International Civil Aviation Organization

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Bangkok, Thailand, 01 – 04 April 2013

Agenda Item 3: Reports from Asia/Pacific RMAs and EMAs

BOBASMA SAFETY MONITORING REPORT

(Presented by BOBASMA)

SUMMARY

This paper presents the horizontal Safety monitoring report of BOBASMA for the period 1st January 2012 to 31st December 2012. The results of the periodic Safety assessment conducted, confirms that the established Target Level of Safety is satisfied in the Bay of Bengal Arabian Sea Indian Ocean airspace.

This paper relates to –

Strategic Objectives:
*A: **Safety** – Enhance global civil aviation safety*

Global Plan Initiatives:

GPI-8 Collaborative airspace design and management
GPI-17 Data link applications
GPI-21 Navigation systems
GPI-22 Communication infrastructure

1. INTRODUCTION

1.1 This working paper is a periodic safety assessment conducted to confirm that the continued use of 50 NM RHS introduced on sixteen RNP10 routes, L301, L507, L509, L510, L759, M300, M770, N563, N571, N877, N895, P570, P574, P628, P762 and P646 in 2011-12 is safe. The report covers the twelve month period from 1st January 2012 to 31st December 2012.

1.2 The report contains a listing of the Large Lateral Deviation and Large Longitudinal Error reports received by BOBASMA for the period 1st January 2012 to 31st December 2012.

2. DISCUSSION

2.1 The lateral separation standard for the RNP10 routes is a minimum of 50NM. Some routes have more than 50NM lateral separation from adjoining routes. Non-RNP10 aircraft are permitted to fly only below FL280 after prior approval from ATC.

2.2 The longitudinal separation standard applied on the sixteen RNP10 routes is as follows:

- 10 minute time based separation;
- 80 NM RNAV distance based separation; or
- 50 NM reduced longitudinal separation between suitably equipped FANS1/A aircraft.

2.3 The one month Traffic Sample Data (TSD) of December 2012 received by BOBASMA from Mumbai, Colombo, Chennai, Kolkata, Kuala Lumpur and Yangon have been used to calculate the collision risk estimates. The TSD of these six Flight Information Regions are fully representative of the traffic movements in the Oceanic airspace of the Bay of Bengal, Arabian Sean and Indian Ocean region.

2.4 Monthly reports of LLDs and LLEs were received by BOBASMA as part of the Monitoring program in place since 1st July 2010, for monitoring of Gross Navigational Errors in the region. **Table 1** contains a summary of the GNE reports received from the different ACCs/OCCs.

Month	Chennai	Mumbai	Kolkata	Kuala Lumpur	Yangon	Bangkok
Jan-12	Yes	Yes	Yes	***No***	Yes	Yes
Feb-12	Yes	Yes	Yes	***No***	Yes	Yes
Mar-12	Yes	Yes	Yes	***No***	Yes	Yes
Apr-12	Yes	Yes	Yes	***No***	Yes	Yes
May-12	Yes	Yes	Yes	***No***	Yes	Yes
Jun-12	Yes	Yes	Yes	***No***	Yes	Yes
Jul-12	Yes	Yes	Yes	Yes	Yes	Yes
Aug-12	Yes	Yes	Yes	Yes	Yes	Yes
Sept-12	Yes	Yes	Yes	Yes	Yes	Yes
Oct-12	Yes	Yes	Yes	Yes	Yes	Yes
Nov-12	Yes	Yes	Yes	Yes	Yes	Yes
Dec-12	Yes	Yes	Yes	Yes	Yes	Yes

Table 1: Record of GNE Reports received

2.5 **Table 2** presents the number of LLD and LLE occurrences reported over the monitoring during the period 1st January to 31st December 2012. There was one report of an LLD event in October 2012 in Chennai FIR.

	LLD		LLE	
	Monthly Count	Cumulative 12 month	Monthly Count	Cumulative 12 month
Jan-12	0	0	0	0
Feb-12	0	0	0	0
Mar-12	0	0	0	0
Apr-12	0	0	0	0
May-12	0	0	0	0

Jun-12	0	0	0	0
Jul-12	0	0	0	0
Aug-12	0	0	0	0
Sep-12	0	0	0	0
Oct-12	1	1	0	0
Nov-12	0	1	0	0
Dec-12	0	1	0	0

Table 2 Monthly Count of LLDs & LLEs.

2.6 **Table 3** is a summary of the LLD & LLE reports received by BOBASMA. The report in Table 4 with an assigned Deviation code 'A' is an LLD reported within Chennai FIR in the month of October. The air crew had deviated without ATC clearance.

Deviation Code	Cause of Deviation	Number of Occurrences
A	Flight crew deviate without ATC Clearance	1

Table 3: Summary of LLD & LLE reports received by BOBASMA

2.7 The estimation of collision risk based on the TSD of December 2012 is attached as **Appendix-A** to this Working Paper. **Table 4** provides the Bay of Bengal Arabian sea Indian Ocean airspace horizontal risk estimates. Both the lateral and longitudinal collision n risk estimates are found to be below the Target Level of Safety (TLS), which is 5.0×10^{-9}

Bay of Bengal Arabian Indian Ocean Airspace – estimated annual flying hours = 6,17,652 hours ***(note: estimated hours based on Dec 2012 traffic sample data)***			
Risk	**Risk Estimation**	**TLS**	**Remarks**
RASMAG 17 Lateral Risk	0.740919×10^{-9}	5.0×10^{-9}	*Below TLS*
RASMAG 17 Longitudinal Risk	1.05943×10^{-9}	5.0×10^{-9}	*Below TLS*
Lateral Risk	$\mathbf{0.74428 \times 10^{-9}}$	5.0×10^{-9}	**Below TLS**
Longitudinal Risk	$\mathbf{0.895888 \times 10^{-9}}$	5.0×10^{-9}	**Below TLS**

Table 4: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

a) note the information contained in this paper; and

b) discuss any relevant matters as appropriate.

