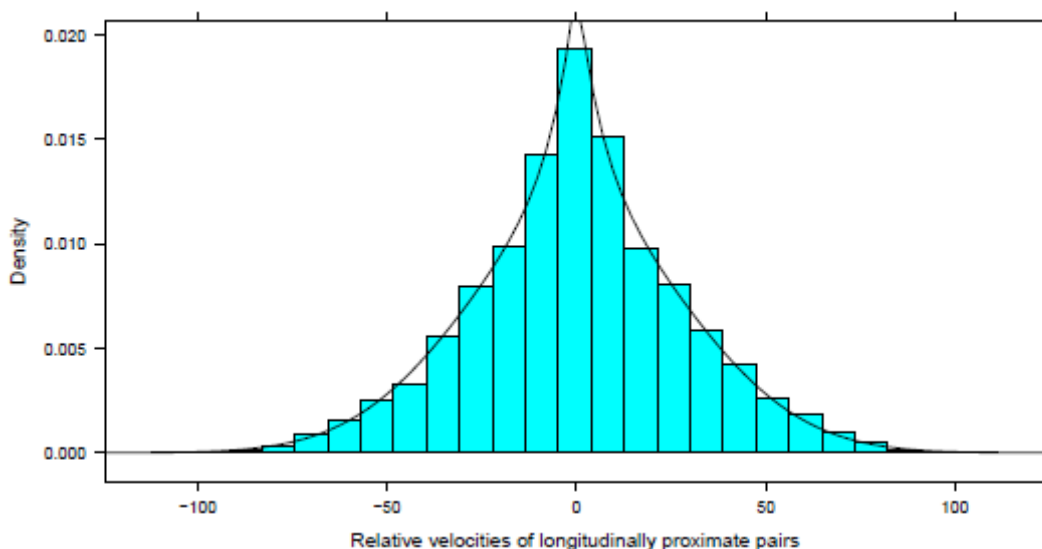


Figure 7: Relative velocities of longitudinally proximate Pairs

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with this value of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

With these we estimate the values of $P(K > k)$ for k ranging between 4 and 14. These are presented in the Table 4.

4. Summary of the Safety Assessment.

The following table gives a summary of the safety assessment of the BOBASIO region for the month of December 2013.

Type of Risk	Estimated Risk	TLS	Remarks
Lateral Risk	0.759155×10^{-9}	5×10^{-9}	Below TLS
Longitudinal Risk	4.0239×10^{-9}	5×10^{-9}	Below TLS

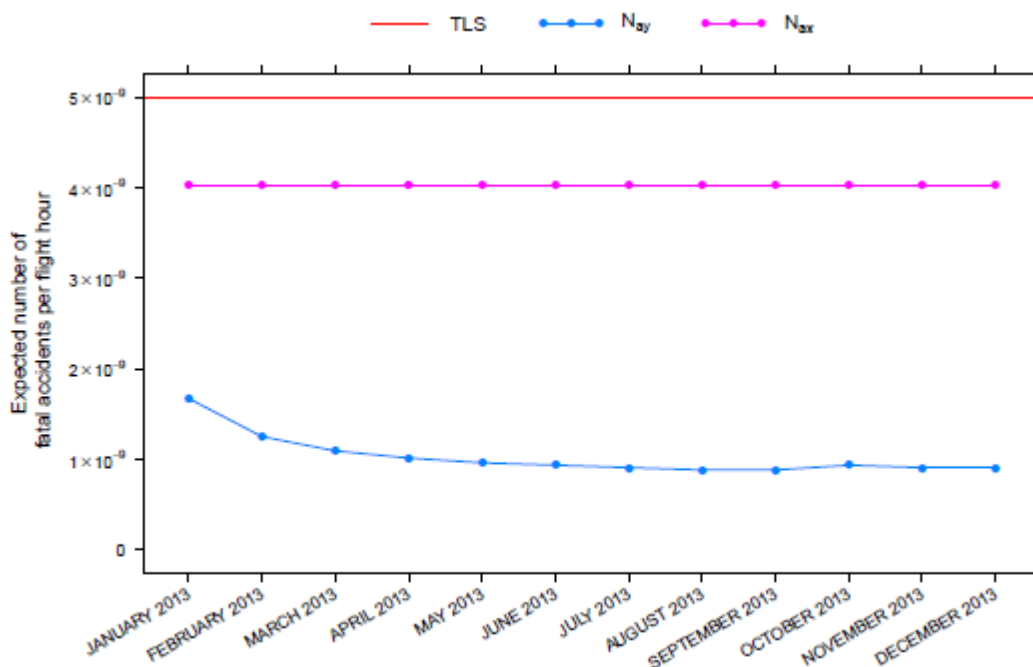


Figure 8: Assessment of Compliance with Lateral and Longitudinal TLS Values.

Figure 8 presents the results of the collision risk estimates for each month using the cumulative 12-month LLD and LLE reports since January 2011.