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Appendix-A

Lateral and Longitudinal Collision Risk Assessment of the BOBASIO Region based on Dec 2012 TSD

1 INTRODUCTION

In this article we reinvestigate the collision risk between two aircraft flying over the Bay of Bengal, Arabian Sea, and Indian Ocean (BOBASIO) region. This safety assessment is undertaken jointly by the Airports Authority of India (AAI) and the Indian Statistical Institute, Delhi Centre under the MoA signed between the two organizations on January 13, 2011. The goal of this study is to confirm that the Target Level of Safety (TLS) which is 5 x 10^{-9} fatal accidents per flight hour is currently met.

Note that in the following routes in the BOBASIO region reduced horizontal separation (RHS) was introduced in two phases:

- Phase 1: On the routes N571 and P762 from July 2011 and
- Phase 2: On the routes L301, L507, L509, L510, L759, M300, M770, N563, N877, N895, P570, P574, P628 and P646 from December 15, 2011.

In this article we carry out the quantitative risk analysis based on two types of data sets supplied by six FIRs.

- **Traffic Sample Data (TSD):**

Traffic sample data from Chennai, Kolkata, Mumbai, Colombo, Yangon and Kuala Lumpur FIRs for the month of December 2012 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

- Duplicate records were removed.
- Records with Exit time less than Entry time were removed.
- Records with flight level less than F280 were removed.

40590 records that were retained after filtering were considered for the subsequent statistical analysis. Figure 1 provides a graphical summary of the number of flights by route and flight level for RHS routes.

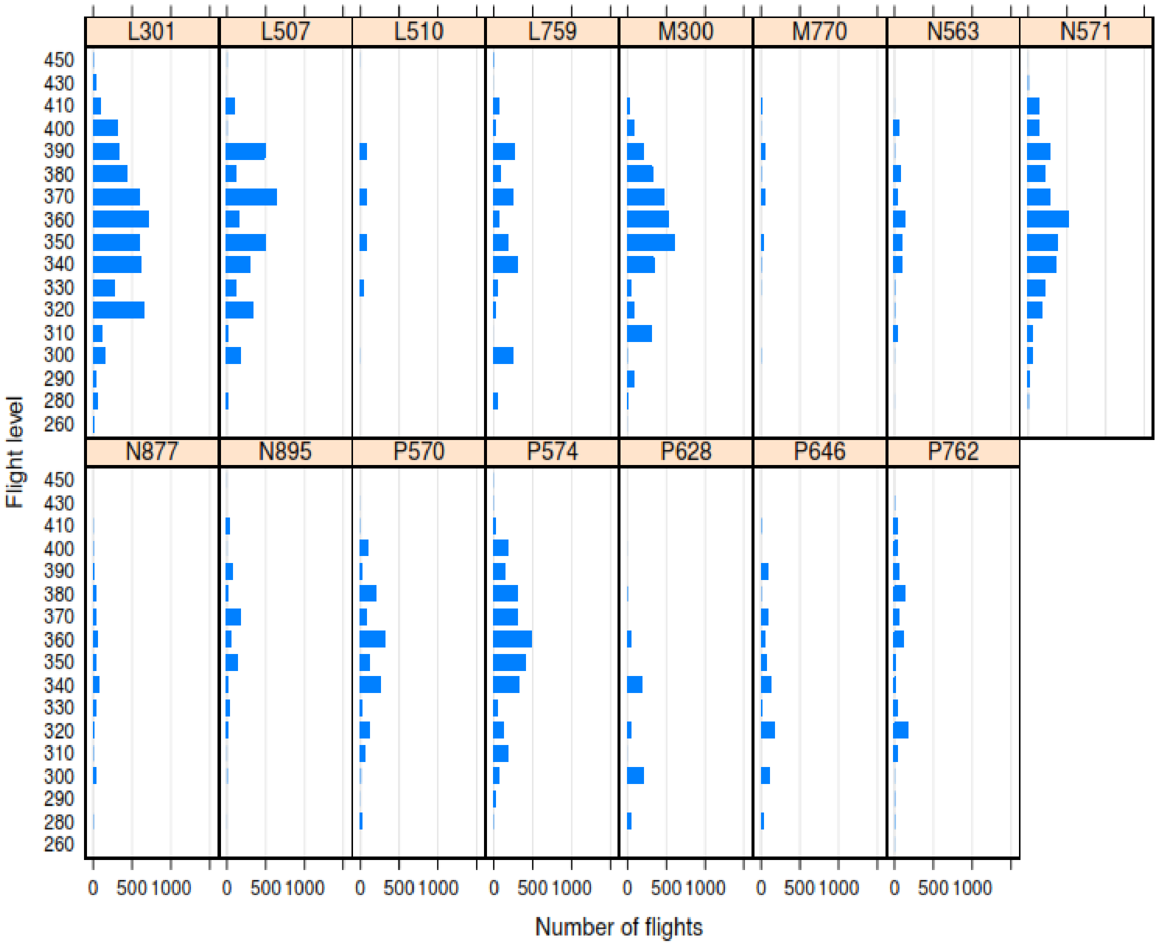

Figure 1: Number of flights by route and flight level based on December 2012 TSD.

- **Gross Navigational Error (GNE) Data:**

  Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata, Mumbai, Colombo, Yangon and Kuala Lumpur FIRs, as summarized in Table 1.

MONTH	FIR	FLIGHTS	LLD	LLD Code	LLE	LLE Code
JANUARY	CHENNAI	9623	0		0	
FEBRUARY	CHENNAI	9058	0		0	
MARCH	CHENNAI	9711	0		0	
APRIL	CHENNAI	9992	0		0	
MAY	CHENNAI	10250	0		0	
JUNE	CHENNAI	7478	0		0	
JULY	CHENNAI	8012	0		0	
AUG	CHENNAI	8263	0		0	
SEP	CHENNAI	8168	0		0	
OCT	CHENNAI	8594	1	A	0	
NOV	CHENNAI	8568	0		0	

DEC	CHENNAI	8824	0	0
JANUARY	KOLKATA	2462	0	0
FEBRUARY	KOLKATA	2251	0	0
MARCH	KOLKATA	2259	0	0
APRIL	KOLKATA	1999	0	0
MAY	KOLKATA	1986	0	0
JUNE	KOLKATA	1790	0	0
JULY	KOLKATA	1923	0	0
AUGUST	KOLKATA	1920	0	0
SEPTEMBER	KOLKATA	1906	0	0
OCTOBER	KOLKATA	2113	0	0
NOVEMBER	KOLKATA	2173	0	0
DECEMBER	KOLKATA	2429	0	0
JANUARY	BANGKOK	7913	0	0
FEBRUARY	BANGKOK	7104	0	0
MARCH	BANGKOK	7361	0	0
APRIL	BANGKOK	6342	0	0
MAY	BANGKOK	6338	0	0
JUNE	BANGKOK	6383	0	0
JULY	BANGKOK	6231	0	0
AUGUST	BANGKOK	6276	0	0
SEPTEMBER	BANGKOK	6008	0	0
OCTOBER	BANGKOK	6910	0	0
NOVEMBER	BANGKOK	7468	0	0
DECEMBER	BANGKOK	10642	0	0
JANUARY	YANGON	10677	0	0
FEBRUARY	YANGON	9568	0	0
MARCH	YANGON	8495	0	0
APRIL	YANGON	9125	0	0
MAY	YANGON	9074	0	0
JUNE	YANGON	8932	0	0
JULY	YANGON	8899	0	0
AUGUST	YANGON	9048	0	0
SEPTEMBER	YANGON	8981	0	0
OCTOBER	YANGON	9278	0	0
NOVEMBER	YANGON	9827	0	0
DECEMBER	YANGON	10454	0	0
JANUARY	MUMBAI	12290	0	0
FEBRUARY	MUMBAI	11531	0	0
MARCH	MUMBAI	12077	0	0
APRIL	MUMBAI	11475	0	0
MAY	MUMBAI	11728	0	0
JUNE	MUMBAI	11458	0	0
JULY	MUMBAI	12302	0	0
AUGUST	MUMBAI	12375	0	0
SEPTEMBER	MUMBAI	11257	0	0
OCTOBER	MUMBAI	14123	0	0
NOVEMBER	MUMBAI	13397	0	0
DECEMBER	MUMBAI	14217	0	0
JULY	KUALALUMPUR	3875	0	0

AUGUST	KUALALUMPUR	3700	0	0
SEPTEMBER	KUALALUMPUR	3750	0	0
OCTOBER	KUALALUMPUR	3736	0	0
NOVEMBER	KUALALUMPUR	3694	0	0
DECEMBER	KUALALUMPUR	3647	0	0
		499718	1	0

Table 1: Summary of reports of Gross Navigational Errors.

In Section 2 we discuss the risk assessment for the lateral direction and Section 3 gives the same for the longitudinal direction.

2 LATERAL COLLISION RISK ASSESSMENT

2.1 Lateral Collision Risk Model

In order to compute the level of safety for lateral deviations of operations on the BOBASIO region we use the Reich Lateral Collision Risk Model. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}\left(S_y\right)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{2\overline{|V|}}{2\lambda_x} + \frac{\overline{|\dot{y}\left(S_y\right)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] \right\} . \tag{1}$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.

The parameters in the equation (1) are defined as follows:

- Nay := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned S_y NM lateral separation.
- Sy: = Minimum planned lateral separation.
- λ_x := Average length of an aircraft flying in BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λz := Average height of an aircraft flying on BOBASIO region.
- $P_y(S_y)$:= The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by Sy.
- $P_z(0)$:= Probability of vertical overlap of aircraft assigned at the same flight path at the same flight level.
- S_x: = Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- E_y(same) := Same direction lateral occupancy at same assigned flight level.
- E_y(opp) := Opposite direction lateral occupancy at same assigned flight level.
- $\overline{|\Delta V|}$:= Average relative speed of two aircraft flying on parallel routes in same direction.
- $\overline{|V|}$:= Average ground speed on an aircraft.

- $|Y(\dot{S}_y)|$:= Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y.
- $\overline{|\dot{z}|}$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, P_z (0) is the probability of vertical overlap; P_y (S_y) is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x}E_y\,(\text{same})$ and $\frac{\lambda_x}{S_x}E_y\,(\text{opp})$ relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\overline{|\Delta V|}}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time $\frac{\overline{V}}{\lambda_x}$ The model is based on the following hypothesis:

- All routes are parallel. [1]
- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.
- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.03127211 NM	Estimated from TSD (see Section 2.3).
λ_y	0.02893539 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008835426 NM	Estimated from TSD (see Section 2.3).
$P_y\ (50)$	2.52989×10^{-8}	Estimated using a mixture model (see Section 2.4).
$P_z\ (0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
S_x	50 NM	Reduced longitudinal separation.
E_y (same)	0.05404064	Estimated from the TSD (see Section 2.6).
E_y (opp)	0	No opposite directional lateral occupancy at same assigned flight level.
$\lvert \Delta V \rvert$	23 knots	Value obtained from TSD (see Section 2.7).
$\lvert \dot{y}(50) \rvert$	75 knots	Conservative value taken from EMA Handbook (see Section 2.8).
$\lvert \dot{z} \rvert$	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk Nay.

$$\mathbf{N_{ay} = 0.74428 \times 10^{-9}}$$

2.3 Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

	Length	Wingspan	Height	Flights
B77W	73.90	64.80	18.50	5161
A320	37.57	34.10	11.76	4230
B738	39.20	34.40	12.57	3692
A332	58.80	60.30	17.40	2967
A333	63.60	60.30	16.85	2674
B772	63.70	60.90	18.40	2663

B744	70.60	64.80	19.40	2642
A388	73.00	79.80	24.10	1629
A343	63.60	60.30	16.85	1154
A321	44.51	34.10	11.76	1118
B763	54.90	47.60	15.90	1099
B773	73.90	60.90	18.40	671
A319	33.84	34.10	11.76	671
A346	75.30	63.45	17.30	515
B77L	63.70	64.80	18.30	425
B752	47.30	38.10	13.60	257
MD11	61.20	51.70	17.60	252
A306	54.10	44.84	16.54	216
A310	46.66	43.90	15.80	212
GLEX	30.30	26.90	7.60	55
B737	33.60	34.30	12.60	55
A345	67.90	63.45	17.10	52
B743	70.60	59.60	19.30	39
GLF5	29.40	28.50	7.50	29
B74S	56.30	59.60	20.00	29
H25B	15.60	15.70	5.40	26
CL60	20.85	19.60	6.30	23
F900	20.20	19.30	7.60	22
GL5T	28.69	28.65	7.70	17
F2TH	20.20	19.30	7.10	17
GLF4	26.90	23.70	7.40	15
B742	70.60	59.60	19.30	13
A342	59.39	60.30	16.70	13
E135	26.30	20.20	6.70	11
C17	53.00	51.80	16.80	9
A318	31.45	34.10	12.56	5

Table 2: Dimensions of aircraft types, along with number of records in the TSD

2.4 Estimating Probability of Lateral Overlap: $P_y(S_y)$

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y, is denoted by $P_y(S_y)$ and is defined as

$$P_y(S_y) := \mathbf{P}\left(|S_y + Y_1 - Y_2| \le \lambda_y\right), \qquad (2)$$

where Y1 and Y2 are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y. We assume that Y1 and Y2 are identically distributed but statistically independent with a distribution F_y.

We model F_y as mixture distribution having a *core* distribution G_y and a *non-core* distribution H_y.

- The *core* distribution G_y, represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.

- The *non-core* distribution H_y, represents *Gross Navigation Errors (GNE),* that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)G_y(y) + \alpha H_y(y)\ . \qquad (3)$$

The mixing parameter α is the probability of a *gross navigational error*.

The core lateral deviation distribution G_y is modeled by a Double Exponential distribution with a parameter $\beta_y > 0$ as the rate, that is, if $Y1 \sim G_y$ then

$$\mathbf{P}\left(|Y_1| > y\right) = e^{-\beta_y y}\,,$$

where $y \geq 0$. In other words we assume that the core distribution has a density of the form

$$g_y\left(y\right) = \frac{\beta_y}{2} e^{-\beta_y |y|}\,.$$

Finally the non-core distribution H_y is modeled by a “Separated Double Exponential” distribution with parameters $\mu_y > 0$, representing the “separation and $\gamma_y > 0$ the rate parameter, that is, if $Y2 \sim H_y$ then

$$\mathbf{P}\left(Y_2 > \mu_y + y\right) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$\mathbf{P}\left(Y_2 < -\mu_y - y\right) = \frac{1}{2} e^{\gamma_y y}\,,$$

where $y \geq 0$. This really means that the non-core distribution H_y gives no mass in $[-\mu_y, \mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y.

The density of this distribution is given by

$$h_y\left(y\right) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y (y + \mu_y)} & \text{if} \quad y < -\mu_y \\ 0 & \text{if} \quad -\mu_y \leq y \leq \mu_y \\ \frac{\gamma_y}{2} e^{-\gamma_y (y - \mu_y)} & \text{if} \quad y > \mu_y \end{cases}\ .$$

This modeling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors.

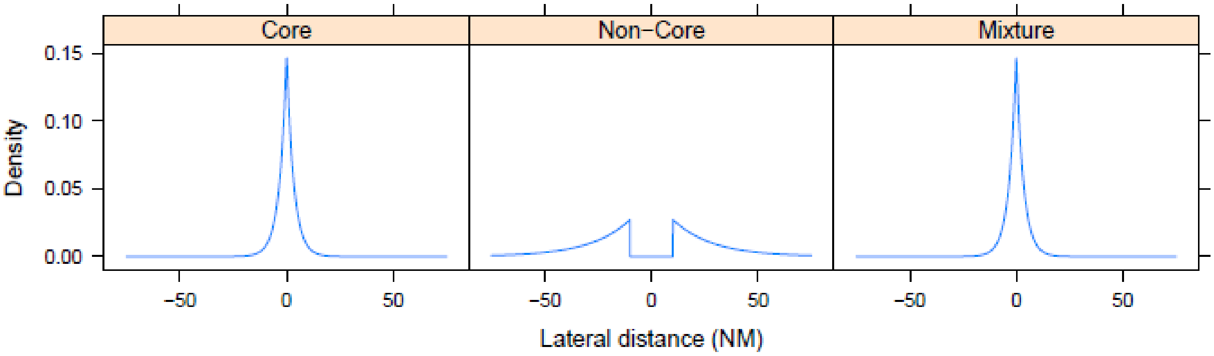

Figure 2: Modeling of lateral deviation.

This in turn gives a better understanding of the mixing parameter α which we estimate by taking the 95% upper confidence limit from the available GNE data. This confidence limit does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods. The value comes out to be

$$\hat{\alpha} = 1 \times 10^{-5},$$

Here we would like to note that even though the non-core distribution H_y has a discontinuous density h_y, it does not create difficulty in this risk assessment.

The parameter β_y is estimated under the RNP10 assumption of ±10 NM deviation with 95% confidence, this leads to the estimate

$$\hat{\beta}_y = -\frac{\log 0.05}{10} = 0.299573227\,.$$

The parameter μ_y is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with S_y=50NM initial separation (see Figure 3). This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γ_y is 0.05489708 leading to the estimated value of P_y (50) as 2.52989 x 10^{-8}.

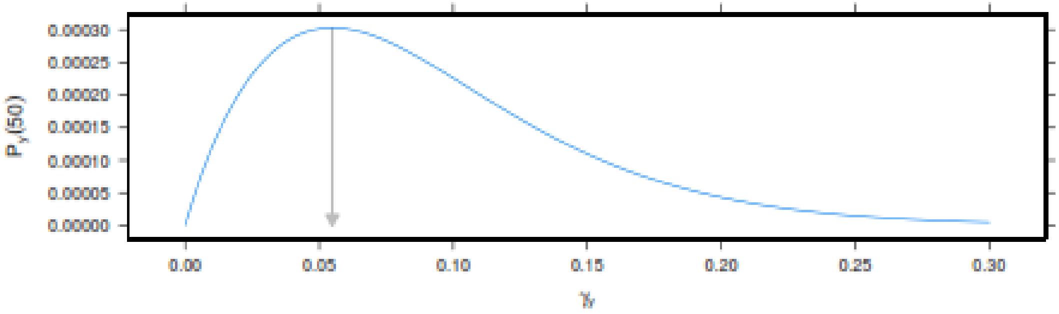

Figure 3: Wingspan overlap probability as a function of γ_y with $S_y = 50$ NM initial separation.

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of P_y (50) and N_{ay} had we observed more GNEs. The results, given below, are still well below the TLS. Note that the actual number of GNEs observed was 1.

No. of GNEs	P_y (50)	N_{ay}
1	2.52989×10^{-8}	0.93035×10^{-9}
2	2.60303×10^{-8}	0.957246×10^{-9}
3	2.74931×10^{-8}	1.01104×10^{-9}
4	2.82245×10^{-8}	1.03793×10^{-9}
5	2.96872×10^{-8}	1.09173×10^{-9}

2.5 Estimating Probability of Vertical Overlap: $P_z(0)$

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by $P_z(0)$. It is defined by

$$P_z(0) = \mathbf{P}(|Z1 - Z2| \leq \lambda_z)\ ,$$

where Z1 and Z2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z1 and Z2 are statistically independent with distribution F_z. Unlike in the computation of $P_y(S_y)$ where we assume the lateral deviations follow a mixture distribution here we may assume that F_z is a Double Exponential distribution with parameter $\beta_z > 0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z |z|}\ .$$

One can then estimate $\beta_z > 0$ by

$$\widehat{\beta}_z = -\frac{\log 0.05}{0.032915} = 91.0142\ .$$

This is under assumption that a typical aircraft stays within ±200 ft = ±0.032915 NM of its assigned flight level 95% of the time. This leads to an estimated value 0.3553035 for P_z (0). Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors

(ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

2.6 Estimating the Lateral Occupancy Parameters: Ey(same) and Ey(opp)

In equation (1) there are two occupancy terms, one for same direction occupancy E_y(same) and another one for opposite direction occupancy E_y(opp).

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing “flying in the same direction” by “flying in the opposite direction”. Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a

WP1	WP2	Proximate	Total
BIDEX	ORARA	0	746
IGOGU	IGREX	72	2208
NOPEK	IGOGU	144	2446
GIRNA	IDASO	156	2884
VATLA	ORARA	42	1040
IGOGU	EMRAN	0	1486
LIBDI	MEPEL	2	1316
RINDA	SAGOD	52	4288
MEPEL	IBITA	8	746
IBITA	TEBOV	356	5706
SAGOD	IBITA	0	1092
POMAN	IGAMA	532	4668
KITAL	LOTAV	226	3858

OPIRA	IGAMA	0	2718
LOTAV	REXOD	218	3652
TOTOX	REXOD	206	2272
TOTOX	PARAR	252	3470
ADPOP	SUGID	0	2820
RASKI	PARAR	424	4758
NOBAT	SUGID	1006	7664
POMAN	ODOLI	0	1950
KITAL	ASPUX	0	486
NISOK	NIXUL	0	112
NIXUL	TOPIN	0	460
SULTO	DUGOS	0	334
ATETA	DEMON	0	282
UDULO	KAKIB	0	520
RIBRO	ELBAB	6	630
VATLA	MABUR	0	784
SAGOD	MEPEL	0	410
DWI	BETNO	0	920
MIPAK	LALAT	0	1778

Table 3: Number of laterally proximate flights per route pair, based on TSD.

parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator n_y is the number of proximate pairs and the denominator, n, is the the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

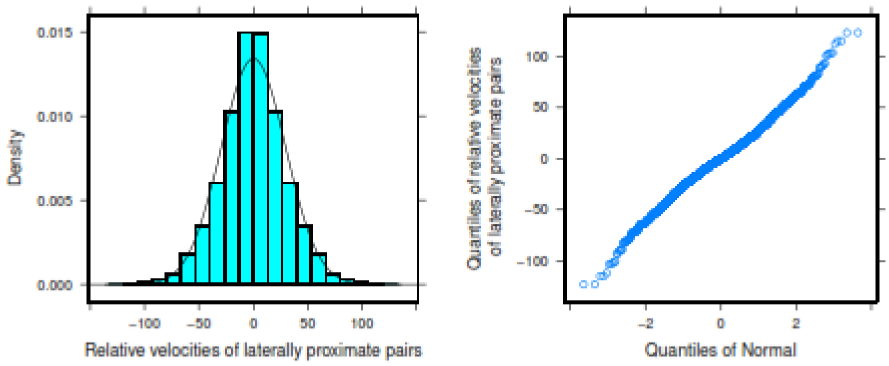

Figure 4: Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

2.7 Estimate of Average Relative Longitudinal Speed: $\overline{|\Delta V|}$

$\overline{|\Delta V|}$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction (see Figure 4). $\overline{|\Delta V|}$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 22.76543. We use the conservative value 23. Here we note that the lateral proximate pairs are already determined while estimating the parameter E_y(same).

2.8 Estimate of Average Relative Lateral Speed: $\overline{|\dot{Y}(S_y)|}$

$\overline{|\dot{Y}(S_y)|}$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.9 Estimate of Average Relative Vertical Speed: $\overline{|\dot{z}|}$

$\overline{|\dot{z}|}$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $\overline{|\dot{z}|}$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $\overline{|\dot{z}|}$ with the vertical separation between the aircraft. As noted by various agencies data on $\overline{|\dot{z}|}$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

3 LONGITUDINAL COLLISION RISK ASSESSMENT

In order to compute the level of safety for longitudinal deviations of operations on the BOBASIO region we use the Longitudinal Collision Risk Model. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)\, P_z(0)\, \frac{2\lambda_x}{\overline{|\dot{x}|}} \left(\frac{\overline{|\dot{x}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(0)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right) \times \left[\sum_{k=m}^{M} 2Q(k)\, \mathbf{P}(K > k) \right] . \qquad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- Nax := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum *m* NM longitudinal separation.
- *m* := Minimum longitudinal separation in NM.
- M: = Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λ_x := Average length of an aircraft flying on BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.

- λ_z := Average height of an aircraft flying on BOBASIO region.
- P_y (0):= Probability of lateral overlap of aircraft assigned same flight route and flight level.
- P_z (0):= Probability of vertical overlap of aircraft assigned at the same flight path at the same flight level.
- $|\dot{x}|$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by *m* NM at a reporting point to overtake lead aircraft at the next reporting point.
- $|\dot{Y}(0)|$: = Relative across-track speed of same route aircraft pair.
- $|\dot{z}|$: = Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.
- Q (k):= Proportion of aircrafts for which the following aircraft has initial longitudinal separation k.
- P (K > k) := Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions. The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events.

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	50 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes
λ_x	0.03127211 NM	Estimated from TSD (See Section 2.3).
λ_y	0.02893539 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008835426 NM	Estimated from TSD (see Section 2.3).
P_y (0)	0.2	Conservative estimate (see Section 3.2).
P_z (0)	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$\|\dot{x}\|$	19 knots	Conservative estimate using speed and distance between way points (see Section 3.3).
$\|\dot{y}(0)\|$	1 knot	RASMAG/9 safety assessment (see Section 3.4).

$\lvert\dot{z}\rvert$	1.5	Conservative value as per EMA Handbook (see Section 2.9).
$Q(k)$	See Table 4	Obtained from TSD (see Section 3.5).
$P(K > k)$	See Table 4	Computed using normal model on speed (see Section 3.6).

Finally this leads to the following estimate for the longitudinal collision risk N_{ax}.

$$\mathbf{N_{ax} = 0.895888 \times 10^{-9}}$$

3.2 Estimating Probability of Lateral Overlap: $P_y(0)$

$P_y(0)$ is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting $S_y = 0$ in the equation (2). This leads to an estimate of $P_y(0)$ as 0.2.

(mins)	k (NM)	Q(k)	P(K > k)
7	56	0.0004832175	1.10253×10^{-5}
8	64	0.0010407761	7.37489×10^{-7}
9	72	0.0018956994	4.47232×10^{-8}
10	80	0.0119689254	3.76914×10^{-9}
11	88	0.0091811322	5.16573×10^{-10}
12	96	0.0083262090	8.51491×10^{-11}
13	104	0.0092926439	1.44569×10^{-11}
14	112	0.0098873732	2.46153×10^{-12}
15	120	0.0104449318	4.19205×10^{-13}
16	128	0.0107422964	7.13916×10^{-14}
17	136	0.0104821024	1.21582×10^{-14}
18	144	0.0092554734	2.07057×10^{-15}
19	152	0.0096643497	3.52623×10^{-16}
20	160	0.0100732260	6.00527×10^{-17}

Table 4: Estimated values of Q(k) and $P(K > k)$

However as noted earlier in the EUR/SAM report, this factor $P_y(0)$ has a significant effect on the risk estimate. Therefore, it should not be underestimated. $P_y(0)$ will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of $P_y(0)$ for the global system performance.

As observed by many monitoring agencies and pointed out to us by AAMA the RGCSP value of $P_y(0)$ does not acknowledge the close track-keeping observed with RNP 4 or GNSS-equipped RNAV 10/RNP 10 aircraft. Further the EMA Handbook recommends to take a conservative value as 0:2. So we take this conservative value for our analysis as well.

3.3 Estimation of the Parameter $\overline{|\dot{x}|}$

$\overline{|\dot{x}|}$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by *m* NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $\overline{|\dot{x}|}$ can be computed by the equation.

$$\frac{d-m}{v_0} = \frac{d}{v_0 + \overline{|\dot{x}|}},$$

leading to

$$\overline{|\dot{x}|} = \frac{m v_0}{d-m}.$$

We conservatively estimate it by taking v0 as the the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is d = 971 NM. With *m* = 50 NM the final estimate turns out to be $\overline{|\dot{x}|}$ = 19.5439739413681 knots. We use a conservative value of 19 knots.

3.4 Estimation of the Parameter: $\overline{|\dot{y}(0)|}$

$\overline{|\dot{y}(0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of 1 knot as given in the EMA Handbook.

3.5 Estimation of the Parameter Q (k)

Q (k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 5), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q (k) as

$$Q(k) = \frac{\text{number of flight pairs with inter-arrival distance } 8k}{\text{total number of flight pairs with at least 80 NM separation}}$$

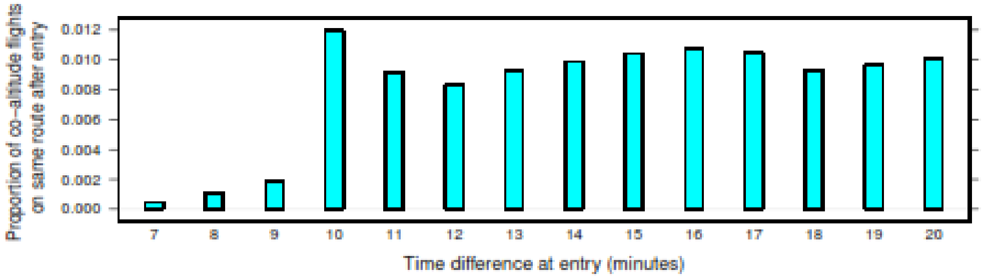

Figure 5: Values of Q(k) estimated from TSD. For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 7, 8, 9,. . . , 20 minutes.

The final estimated values of Q (k) for k ranging between 7 and 20 minutes are given in the Table 4.

3.6 Estimation of the Parameter P (K > k)

To estimate **P** (K > k) we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and $V^{/}$ ' be their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T0 be the maximum duration of time before ATC intervenes. Then

$$\mathbf{P}\left(K > k\right) = \mathbf{P}\left(0 < \frac{k}{V' - V} < T_0\right) = \mathbf{P}\left(V' - V > \frac{k}{T_0}\right).$$

We note here that the value of T0 is conservatively taken to be 0.5 hours.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 6) it is not clear that these differences necessarily Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v\left(v\right) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + \left(1 - p\right)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

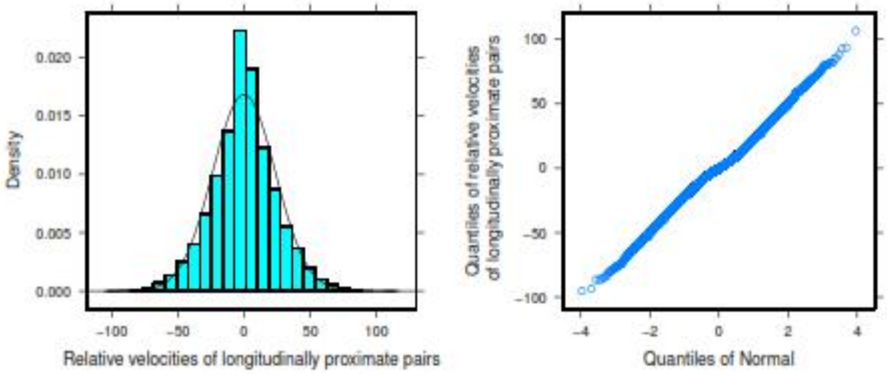

Figure 6: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does not necessarily seem to be a reasonable fit.

which is a mixture of Double Exponential and Normal densities with mixing proportion p. We then estimate the parameters of this mixture model by their maximum likelihood estimates (MLEs). Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates:

$$\widehat{p} = 0.2851915,\ \widehat{\beta}_v = 0.1106362,\ \text{ and } \widehat{\sigma}_v = 26.79624\,.$$

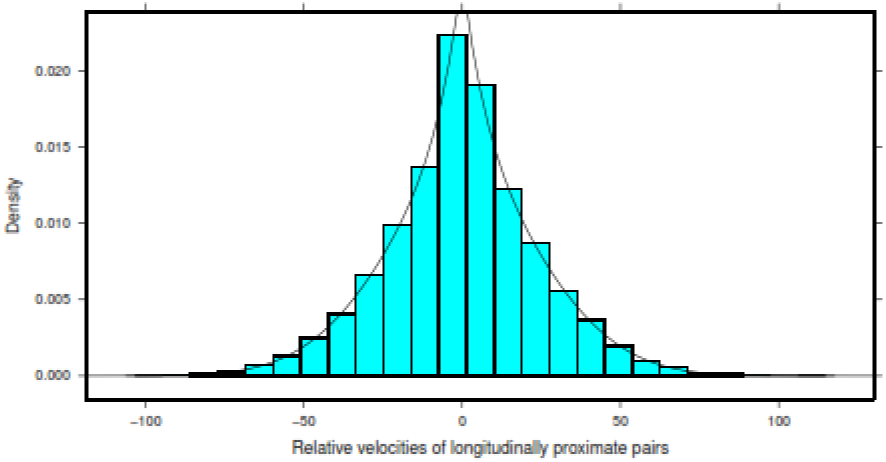

Figure 7: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density (estimated using the EM algorithm).

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with these values of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

With these we estimate the values of $\mathbf{P}$ (K > k) for k ranging between 7 and 20. These are presented in the Table 4.

4 Summary of the Safety Assessment

The following table gives a summary of the safety assessment of the BOBASIO region for the month of December 2012.

Type of Risk	Estimated Risk	TLS	Remarks
Lateral Risk	0.74428×10^{-9}	5×10^{-9}	Below TLS
Longitudinal Risk	0.895888×10^{-9}	5×10^{-9}	Below TLS

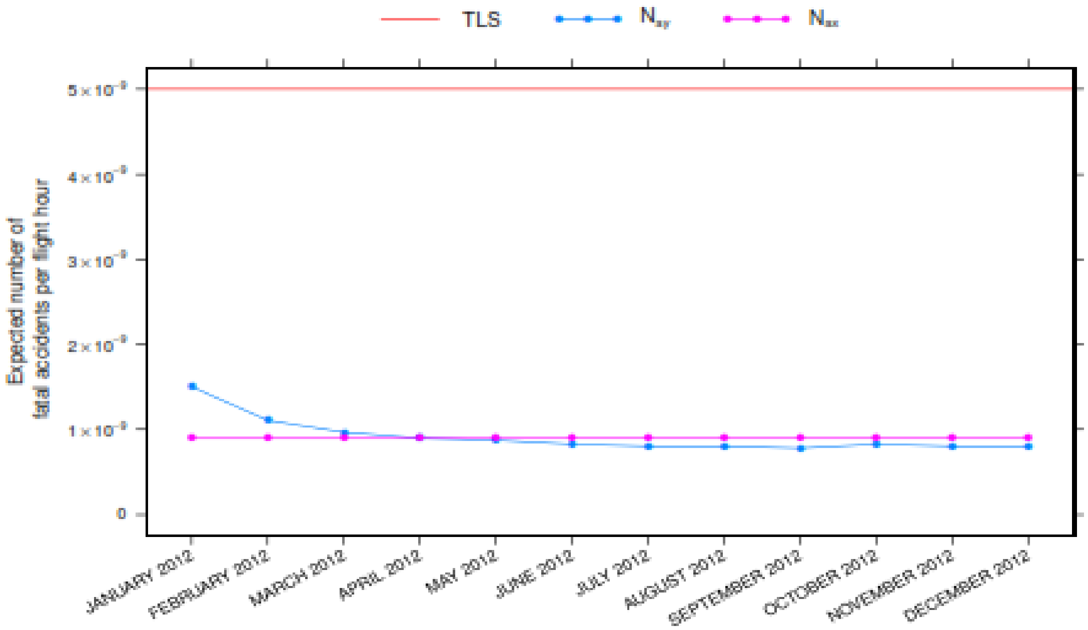

Figure 8: Assessment of Compliance with Lateral and Longitudinal TLS Values.

Figure 8 presents the results of the collision risk estimates for each month using the cumulative 12-month LLD and LLE reports since January 2012.